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# On the Theory
# of Double Products and
# Strains in Hyperspace

BY

EDWIN BIDWELL WILSON

# I.—ON THE THEORY OF DOUBLE PRODUCTS AND STRAINS IN HYPERSPACE.

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### PART I. ON A MULTIPLE ALGEBRA AS SET FORTH BY GIBBS

## *Introduction.*

1. *History and apologies.*—During the academic year 1899–1900, I followed a course of lectures on vector analysis by the late professor J. Willard Gibbs. These lectures, with some alterations as to additions and retrenchments, I published with his permission in the year 1901.[1]  Previously to this, during the academic year 1900–1901, I had the opportunity of following a short course of twenty-five or thirty lectures on multiple algebra under the same master. These lectures have never been published, and very likely never can be. My own notes were very meager and most of them have unfortunately been lost. There remains, however, a set of notes taken by the late professor G. P. Starkweather of Yale University. These notes were of a similar course given during the academic year 1895–1896 ; and as nearly as my memory and the fragments of my own notes will serve me, it appears that Gibbs had not materially changed his course during the intervening five years. It is therefore from Starkweather's notes that the following articles on multiple algebra are drawn, with the practical certainty that the presentation is essentially that followed by Gibbs in his later years.

Gibbs's course on multiple algebra, following immediately upon his lectures on vector analysis, began with a discussion of quaternions defined as the sum of the vector product and the negative of the scalar product of two vectors, and then turned to the geometric

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[1] Vector Analysis, a textbook for the use of students of mathematics and physics, founded upon the lectures of J. Willard Gibbs. Yale Bicentennial Publications, Charles Scribner's Sons, New York, and Edwin Arnold, London.  xviii + 436 pp.  A second edition, merely corrected, has recently appeared.  References in the text to Vector Analysis are to either edition. Gibbs's original pamphlet on vector analysis, printed privately in 1881–1884 but never published, may now be found reprinted in the collection of The Scientific Papers of J. Willard Gibbs, volume 2, pp. 17–90.

algebras, that is, the point-, line-, and plane-analyses of Grassmann
These subjects occupied approximately half the time of the course
During the remaining half, he took up the theory of dyadics, which
is in immediate and intimate connection with the theory of matrices,
and concluded with C. S. Peirce's thorem that any linear associative
algebra may be put in quadrate, that is, in matricular form.   This
brief series of lectures by no means contained all of Gibbs's ideas
and developments in multiple algebra.   Indeed he had published
at a much earlier date some reflections and theories on the subject[1]
which found no place in his course.   An examination of the notes
which he left at his death shows, however, that he followed his
usual custom of not committing his results to paper except in so
far as they were immediately needed for the lectures in his course.
The reason for my being so bold at this time as to publish some
of the most essential extracts from Gibbs's lectures on multiple
algebra is partly because they may be of interest to mathematicians
who may be concerned with the theory of matrices or with mul-
tiple algebra, and partly because I desire to make use of the ab-
breviations which his notation and methods afford in discussing some
geometric problems in connection with the theory of strains.   If at
any time in developing the multiple algebra I take the liberty of
adding to what I find in Starkweather's notes or in my own, or
if I depart from the methods of Gibbs, I shall try to make the
fact evident—not for the purpose of claiming any originality of my
own, but that the reader may have as definite as possible an idea
of what Gibbs did in his course on multiple algebra, in so far as
I find it necessary or advisable to print it at this time.

2.  *Preliminary notions and notations.*—Let the primary elements
of the algebra be denoted by Greek small letters, $\alpha$, $\beta$, $\gamma$, . . . .
If the algebra is $n$-dimensional, any $n + 1$ of the elements will be
connected by a linear relation.

$$\Sigma \; a\,\alpha + b\,\beta + c\,\gamma + \ldots \text{ to } n + 1 \text{ terms} = 0,$$

with scalar coefficients $a$, $b$, $c$, . . . not all of which are zero; and
any element of the system may be expressed linearly with scalar
coefficients in terms of any given $n$ linearly independent elements, as
(1)            $\varrho = \Sigma \; a\,\alpha + b\,\beta + c\,\gamma + \ldots$ to $n$ terms.

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[1] On Multiple Algebra, an address before the section of mathematics
and physics of the American Association for the Advancement of Science,
by the Vice-President.  Proceedings of the American Association for the
Advancement of Science, volume 35, pp. 37—66   This address is reprinted
in The Scientific Papers, volume 2, pp. 91—117.

Moreover, the system will be supposed, as usual, to contain all the elements which may be linearly derived from any given elements; and it may be assumed that the coefficients in this derivation are any real or complex scalars. It will not be necessary to go into further details as regards these matters which are the same in all linear associative or Grassmannian algebras. So much in regard to addition, subtraction, and linear dependence or independence.

The primary elements $\alpha$, $\beta$, $\gamma$, ... may be interpreted either as vectors issuing from a fixed point in Euclidean space of $n$ dimensions or as points lying in a Euclidean space (supposed flat, of course) of $n-1$ dimensions. It is the former interpretation which will be most used in what follows. It should be noted however, that the algebraic system is independent of any geometric interpretation. If proofs are given by means of either of the said interpretations, it is merely because the geometric language facilitates expression. As a matter of fact in a Grassmannian algebra where the combinatory products lead to elements of different types from the elements which constitute the factors, the geometric language and conception are far more fruitful and convenient than in those algebras in which the product is always of the same type as the factors; and hence it will be used constantly in what follows.

Two primary elements may be multiplied according to the combinatory law

(2) $$\alpha \times \beta = -\beta \times \alpha$$

to form a product which is an element of another type and may be called a secondary element or element of the second class. The use of the cross $\times$ for combinatory multiplication it in accord with Gibbs's usage in his address on multiple algebra. In like manner $k$ elements, $k \leqq n$, may be combined to form an element of the $k$th class. Such multiplication is called progressive; it is associative and it is distributive relative to expressions such as (1). If an element of class $k$ be multiplied into an element of class $l$, $k + l \leq n$, the multiplication remains progressive; if $k + l$ $n$, the product is of class $n$ and is a scalar. The properties of progressive multiplication as contained in (2) and in the associative and distributive laws are simple and are treated in a variety of places.[1] If the sum of the classes, $k + l$, of two elements is greater than $n$, the rules of progressive multiplication give a zero value for the product and it

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[1] For instance, in either of Grassmann's Ausdehnungslehren, 1844 and 1862, or in Whitehead's Universal Algebra, volume 1, Cambridge University Press.

becomes necessary to redefine the method of evaluating the product. Thus arises the theory of regressive multiplication in which the class of the product is $k + l - n$ instead of $k + l$. This product is also treated in detail in the references just cited.

It should be noted, however, that the theory of regressive multiplication, which is usually based upon the theory of supplements in the Grassmannian sense, was treated in an entirely different manner by Gibbs. His point of view and method of procedure were outlined in his address on multiple algebra already cited: but as that presentation is extremely brief, it may be well to recapitulate his method in some detail. Let $\alpha, \beta, \gamma, \ldots$ be any number of elements of the first class. Consider the product of two factors (the cross introduced in (2) may be omitted for brevity in writing)

$$(\alpha \beta \gamma \delta \ldots) (\ldots \lambda \mu \nu \pi)$$

each of which contains not more than $n$ elements, say $k$ and $l$ respectively, but which together contain more than $n$ elements. The product of two such factors is called regressive when computed by either of the following rules:

$1^0$. From the second factor $(\ldots \lambda \mu \nu \pi)$ take enough, that is, $n - k$, of the remoter (last) elements to form a total of $n$ with all the elements of the first factor $(\alpha \beta \gamma \delta \ldots)$, thus obtaining a scalar (a product of the $n$th class) to serve as coefficient to the remaining elements of the second factor. Do this for every permutation of the $l$ elements in the second factor $(\ldots \lambda \mu \nu \pi)$ which may be necessary to bring every combination (not permutation) of $n - k$ of them once and only once to the end of the factor, and add the results thus obtained with the positive or negative sign according as the number of simple transpositions of the $l$ elements in any permutation is even or odd.

$2^0$. From the first factor take enough, that is, $n - l$, of the remoter (first) elements to form a total of $n$ with all the elements of the second factor, thus obtaining a scalar to serve as coefficient to the remaining elements of the first factor. Do this ... and so on, as before.

Thus if $n = 4$, the following expansions of regressive products are in accordance with the rules just stated.

$$\alpha(\beta \gamma \delta \varepsilon) = (\alpha \gamma \delta \varepsilon) \beta - (\alpha \delta \varepsilon \beta) \gamma + (\alpha \varepsilon \beta \gamma) \delta - (\alpha \beta \gamma \delta) \varepsilon,$$

$$(\alpha \beta \gamma) (\delta \varepsilon) = (\alpha \beta \gamma \varepsilon) \delta - (\alpha \beta \gamma \delta) \varepsilon = (\alpha \beta \delta \varepsilon) \gamma + (\beta \gamma \delta \varepsilon) \alpha + (\gamma \alpha \delta \varepsilon) \beta,$$

$$(\alpha \beta \gamma) (\delta \varepsilon \zeta) = (\alpha \beta \gamma \zeta) \delta \varepsilon + (\alpha \beta \gamma \varepsilon) \zeta \delta + (\alpha \beta \gamma \delta) \varepsilon \zeta = (\alpha \delta \varepsilon \zeta) \beta \gamma$$
$$+ (\beta \delta \varepsilon \zeta) \gamma \alpha + (\gamma \delta \varepsilon \zeta) \alpha \beta.$$

It may be remarked that in the first line, the product on the left of the sign of equality is already expanded as far as possible by the second of the rules. Furthermore, if $n$ had been 5, the last product would have been

$$(\alpha \beta \gamma) (\delta \varepsilon \zeta) = (\alpha \beta \gamma \varepsilon \zeta) \delta + (\alpha \beta \gamma \delta \varepsilon) \zeta + (\alpha \beta \gamma \zeta \delta) \varepsilon = (\alpha \beta \delta \varepsilon \zeta) \gamma$$
$$+ (\beta \gamma \delta \varepsilon \zeta) \alpha + (\gamma \alpha \delta \varepsilon \zeta) \beta.$$

It is hardly necessary to note that the signs in the expansions may all be taken positive by properly arranging the permutations on the letters.

Obviously, if the numbers of the elements in the actors are $m$ and $m'$, the product belongs to class $k + l - n$. It is of fundamental importance to observe that the way in which the regressive product is defined by reference to the progressive product of $n$ elements, is sufficient to insure the distributivity of the regressive product relative to sums.

To justify the double definition, it is necessary to show that the two rules lead to the same result. For this, it is convenient to consider all the primary elements as expressed linearly in terms of $n$ independent primary unit elements, all elements of higher classes as expressed in terms of the unit elements of those classes. Then, inasmuch as the distributive law applies to the regressive product and the product of two sums of terms may be resolved into the sum of the products of each pair of terms, of which one is selected from the first factor and one from the second, it is sufficient to prove the equivalence of the two rules for factors made up of certain of the units. As the factors contain respectively $k$ and $l$ units, and as there are only $n$ units in all, there must be $k + l - n$ units common to the two factors; it being understood that if any unit is repeated in both factors, it may be repeated in the list of common units as many times as it occurs in the factor in which it occurs least frequently, unless so great a repetition is not required to make up a total of $k + l - n$ common units, after all those which are common to both factors, but are not repeated in more than one of the factors have been counted. Let the product of the $k + l - n$ units common to the two factors be $M$, and let the two factors be written as $AM$ and $MB$. Consider the product $AM \times MB$, where it is clear that $A$ contains $n - l$ and $B$ contains $n - k$ units. According to the first rule it is necessary to select $n - k$ elements from $MB$ to form with the $k$ elements in $AM$ a scalar coefficient for the remaining elements of $MB$. If any of these $n - k$ are taken from $M$, the resulting scalar will surely have a repeated unit and will vanish. Hence all $n - k$ should be taken from $B$, and according to the first rule the product is

(3) $$AM \times MB = (AMB)\, M$$

The same result is obtained by a similar application of the second rule.

A further word on the geometrical interpretation will considerably facilitate the expression of some of the following remarks. If the primary elements be interpreted as vectors issuing from a fixed point in Euclidean space of $n$ dimensions, the elements of the second class will be conceived as plane areas and in particular the product of two vectors will be the parallelogram included by them, the elements of the third class will be three-dimensional volumes, and so on until the scalars which are elements of the $n$th class will be $n$-dimensional volumes, and in particular the product of $n$ vectors will be the $n$-dimensional parallelepiped constructed upon them. It appears therefore that the necessary and sufficient condition that $k$ vectors $k \leq n$, be linearly dependent is that their (progressive) product be zero. The regressive product is the product of two factors, which, regarded as spaces, have a total dimensionality greater,

than $n$. Formula (3) shows that in case the factors are made up of units, the regressive product is the space common to the two spaces of the factors, that is, it is the intersection of the factors, taken, of course, with a certain magnitude. An examination of the rules for expanding the regressive product, especially as illustrated by the examples there given, shows at once that the result is true in general, and that the regressive product of two spaces is always the intersection of the spaces, taken with a proper numerical value. It should be noted that if the spaces of two factors in a regressive product do not exhaust the dimensionality of all space, that is, if the spaces of both factors lie in a subspace of the $n$-dimensional space, then the scalar coefficients which occur in the expansion of the product will be products of $n$ vectors lying in that subspace and will therefore all vanish. That is to say, if the factors lie in a subspace, the regressive product must be zero.

This fact may serve as foundation for the proof of the associative law for the multiplication of three factors, which may be denoted by $X$, $Y$, $Z$. If $X$ and $Y$ lie in a subspace of the $n$-dimensional space, the regressive product $XY$ is zero and hence $XY$ times $Z$ is necessarily zero. But if $X$ and $Y$ lie in a subspace, so must $X$ and the product $YZ$, which is the space common to $Y$ and $Z$. Hence $X$ times $YZ$ is also zero; and the associative law holds in this case. To prove the law in general it is sufficient, owing to the applicability of the distributive law, to prove it for the case that $X$, $Y$, $Z$ are products of the units. Furthermore, it may be assumed that $X$ and $Y$, and also $Y$ and $Z$, exhaust the $n$ dimensions of space. Let $M$ represent the product of the units common to $X$, $Y$, $Z$; and let $A$ be the product of the units other than those in $M$ which are common to $Y$ and $Z$; and similarly $B$ and $C$ for the pairs $Z$, $X$ and $X$, $Y$. Then as $X$ and $Y$ and also $Y$ and $Z$ must exhaust all $n$-dimensions of space, it is obvious that every unit which occurs in $X$ must occur in $Y$ or $Z$, and similarly for $Y$ and $Z$. Hence the factors may be written as

$$X = B M C \qquad\qquad Y = C M A \qquad\qquad Z = A M B$$

and the two groupings of the factors give

$$[X Y] Z = [(B M C)(C M A)] (A M B) = [(B M C A) C M] (A M B)$$
$$= (B M C A)(C A M B) M,$$
$$X [Y Z] = (B M C) [(C M A)(A M B)] = (B M C) [(C A M B) M A]$$
$$= (B M C A)(C A M B) M,$$

which are equal; and the associative law is proved. Care should, however, be exercised against applying the law to cases to which it cannot apply, such as

$$n = 5, \qquad [(\alpha \beta \gamma) \delta \varepsilon] (\delta \varepsilon \alpha) = 1 (\delta \varepsilon \alpha), \qquad (\alpha \beta \gamma) [(\delta \varepsilon)(\delta \varepsilon \alpha)] = 0.$$

Here the products are not regressive, but progressive.

In addition to the combinatorial product $\alpha \times \beta$ of two elements there is the dyadic product $\alpha \beta$. This corresponds to the simplest type of Grassmann's Lückenausdrücke. It is, according to Gibbs's

definition, a mere formal juxtaposition of two letters subject to the distributive law; that is,

$$\alpha\,(\beta+\gamma) = \alpha\,\beta + \alpha\,\gamma \quad \text{and} \quad (\alpha+\beta)\,\gamma = \alpha\,\gamma + \beta\,\gamma.$$

Hence if $\alpha$ and $\beta$ are each expressed in terms of $n$ given independent elements (which need not be the same set of $n$ for both vectors), the product $\alpha\beta$ may be expanded into a block of $n^2$ terms or dyads. It is through this fact that connection is made with the theory of matrices. There is no necessity that the two elements in a dyadic product should belong to the same class, whether primary or otherwise. If $\Gamma$ and $\varDelta$ belong respectively to the $k$th and $l$th classes, the dyad $\Gamma\varDelta$ may be defined in a manner similar to $\alpha\beta$ as a formal juxtaposition of two elements subject to the distributive law. As the $k$th and $l$th classes contain respectively

$$\frac{n\,(n-1)\ldots(n-k+1)}{k!} \quad \text{and} \quad \frac{n\,(n-1)\ldots(n-l+1)}{l!}$$

linearly independent elements, the dyad $\Gamma\varDelta$ may be expanded into a block of $n\ldots(n-k+1)\,.\,n\ldots(n-l+1)\,/\,k!\,l!$ terms. These terms will not form a square matrix unless $k=l$ or $k+l=n$; in other cases the matrix will be rectangular.

Gibbs applied also the name indeterminate product to the combination $\alpha\beta$ or $\Gamma\varDelta$, and he was very particular to state that he considered it the most general and most essential product with which multiple algebra has to deal.[1] Other products may be regarded as functions of the dyadic product. This product determines its constituent elements $\alpha$ and $\beta$, or $\Gamma$ and $\varDelta$, except that a scalar factor may be transferred from one to the other. The proof of this is not essentially different from that given for the simple case of vectors in the Vector Analysis, page 272. In what follows, the only dyadic products which will be considered are those in which the sum of the class-numbers $k$ and $l$ is equal to $n$. In this case the combinatorial product of two like dyads $\Gamma\varDelta$ and $\Gamma'\varDelta'$ is defined by the simple equation

$$(4) \qquad (\Gamma\varDelta) \times (\Gamma'\varDelta') = \Gamma\,(\varDelta \times \Gamma')\,\varDelta' = (\varDelta \times \Gamma')\,\Gamma\varDelta'$$

where $\varDelta \times \Gamma'$ is necessarily a scalar. The product therefore reduces to a similar dyad $\Gamma\varDelta'$ modified by a scalar factor. In like manner the product of a dyad into an element of the same class as the first member of the dyad is defined by the equation

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[1] See his address On Multiple Algebra, pp. 23–25; The Scientific Papers, volume 2, pp. 109–111: also Vector Analysis, article 102, pp. 271–275. The question is also treated in my communication On Products in Additive Fields: Verhandlungen des dritten internationalen Mathematiker-Kongresses. Teubner, 1905, pp. 202–215.

(4')                $(\Gamma \ 1) \times \Gamma'' = \Gamma'( \ 1 \times \Gamma'') = ( \ 1 \times \Gamma') \Gamma'$

where $1 \times \Gamma''$ is a scalar, and the result is therefore an element of the same class as that which was multiplied by the dyad.

3. *Reciprocal sets of elements.*—The theory of reciprocal sets is fundamental to the entire treatment of multiple algebra as here given. To a large extent it obviates the necessity of discussing the theory of supplements in the Grassmannian sense. In fact, by his definition and treatment of regressive multiplication and by his theory of reciprocal sets, Gibbs entirely avoided the supplements in his course. Before proceeding, however, to the reciprocal sets, it will be well to introduce once for all the change of notation already adopted in the discussion of regressive multiplication. The sign of the combinatorial product, the cross $\times$, occurs so frequently as to render the formulas too bulky. I shall therefore write

$$\Gamma \Delta \text{ instead of } \Gamma \times \Delta$$

for the combinatorial product. This necessitates a different notation for the dyadic product, and I shall write for this product

$$\Gamma \,|\, \Delta \text{ instead of } \Gamma \Delta,$$

where it should be noted that the vertical bar has no relation to the Ergänzung of Grassmann. This is in entire accord with Gibbs's procedure in his lectures; the change is made purely for convenience. The reason that this notation was not adopted from the start was to emphasize the fact that the dyadic product was fundamental and the combinatory product merely a function of it.

Let there be given $n$ independent primary vectors or elements

$$\alpha_1, \alpha_2, \ldots, \alpha_n, \qquad \alpha_1 \ \alpha_2 \ldots \alpha_n \neq 0.$$

Form the $n$ expressions

(5)        $\alpha'_i = \dfrac{\alpha_{i+1} \ \alpha_{i+2} \ldots \alpha_n \ \alpha_1 \ \alpha_2 \ldots \alpha_{i-1}}{\alpha_{i+1} \ \alpha_{i+2} \ldots \alpha_n \ \alpha_1 \ \alpha_2 \ldots \alpha_{i-1} \ \alpha_i}, \quad i = 1, 2, \ldots, n.$

The $n$ quantities $\alpha'_1, \alpha'_2, \ldots \alpha'_n$ thus obtained are elements of the $(n{-}1)$st class. Taken as a set, they are called the reciprocal set to the $n$ elements $\alpha_1, \alpha_2, \ldots, \alpha_n$. For brevity $\alpha'_i$ is sometimes called the reciprocal of $\alpha_i$. From the definition (5) of the reciprocal set it appears that the elements and their reciprocals satisfy the equations

(6)            $\alpha'_i \ \alpha_i = 1, \qquad \alpha'_i \ \alpha_j = 0, \qquad i = j.$

By the laws of regressive multiplication it follows that the $n(n{-}1)/2$ elements $\alpha'_i \ \alpha'_j$ of the $(n{-}2)$nd class and the equal number of elements $\alpha_i \ \alpha_j$ of the second class satisfy the equations

(6')        $\alpha'_i \ \alpha'_j \ \alpha_i \ \alpha_j = 1, \qquad \alpha'_i \ \alpha'_j \ \alpha_k \ \alpha_l = 0, \qquad k$ and $l$ not both
                                                                equal to $i$ and $j$.

Similar equations are satisfied by the elements $\alpha'_i \ \alpha'_j \ \alpha'_k$ of the

follows the theorem: If any set of $n$ quantities of the $(n-1)$st class, $\bar{\beta}_1, \bar{\beta}_2, \ldots, \bar{\beta}_n$, satisfy the equations (6), the elements $\bar{\beta}_i$ and $\alpha'_i$ are identical. For the $\bar{\beta}$'s may be expressed linearly in terms of the $\alpha$'s as

$$\bar{\beta}_i = a_{11} \, \alpha'_1 + a_{2i} \, \alpha'_2 + \ldots + a_{ni} \, \alpha'_n$$

and then equations (6) give the equations $a_{ii} = 1$, $a_{ji} = 0$. Thus the uniqueness of the reciprocal set is established. Furthermore equations (6), (6'), ... may be written in the form

$$\alpha_i \, \alpha'_i = \pm 1 \qquad\qquad \alpha_j \, \alpha'_i = 0$$

(6''')  $\qquad \alpha_i \, \alpha_j \, \alpha'_i \, \alpha'_j = + 1 \qquad \alpha_k \, \alpha_l \, \alpha'_i \, \alpha'_j = 0,$

. . . . . . . .        . . . . . . . ,

where the negative signs hold when and only when $n$ is even, and then only in every alternate equation. From the uniqueness of sets of reciprocals the theorem may therefore be stated that: The reciprocals of a given set of elements are equal to the given set except when their class-number is odd and $n$ is even, in which case they are the negative of the given set.

One of the prime uses of the reciprocals is to express the idemfactors. To avoid the introduction of subscripts, let $\alpha$, $\beta$, $\gamma$, ... be a set of $n$ independent primary elements and $\alpha'$, $\beta'$, $\gamma'$, ... their reciprocals. The dyadic expression

(7)  $\qquad\qquad I = \alpha \mid \alpha' + \beta \mid \beta' + \gamma \mid \gamma' + \ldots,$  $\quad$ $n$ terms,

is called an idemfactor for primary elements. It has the property that when multiplied combinatorially, see (4'), into a primary element, it reproduces that element: that is,

$$I \varrho = (\alpha \mid \alpha' + \beta \mid \beta' + \gamma \mid \gamma' + \ldots) \, \varrho = \alpha \, (\alpha' \, \beta) + \beta \, (\beta' \, \varrho) + \gamma \, (\gamma' \, \varrho)$$
$$+ \ldots = \varrho.$$

This may be seen by considering $\varrho$ as expressed in terms of $\alpha$, $\beta$, $\gamma$, .... In like manner the dyadic expression

(7')  $\qquad\qquad I_2 = \Sigma \, \alpha \, \beta \mid \alpha' \, \beta',$  $\qquad$ $n(n-1)/2$ terms,

is an idemfactor for elements of the second class, that is, $I_2 \, \varrho \, \sigma = \varrho \, \sigma$. And

(7'')  $\qquad\qquad I_3 = \Sigma \, \alpha \, \beta \, \gamma \mid \alpha' \, \beta' \, \gamma'$  $\qquad$ $n(n-1)(n-2)/6$ terms,

is an idemfactor for elements of the third class. And so on.

### *Elementary Properties of Dyadics.*

**4.** *Various representations of dyadics.*—For the present purposes, the primary dyad will be defined as one whose first factor is a primary element and whose second factor is an element of the $(n-1)$st class. For brevity, these factors will be designated respectively as the antecendent and the consequent. To bring out

more clearly the different classes of the antecedent and the consequent, the former will be denoted by a Greek small letter and the latter by a Greek small letter carrying a dash—thus $\alpha \mid \bar{\lambda}$. Dyads of the form $\alpha \beta \mid \bar{\lambda} \bar{\mu}$ may be called secondary, and so on. The notation of the reciprocal set to $n$ elements $\alpha$, $\beta$, $\gamma$, ... has been $\alpha'$, $\beta'$, $\gamma'$, ...; and this will be adhered to. The dashes will not be introduced to call additional attention to the class of the reciprocals. The set of reciprocals of $n$ independent elements of the $(n-1)$st class $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$, ... will be represented by $\bar{\alpha}'$, $\bar{\beta}'$, $\bar{\gamma}'$, ... and will be of the first class.

The sum of any number of primary dyads

(8) $$\Phi = \alpha_0 \mid \bar{\alpha}_0 + \beta_0 \mid \bar{\beta}_0 + \gamma_0 \mid \bar{\gamma}_0 + \cdots$$

is called a dyadic polynomial or simply a dyadic. As was stated in article 2, each of the dyads may be expanded as a block of $n^2$ terms by expressing the antecedents and the consequents in terms of a set of independent elements $\alpha$, $\beta$, $\gamma$. ..., $r$ and $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$, ..., $\bar{r}$. If this be done the dyadic $\Phi$ takes the form

(9) $$\Phi = c_{aa} \, \alpha \mid \bar{\alpha} + c_{ab} \, \alpha \mid \bar{\beta} + \ldots + c_{an} \, \alpha \mid \bar{r}$$
$$+ c_{ba} \, \beta \mid \bar{\alpha} \ldots \ldots \ldots + c_{bn} \, \beta \mid \bar{r}$$
$$+ \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$
$$+ c_{na} \, r \mid \bar{\alpha} + \ldots \ldots \ldots + c_{nn} \, r \mid \bar{r}.$$

If these terms be added according to rows or according to columns, $\Phi$ reduces to a sum of $n$ terms:

(9′) $$\Phi = \alpha \mid \bar{\alpha}_1 + \beta \mid \bar{\beta}_1 + \ldots + r \mid \bar{r}_1,$$
$$\Phi = \alpha_1 \mid \bar{\alpha} + \beta_1 \mid \bar{\beta} + \ldots + r_1 \mid \bar{r}.$$

In this reduction to a sum of $n$ terms, either the antecedents or the consequents may be chosen arbitrarily, but not both. The most useful reduction to the form (9) is when the antecedents and consequents are reciprocal sets.

Two dyadics $\Phi$ and $\Psi$ may be said to be equal if the $n^2$ coefficients $c_{ij}$ are equal when the dyadics are both reduced to the form (9) in terms of the same antecedents and the same consequents. Another definition, which is preferable and obviously reducible to this, is contained in any one of the three statements: Two dyadics $\Phi$ and $\Psi$ are equal when and only when

$$\Phi \rho = \Psi \rho \qquad \text{for all values of } \rho$$
$$\text{or} \quad \bar{\sigma} \, \Psi = \bar{\sigma} \, \Psi \qquad \text{for all values of } \bar{\sigma}$$
$$\text{or} \quad \bar{\sigma} \, \Phi \rho = \bar{\sigma} \, \Psi \rho \qquad \text{for all values of } \rho \text{ and } \bar{\sigma}.$$

To insure the equality, it is not necessary to verify these equations for all values of $\rho$ or of $\bar{\sigma}$ or of $\rho$ and $\bar{\sigma}$. If the equations hold respectively for $n$ independent values of the elements in question, they will hold for all values. In fact, by the aid of reciprocals the

dyadic which converts $n$ given independent elements $a, \beta, \ldots, \nu$ into $n$ elements $a_1, \beta_1, \ldots, \nu_1$, not necessarily independent. may be written down as

(10)    $\Phi = a_1 \,|\, a' + \beta_1 \,|\, \beta' \ldots + \nu_1 \,|\, \nu'$,    $\Phi \, a \quad a_1$, etc.

If the antecedents of this dyadic are not linearly independent, the expression may be reduced to a sum of $l$ dyads where $l < n$. In general if the antecedents or consequents or both, which occur in the reduction of a dyadic to a sum of $n$ terms, be not independent, the dyadic may be reduced to a sum of dyads less than $n$ in number. If $l$ be the least number of dyads which may be obtained in the reduction

(11)    $\Phi = a \,|\, \bar{a} + \beta \,|\, \bar{\beta} + \ldots + \lambda \,|\, \bar{\lambda}$    ($l$ terms),

where the antecendents and consequents are now linearly independent, the dyadic is said to have nullity of degree $n-l$. If the elements be interpreted as vectors issuing from an assumed origin in space of $n$ dimensions, the nullity may be stated geometrically by saying that by the operation of the dyadic the $n$-dimensional space has been converted into a flat subspace of $l$ dimensions passing through the assumed origin.

In like manner the dyadic which, used as a postfactor, converts the $n$ independent elements $\bar{a}, \bar{\beta}, \ldots, \bar{\nu}$ into $\bar{a}_1, \bar{\beta}_1, \ldots, \bar{\nu}_1$ is

(10')    $(-1)^{n-1} \, \Phi = \bar{a}' \,|\, \bar{a}_1 + \bar{\beta}' \,|\, \bar{\beta}_1 + \ldots + \bar{\gamma}' \,|\, \bar{\gamma}_1$,    $\bar{a} \, \Phi = \bar{a}_1$, etc.,

where the consequents need not be independent. If the dyadic reduces to a sum of $l$ terms, of which the antecedents and consequents are then linearly independent, the degree of nullity is again $n-l$. The application of the dyadic has converted space into a subspace of dimensions $l$ in hyperplanes. This subspace may or may not be identical with that previously obtained by using the dyadic as a prefactor to elements of the first class. In general the two subspaces will not be identical.

5. *Combinatory products of dyadics.*—As the individual dyads satisfy the distributive law, the definition of the product of two dyadics follows immediately from the definition of the product of the dyads as given in article 2. It also follows that the product of dyadics is itself a distributive operation. An examination of the definition shows, however, that the product is not in general commutative but that it is associative, that is,

(12)    $\Phi(\Psi\Omega) = (\Phi\Psi)\Omega = \Phi\Psi\Omega$.

The associative property is not lost if elements of the proper class are multiplied into the products at either end or at both ends, that is,

(12')    $(\bar{\sigma}\Phi)(\Psi\varrho) = \bar{\sigma}(\Phi\Psi\varrho) = (\bar{\sigma}\Phi\Psi)\varrho = \bar{\sigma}\Phi\Psi\varrho$.

If, however, elements were inserted between the dyadics in the product, the associative property would be lost[1]

It $\Phi$ and $\Psi$ are two dyadics which have respectively the nullities $n-k$ and $n-l$, so that

$$\Phi \quad c_1 \bar{c}_1 + \beta_1|\bar{\beta}_1 + \ldots + \varkappa_1|\bar{z}_1 \quad (k \text{ terms})$$
and $$\Psi - c_2|\bar{c}_2 + \beta_2|\bar{\beta}_2 + \ldots + \lambda_2 \bar{\lambda}_2 \quad (l \text{ terms}),$$

the product $\Phi\Psi$ or the product $\Psi\Phi$ cannot have a nullity less than the greater of the two nullities $n-k$ and $n-l$, nor a nullity greater than the sum, $2n-k-l$, of the nullities. To show that the nullity of $\Phi\Psi$ is at least $n-l$, it is merely necessary to inspect the product

$$\Phi\Psi = (\Phi c_2)|\bar{c}_2 + (\Phi \beta_2)|\bar{\beta}_2 + \ldots + (\Phi \lambda_2) \bar{\lambda}_2.$$

To show that the nullity is at least as great as $n-k$, consider

$$\Phi \Psi - c_1|(\bar{c}_1 \Psi) + \beta_1|(\bar{\beta}_1 \Psi) + \ldots + \varkappa_1|(\bar{z}_1 \Psi).$$

To see that the nullity of $\Phi \Psi$ is not greater than $2n-k-l$, consider the antecedents $\Phi c_2, \Phi \beta_2, \ldots, \Phi \lambda_2$ in the first expression of the product. As the nullity of $\Phi$ is only $n-k$, not more than $n-k$ dimensions of space are annihilated and hence in the most unfavorable case at least $l-n+k$ of these antecedents must be linearly independent. Hence the nullity of $\Phi \Psi$ is not greater than $n-l+n-k$, and the proposition is proved. It is, of course, obvious that the nullity of the product could not be greater than $n$. With this understanding the generalisation to a product of any number of factors is immediate. The theorem is due to Sylvester.

If either $\Phi$ or $\Psi$ has any degree of nullity, the product cannot equal the idemfactor $I$, which has no degree of nullity. Dyadics which have no degree of nullity will be called complete. If $\Phi$ and $\Psi$ are two complete dyadics which satisfy the relation $\Phi \Psi - I$, they will be called reciprocal dyadics—

(13) $$\Phi\Psi \quad I \quad \Psi\Phi, \quad \Psi \quad \Phi^{-1}, \quad \Phi - \Psi^{-1}.$$

It may be shown that in this case the product of $\Phi$ and $\Psi$ is commutative as indicated and that the reciprocal of either is uniquely determined.

The reciprocal of a product may be shown to be the product of the reciprocals taken in inverse order, that is,

(14) $$(\Phi \Psi \Omega)^{-1} - \Omega^{-1} \Psi^{-1} \Phi^{-1}.$$

The reciprocal may be written down immediately. For if

(15) $$\Phi = \alpha|\bar{\alpha}+\beta|\bar{\beta}+\ldots+\nu|\bar{\nu}, \quad (-1)^{n-1}\Phi^{-1} = \bar{\alpha}'|\alpha'+\bar{\beta}'|\beta'+\ldots+\bar{\gamma}'|\nu'.$$

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[1] The proofs of these statements are so simple and so like those given for the simpler case in the Vector Analysis, chapter 5, that there is no need of giving them here. The same is true of a large number of propositions which follow. See also Whitehead's Universal Algebra.

The existence of a reciprocal for any complete dyadic establishes the principle of cancelation for such dyadics. Thus

   if $\Phi \Psi = \Phi \Omega$ then $\Phi^{-1} \Phi \Psi = \Phi^{-1} \Phi \Omega$ and $\Psi \quad \Omega$

   or if $\Phi \rho = \Phi \sigma$ then $\Phi^{-1} \Phi \rho = \Phi^{-1} \Phi \sigma$ and $\rho \quad \sigma$.

In the first of these equations the second dyadic $\Phi$ or $\Omega$ need not be complete. Although a complete dyadic may be canceled from an equation, an incomplete generally cannot be canceled.

A dyadic may be multiplied into itself; the product $\Phi \Phi$ will be denoted by $\Phi^2$. In like manner all the successive powers may be formed. From the theorem on nullity, it follows that if a dyadic is complete, all its powers will be complete. As the reciprocal of any power of a dyadic is the same as that power of the reciprocal, it is seen that negative as well as positive integral exponents are applicable to complete dyadics. Incomplete dyadics will be considered to have only positive powers. It may happen that the successive powers have an increasing degree of nullity, so that there is a certain least power $p$ such that $\Phi^p = 0$. In this case $\Phi$ is said to be a nilpotent dyadic. It is not necessary, however, that the nullity of the successive powers should increase to the value $n$. This may be seen by a simple example. Consider the dyadic

   $$\Phi = \alpha \,|\, \alpha' + \beta \,|\, \beta' + \ldots + \lambda \,|\, \lambda', \quad (l < n \text{ terms}).$$

This has nullity of degree $n-l$, and as all its powers are identical with it, they also have nullity of degree $n-l$. In general, however, the powers of an incomplete dyadic have increasing nullities up to a certain power, from which on they all have the same nullity. And by reasoning like that employed in proving Sylvester's theorem on nullities, it is seen at once that if $n-l$, $n-l+l_1$, $n-l+l_1+l_2$, ... are the respective nullities of $\Phi$, $\Phi^2$, $\Phi^3$, ..., then $n-l \geqq l_1 \geqq l_2 \geqq \ldots$. Gibbs apparently did not state this last fact in his lectures.

6. *Homologous dyadics.*—It has been stated that in general, dyadics are not commutative in their multiplication. If two dyadics $\Phi$ and $\Psi$ are such that $\Phi \Psi = \Psi \Phi$, they will be said to be homologous. Any dyadic is homologous with the idemfactor $I$, and all powers of a dyadic are homologous with one another. Moreover, if two or more dyadics are homologous, any dyadics which may be obtained from them by the algebraic processes of addition, multiplication, and so forth, are also homologous with the original dyadics and with each other. Thus the algebra of homologous dyadics does not differ essentially from the algebra of ordinary real and complex numbers except as regards the extraction of roots. It will be seen later, in article 14, that even in such simple cases as the

square roots of the idemfactor, two square roots are not generally homologous. It is possible to define logarithms and exponentials of dyadics, and to show that these are homologous with the original dyadic; but this does not appear to be very useful.[1]

The system of homologous dyadics which is most useful is that which consists of a given dyadic, the idemfactor, and the dyadics derivable by means of rational operations on these two. For instance, let a dyadic $\Phi$ satisfy an equation, with scalar coefficients, of the type

(16) $\qquad \Phi^p + a_1 \Phi^{p-1} + \ldots + a_{p-1} \Phi + a_p I = 0,$

and consider the scalar equation

$$x^p + a_1 x^{p-1} + \ldots + a_{p-1} x + a_p = 0.$$

The roots of this equation may be found and the equation factored into

$$(x - r_1)(x - r_2) \ldots (x - r_p) = 0.$$

So likewise the equation involving $\Phi$ may be factored into

(16') $\qquad (\Phi - r_1 I)(\Phi - r_2 I) \ldots (\Phi - r_p I) = 0.$

Again, two polynomials in $\Phi$:

$$\Gamma(\Phi) = \Phi^p + a_1 \Phi^{p-1} + \ldots + a_{p-1} \Phi + a_p I$$

and $\qquad \mathit{1}(\Phi) = \Phi^m + a_1 \Phi^{m-1} + \ldots + a_{m-1} \Phi + a_m I$

may be divided according to the usual algorithm. If $\mathit{\Delta}$ is of lower degree than $\Gamma$, the result of the division may be written

(17) $\qquad \Gamma(\Phi) = B(\Phi)\, \mathit{\Delta}(\Phi) + P(\Phi)$

where the remainder $P(\Phi)$ is a polynomial of degree less than $m$. Thus the Euclidean algorithm for the highest common factor may be applied to two such polynomials.

Any dyadic $\Phi$ may be shown to satisfy a polynomial of degree not greater than $n^2$. To see this, let $\Phi$ be expressed as a block of $n^2$ terms in the form (9). where the antecedents and consequents are chosen as reciprocal sets. The higher powers of $\Phi$ are likewise expressible in terms of the same $n^2$ dyads and certain combinations of the coefficients $c_{ij}$. Consider the system of equations, $n^2 + 1$ in number, formed by the first $n^2$ powers of $\Phi$ and the idemfactor. From these equations the $n^2$ dyads may be eliminated as if they where ordinary variables in $n^2 + 1$ linear equations in $n^2$ unknowns. The result is obviously an equation of the form

(18) $\qquad c_1 \Phi^{n^2} + c_2 \Phi^{n^2-1} + \ldots + c_{n^2} \Phi + c_{n^2} I = 0.$

As a matter of fact, it will be shown in article 11 that any dyadic $\Phi$ satisfies an equation of degree $n$—the Hamilton-Cayley equation—

[1] See, for instance, The Scientific Papers, volume 2, pp. 78–84. Some simple differential equations are also solved in these pages.

but the existence of this equation is not necessary for many of the theorems concerning polynomials.

As $\Phi$ satisfies an equation of degree $n^2$, it may be interred that $\Phi$ must satisfy an equation of least degree. Let this equation be of degree $p$, so that

(16″) $\qquad A(\Phi) = \Phi^p + a_1 \Phi^{p-1} + \ldots + a_{p-1} \Phi + a_p I = 0.$

The equation of least degree is unique. For if there were two different equations of least degree, their difference would be an equation of less degree—which is absurd. It follows that any equation $E(\Phi) = 0$ in $\Phi$ must be the product of the equation of least degree and a polynomial in $\Phi$. For it is possible to write, in accordance with (17),

(17′) $\qquad\qquad 0 = E(\Phi) = B(\Phi)\, A(\Phi) + P(\Phi).$

Hence $P(\Phi)$ vanishes and the statement is proved. The equation of least degree is therefore a necessary factor of any equation in $\Phi$.

## Double Multiplication.

7. *Introduction to double products.*—The developments of the two preceding sections do not differ materially from the ordinary treatments of the generalized linear vector function (Hamilton) or the simplest type of Lückenausdrücke and quotients (Grassmann) or the theory of matrices (Cayley, Sylvester, Frobenius, and others). They have been passed hastily in review, partly for the purpose of outlining Gibbs's course on multiple algebra, partly for the purpose of establishing the notations, methods, and fundamental theorems which will be useful in the future. With his usual reticence, Gibbs apparently did not think that this part of his work on multiple algebra was of sufficient importance and originality to warrant his printing it. With regard to double multiplication it was different. He seemed to feel that here he had introduced a new idea and a new set of methods, which might be of considerable importance in a complete treatment of multiple algebra. In fact I remember that he once·told me that he had in mind several points in multiple algebra which he hoped to find time to publish after he had completed the revision of his published papers on thermodynamics. Very likely he was thinking of his theory of double multiplication. Unfortunately, however, the revision of his thermodynamic papers was cut short, almost before it had begun, by his sudden death; and the only portions of his work on double multiplication which were published during his life consist of the few words on the subject in his address On Multiple Algebra and of the discussion

given for the simple case of vectors in the fifth and sixth chapters of the Vector Analysis.

Given two dyadics

$$\Phi = \alpha \mid \bar{\alpha} + \beta \mid \bar{\beta} + \gamma \mid \bar{\gamma} + \ldots,$$
$$\Psi = \alpha_1 \mid \bar{\alpha}_1 + \beta_1 \mid \bar{\beta}_1 + \gamma_1 \mid \bar{\gamma}_1 + \ldots,$$

the combination

$$\Phi \overset{\times}{\times} \Psi = \alpha \, \alpha_1 \mid \bar{\alpha} \, \bar{\alpha}_1 + \alpha \, \beta_1 \mid \bar{\alpha} \, \bar{\beta}_1 + \alpha \, \gamma_1 \mid \bar{\alpha} \, \bar{\gamma}_1 + \ldots$$
$$+ \, \beta \, \alpha_1 \mid \bar{\beta} \, \bar{\alpha}_1 + \beta \, \beta_1 \mid \bar{\beta} \, \bar{\beta}_1 + \beta \, \gamma_1 \mid \bar{\beta} \, \bar{\gamma}_1 + \ldots$$
$$+ \, \gamma \, \alpha_1 \mid \bar{\gamma} \, \bar{\alpha}_1 + \gamma \, \beta_1 \mid \bar{\gamma} \, \bar{\beta}_1 + \gamma \, \gamma_1 \mid \bar{\gamma} \, \bar{\gamma}_1 + \ldots$$
$$+ \, \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot$$

is called the double (combinatory) product of $\Phi$ into $\Psi$. This product will be denoted, as indicated, by inserting a double cross between the dyadics. The value of using a definitive symbol for the combinatory product is thus brought clearly into the foreground as soon as the question of these double products is taken up. Turning the fact that the progressive and regressive products obey the distributive law, it is clear that the value of $\Phi \overset{\times}{\times} \Psi$ does not depend on the particular representation of $\Phi$ and $\Psi$ which may be adopted.

From the definition, the double product is obviously distributive. Moreover it is commutative. For the combinatory product of the elements is commutative except for a change of sign, and in the double product there are two changes of sign. Furthermore, the double product of several dyadics is associative, that is,

(20) $\qquad (\Phi \overset{\times}{\times} \Psi) \overset{\times}{\times} \Omega = \Phi \overset{\times}{\times} (\Psi \overset{\times}{\times} \Omega) = \Phi \overset{\times}{\times} \Psi \overset{\times}{\times} \Omega.$

This follows from the associative property of the combinatory product of the elements. If a double product of more than $n$ dyadics were formed, the laws of regressive multiplication would have to be brought in to determine the meaning of the product. The work that follows will, therefore, be restricted to the consideration of double products of $n$ or fewer dyadics. In accordance with the definitions given in article 4, the double product of two (primary) dyadics is a secondary dyadic; the double product of three dyadics is a dyadic of the third class; and so on. The double product of $n$ dyadics is a scalar. The definition of double products may clearly be extended to the product of dyadics other than primary, provided that the class of the product does not exceed $n$.[1])

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[1] It may be noted that in the Vector Analysis, p. 308, the double product (with a cross) of two dyadics is stated to be non-associative. This is because, from the point of view of the Vector Analysis, the combinatory product of two vectors is not regarded as a quantity of the

**8.** *Double powers.*—A dyadic may be multiplied doubly into it-
self. Thus if

$$\Phi = \alpha_1 \bar{\alpha} + \beta | \bar{\beta} + \gamma | \bar{\gamma} + \dots$$

then

(21)
$$\begin{aligned}
\Phi \underset{\times}{\times} \Phi &= \alpha\,\beta\,|\,\bar{\alpha}\bar{\beta} + \alpha\,\gamma\,|\,\bar{\alpha}\,\bar{\gamma} + \dots \\
&+ \beta\,\alpha\,|\,\bar{\beta}\,\bar{\alpha} + \beta\,\gamma\,|\,\bar{\beta}\,\bar{\gamma} + \dots \\
&+ \gamma\,\alpha\,|\,\bar{\gamma}\,\bar{\alpha} + \gamma\,\beta\,|\,\bar{\gamma}\,\bar{\beta} + \dots \\
&+\dots \dots \dots
\end{aligned}$$

It will be noticed that the terms in $\Phi \underset{\times}{\times} \Phi$ occur in pairs. The ex-
pression

(22)
$$\Phi_2 = \tfrac{1}{2}\,\Phi \underset{\times}{\times} \Phi = \tfrac{1}{2}\,\Phi \underset{\times}{\overset{2}{\times}}.$$

which is one-half of $\Phi\underset{\times}{\overset{2}{\times}}$, will be denoted, as indicated, by $\Phi_2$. It
may be called the double square of $\Phi$ or, more briefly and properly,
the second of $\Phi$. In like manner the expressions

(22′)
$$\Phi_3 = \tfrac{1}{3!}\,\Phi \underset{\times}{\times} \Phi \underset{\times}{\times} \Phi = \tfrac{1}{3!}\,\Phi \underset{\times}{\overset{3}{\times}}, \quad \Phi_4 = \tfrac{1}{4!}\,\Phi \underset{\times}{\overset{4}{\times}}, \quad \dots, \quad \Phi_n = \tfrac{1}{n!}\,\Phi \underset{\times}{\overset{n}{\times}}$$

may be formed and will be called the third of $\Phi$, the fourth of $\Phi$,
..., the $n$th of $\Phi$. Collectively the set $\Phi_2$, $\Phi_3$, ..., $\Phi_n$ may be
called the double powers of $\Phi$, although it should be remembered

that in $\Phi_k$ the factor $\dfrac{1}{k!}$ has been inserted.

The double powers afford a ready means of formulating the con-
ditions that a dyadic $\Phi$ possess a certain degree of nullity without
the necessity of reducing $\Phi$ to the sum of the fewest possible
dyads—a reduction which is by no means easily carried out on any
assigned dyadic. If the dyadic has $n-l$ degrees of nullity, it may
be written as

$$\Phi = \alpha|\bar{\alpha} + \beta|\bar{\beta} + \gamma|\bar{\gamma} + \dots + \lambda|\bar{\lambda} \quad \text{(}l\text{ terms)},$$

where the antecedents and consequents are independent. In this
case $\Phi_l$ takes the form

(23)
$$\Phi_l = (\alpha\,\beta\,\gamma \dots \lambda)\,(\bar{\alpha}\,\bar{\beta}\,\bar{\gamma} \dots \bar{\lambda})$$

and does not vanish. All the higher double powers will vanish be-
cause one element will have to be repeated in each antecedent and
consequent. The lower powers cannot vanish; for the double pro-

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second class, but as a vector, and vector multiplication is not associa-
tive. Moreover, in the Vector Analysis, the scalar product of two vec-
tors occurs, and hence there is the double scalar product of two dyadics.
If Grassmann's inner product were introduced into the system in addition
to his outer product (the combinatorial product), there would be double
inner products of dyadics. These were not taken up in Gibbs's course,
and they will be omitted at this point.

duct of a zero dyadic into any dyadic is zero, and $\Phi_l$ is not zero. Hence the necessary and sufficient condition that $\Phi$ have $n-l$ degrees of nullity is that

(23') $$\Phi_l \neq 0, \qquad \Phi_{l+1} = 0.$$

This condition may be applied directly to $\Phi$ without any previous reduction.

The geometric interpretation of the successive double powers is important. Suppose the dyadic is written as a sum of $n$ terms with independent consequents, so that

$$\Phi = \alpha \,|\, \bar{\alpha} + \beta \,|\, \bar{\beta} + \gamma \,|\, \bar{\gamma} + \dots .$$

This dyadic converts the vectors $\bar{\alpha}'$, $\bar{\beta}'$, $\bar{\gamma}'$, ... into the vectors $\alpha$, $\beta$, $\gamma$, ... (or their negatives).[1] The second of $\Phi$ has the form

(21') $$\Phi^2 = \alpha\beta \,|\, \bar{\alpha}\bar{\beta} + \alpha\gamma \,|\, \bar{\alpha}\bar{\gamma} + \dots$$
$$+ \beta\gamma \,|\, \bar{\beta}\bar{\gamma} + \dots$$
$$+ \dots .$$

This (secondary) dyadic converts the elements $\bar{\alpha}'\bar{\beta}'$, $\bar{\alpha}'\bar{\gamma}'$, $\bar{\beta}'\bar{\gamma}'$, .... of the second class into the elements $\alpha\beta$, $\alpha\gamma$, $\beta\gamma$, .... . Elements of the second class are the geometric counterpart of plane areas, namely the area of the parallelograms of which the vectors which correspond to the primary elements are the sides. Hence, if $\Phi$ represents the transformation of vectors in space of $n$ dimensions, $\Phi_2$ represents the transformation of two dimensional plane areas in that space. In like manner $\Phi_3$ represents the transformation of three-dimensional volumes in the space, and so on until $\Phi_n$ which gives the ratio in which $n$-dimensional volumes are changed.

A considerable number of formulas for operation with double powers may be readily deduced. To show that

(24) $$(\Phi \, \Psi)_2 = \Phi_2 \, \Psi_2$$

let $\Phi$ be expressed as a sum of $n$ dyads with independent consequents $\bar{\alpha}$, $\bar{\beta}$, $\bar{\gamma}$, ... and let $\Psi$ be expressed as a sum of $n$ dyads with the reciprocals of these consequents as antecedents.

$$\Phi = \alpha \,|\, \bar{a} + \beta \,|\, \bar{\beta} + \gamma \,|\, \bar{\gamma} + \dots ,$$
$$\Psi = \bar{\alpha}' \,|\, \bar{\lambda} + \bar{\beta}' \,|\, \bar{\mu} + \bar{\gamma}' \,|\, \bar{\nu} + \dots .$$

Then $$(-1)^{n-1} \, \Phi \, \Psi = \alpha \,|\, \bar{\lambda} + \beta \,|\, \bar{\mu} + \gamma \,|\, \bar{\nu} + \dots .$$

It is merely necessary to form the expression for $\Phi_2 \, \Psi_2$ and $(\Phi\Psi)_2$ to see that they are immediately identical. The same method may be used to show that $(\Phi\Psi)_k = \Phi_k \, \Psi_k$. By an obvious generalisation it follows that the $k$th of a product of any number of factors is the product of the $k$ths of those factors, that is,

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[1] It is scarcely necessary to mention that, geometrically speaking, the dyadic $\Phi$ represents a homogeneous strain about fixed origin.

(24')                    $(\Phi \Psi \Omega \ldots)_k = \Phi_k \Psi_k \Omega_k \ldots$

As a corollary it is seen that $(\Phi^n)_k = (\Phi_k)^n = \Phi_k^n$. The formula given in article 5 for the reciprocal of a dyadic may be used to extend this result to negative exponents in case $\Phi$ is complete.

If $\Phi$ and $\Psi$ are homologous dyadics, the developments of article 6 show that the expansion of

$$(\Phi + \Psi)^n = \Phi^n + n\, \Phi^{n-1}\, \Psi + \frac{n(n-1)}{2!}\, \Phi^{n-2}\, \Psi^2 + \ldots + \Psi^n$$

may be carried on by the ordinary binomial theorem. It the dyadics are not homologous, this will no longer be true: the second term, for instance will consist of $n$ terms in which $\Phi$ occurs $n-1$ times and $\Psi$ once, but the rearrangement which permits of writing $n\, \Phi^{n-1}\, \Psi$ will be impossible. There is, however, a binomial theorem for the $k$th of a sum, namely,

(25)      $(\Phi + \Psi)_k = \Phi_k + \Phi_{k-1} \underset{\times}{\times} \Psi + \Phi_{k-2} \underset{\times}{\times} \Psi_2 + \ldots + \Psi_k.$

The proof consists in considering the expansion of $(\Phi + \Psi)\overset{k}{\underset{\times}{\times}}$. As the commutative and associative laws hold for double products, it is possible to write

$$(\Phi + \Psi)\overset{k}{\underset{\times}{\times}} = \Phi\overset{k}{\underset{\times}{\times}} + k\, \Phi\overset{k-1,}{\underset{\times}{\times}} \underset{\times}{\times} \Psi + \frac{k(k-1)}{2!}\, \Phi\overset{k-2}{\underset{\times}{\times}} \underset{\times}{\times} \Psi\overset{2}{\underset{\times}{\times}} + \ldots$$

$$= k!\, \Phi_k + k.(k-1)!\, \Phi_{k-1} \underset{\times}{\times} \Psi + k(k-1).(k-2)!\, \Phi_{k-2} \underset{\times}{\times} \Psi_2 + \ldots.$$

On dividing through by $\frac{1}{k!}$ the theorem is proved. It will be noticed that the usual binomial coefficients are lacking in the binomial theorem for double products.

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9. *Conjugate dyadics.*—The conjugate of a given primary dyadic is a dyadic which satisfies the condition

(26) $\varrho\, \Phi_c = \Phi\, \varrho,\ \ \Phi_c = (-1)^{n-1}(\bar{\alpha}|\alpha + \bar{\beta}|\beta + \ldots)$ if $\Phi = \alpha|\bar{\alpha} + \beta|\bar{\beta} + \ldots$

for all values of the quantity $\varrho$. It is denoted by a subscript $c$. The dyadic $\Phi_c$ is not primary, but of the class $n-1$. The necessity for the negative sign arises when $n$ is even, because then $\bar{\alpha}\,\varrho = -\varrho\,\bar{\alpha}$. In the definition of conjugates for dyadics of the second and higher classes, the factor $(-1)^{n-1}$ is applied only in the case of dyadics of odd class; for it is only in such cases that the reversed of the order of the factors changes the sign. The idemfactors $I, I_2, I_3, \ldots, I_{n-1}, I_n = 1$, which are mentioned in article 8, and which are the appropriate idemfactors respectively for elements of the first, second, third, $\ldots$ $(n-1)$st, and $n$th classes, as may be

seen from the work on double multiplication, satisfy the obvious equations

(27)    $I = I_{n-1,\,c},\ I_2 = I_{n-2,\,c},\ \ldots,\ I_{n-2} = I_{2c},\ I_{n-1} = I_c$

The conjugate of the conjugate of any dyadic is the given dyadic, that is $(\Phi_c)_c = \Phi$. The process of taking the conjugate is involutory.

As to the rules of operation with conjugates, a number of theorems may be stated. The conjugate of the sum of any dyadics is the sum of the conjugates. The conjugate of the product of two dyadics is the product of the conjugates taken in inverse order. For let

$$\Phi = \alpha|\bar{\alpha} + \beta|\bar{\beta} + \gamma|\bar{\gamma} + \ldots,$$
$$\Psi = \bar{\alpha}'|\lambda + \bar{\beta}'|\mu + \bar{\gamma}'|\nu + \ldots.$$

By merely forming the expression for $(\Phi\,\Psi)_c$ and $\Psi_c\,\Phi_c$ the truth of the theorem is evident in this case. The proof for dyadics other than primary would be similar, and the theorem may evidently be extended to any number of factors by induction. Hence the conjugate of any power of a dyadic is that same power of the conjugate of the dyadic, and the result may be extended to negative powers if a reciprocal exists. It may also be seen that the double product of the conjugates of two or more dyadics is the conjugate of the double product of the dyadics. Here the order of the factors is immaterial. As a special case, the conjugate of a double power is that double power of the conjugate.

As $\Phi_c$ and $\Phi_{n-1}$ are both of the $(n-1)$st class, it is natural to seek a relation between them. Let

$$\Phi = \alpha|\bar{\alpha} + \beta|\bar{\beta} + \gamma|\bar{\gamma} + \ldots + \nu|\bar{\nu},\ (n \text{ terms})$$

Now $\Phi_{n-1}$ is the sum of all combinations (not permutations, for the factor $\dfrac{1}{(n-1)!}$ has been thrown out) of $n-1$ antecedents and their corresponding consequents. These combinations may be represented in terms of the reciprocal sets provided that the dyadic $\Phi$ is complete so that the antecedents and consequents are linearly independent. Thus in this case

$$\Phi_{n-1} = (\alpha\beta\gamma\ldots\nu)(\bar{\alpha}\bar{\beta}\bar{\gamma}\ldots\bar{\nu})(\ \alpha'|\bar{\alpha}' + \beta'|\bar{\beta}' + \gamma'|\bar{\gamma}' + \ldots$$
$$+ \nu'|\bar{\nu}')$$

and  $\Phi_c = (-1)^{n-1}(\bar{\alpha}|\alpha + \bar{\beta}|\beta + \bar{\gamma}|\gamma + \ldots + \bar{\nu}|\nu)$

The negative sign occurs in precisely those cases where the theory of reciprocals in article 3 requires it. Hence

(28)    $\Phi_c\,\Phi_{n-1} = \Phi_n\,I_{n-1}.$

If $\Phi$ has one or more degrees of nullity, it may be written as the sum of $n-1$ dyads, which need not have independent antecedents and consequents, unless the degree of nullity is one; and hence

$\Phi_{n-1}$ consists of a single term containing all the antecedents. The product $\Phi_c \Phi_{n-1}$ is therefore null. In this case $\Phi_n$ is evidently also null, and equation (28) must therefore hold for all cases.

The demonstration just given for the relation between $\Phi_c$ and $\Phi_{n-1}$ would evidently apply with only insignificant alterations to establishing a similar relation between $\Phi_{2c}$ and $\Phi_{n-2}$, $\Phi_{3c}$ and $\Phi_{n-3}$, and so on. Hence the general formula

(29)                         $(\Phi_l)_c \Phi_{n-l} = \Phi_n I_{n-l}$

On taking the conjugate of each side, the formula

(29')                        $\Phi_k (\Phi_{n-k})_c = \Phi_n I_l$

is found. In case $\Phi$ has no degree of nullity, these equations may be solved. Hence

(30)           $\Phi_k = \Phi_n \left(\Phi_{n-k}^{-1}\right)_c$  or  $\Phi_k^{-1} = (\Phi_{n-l})_c / \Phi_n.$

The formulas represented in (29') look much like the successive double powers of the formula for $k = 1$, which is

(28')                        $\Phi (\Phi_{n-1})_c = \Phi_n I.$

If the ordinary rules of forming successive double powers be applied formally, the result is

$$(\Phi \Phi_{n-1}, c)_2 = \Phi_n^2 I_2, \ (\Phi \Phi_{n-1}, c)_3 = \Phi_n^3 I_3, \ldots$$

or             $\Phi_2 (\Phi_{n-1})_{2, c} = \Phi_n^2 I_2, \ \Phi_3 (\Phi_{n-1})_{3, c} = \Phi_n^3 I_3, \ldots.$

A comparison with (29') would apparently yield the result

(31)        $(\Phi_{n-1})_2 = \Phi_n \Phi_{n-2}, \qquad (\Phi_{n-1})_3 = \Phi_n^2 \Phi_{n-3}, \ldots.$

The justification for such procedure, however, would involve the discussion of double powers of order greater than $n$.


## Invariant Properties of Dyadics.

10.  *The scalar invariants.*—Let a dyadic $\Phi$ be written as the sum of any number of dyads:

$$\Phi = \alpha | \bar{\alpha} + \beta | \bar{\beta} + \gamma | \bar{\gamma} + \ldots.$$

Suppose that the vertical bar which serves to keep the elements of the dyads apart, be removed and the sign of the combinatorial product be inserted in its place (again the value of having a sign such as $\times$ for the combinatorial product is brought out) so that the antecedent and consequent of the dyad coalesce into a scalar. The sum of these scalars, taken with the proper sign, obtained from each dyad will be called the scalar of the dyadic and written [1]

(32)           $(-1)^{n-1} \Phi_s = \alpha \bar{\alpha} + \beta \bar{\beta} + \gamma \bar{\gamma} + \ldots.$

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[1] The sign is negative when the negative sign is called for in equation (5").

The value of this scalar depends only on the dyadic and not at all on the particular representation which has been used. This is due to the fact that both dyadic and combinatorial products obey the distributive law. Such scalars may be obtained regardless of the class to which the dyadic belongs. In particular, the double powers yield the $n$ scalars

(33)          $\Phi_s,\ \Phi_{2s}\ldots\ldots,\ \Phi_{n-1,\,s},\ \Phi_n,$

where the subscript $s$ has been omitted in the case of $\Phi_n$ which is a scalar.

The scalar of the conjugate of a dyadic is the same as the scalar of the dyadic: for the negative sign which sometimes enters into the definition of the conjugate occurs in precisely those instances in which the reversal of the order of the factors would introduce a change in the sign of the combinatory product. The scalar of the product of two dyadics satisfies the equation

(34)          $(\Phi\,\Psi)_s = \Phi\quad \Psi_c$

Perhaps the easiest way to see this is to consider both $\Phi$ and $\Psi$ expanded into a block of $n^2$ terms of the form (9) where the antecedents and consequents are reciprocal sets and are the same for both expansions. Then $(\Phi\,\Psi)_s$ is obviously the sum of the products of pairs of coefficients symmetrically situated with respect to the main diagonals, one taken from one of the dyadics and the other from the other. The same rule applies for evaluating $\Phi\,{\underset{\times}{\overset{\times}{\phantom{.}}}}\,\Psi_c$, and hence the two expressions are equal. It may be seen directly, or by the application of the rules for conjugates and double products, that

(35)          $(\Psi\,\Phi)_s = (\Phi\,\Psi)_s\,.$

A more general theorem is that the scalar of the product of any number of factors is unchanged by a cyclic permutation of the factors. The proof in the case of three factors is contained in the equations

(35′)          $(\Phi\,\Psi\,\Omega)_s = [(\Phi\,\Psi)\,\Omega]_s = (\Phi\,\Psi)\,{\underset{\times}{\overset{\times}{\phantom{.}}}}\,\Omega_c = \Omega_c\,{\underset{\times}{\overset{\times}{\phantom{.}}}}\,(\Phi\,\Psi) =$

$\Omega\,{\underset{\times}{\overset{\times}{\phantom{.}}}}\,(\Phi\,\Psi)_c = (\Omega\,\Phi\,\Psi)_s\,;$

the proof for a greater number of factors is by induction. This result may be used to put the matter of invariance of the scalar of a dyadic in a different light. Consider any linear transformation of coordinates. This may be represented by a dyadic $\Psi$. Under this transformation, the strain represented by $\Phi$ becomes

(36)          $\Omega = \Psi\,\Phi\,\Psi^{-1}$

and          $\Omega_s = (\Psi\,\Phi\,\Psi^{-1})_s = (\Phi\,\Psi^{-1}\Psi)_s = \Phi_s,$

and hence it appears that none of the $n$ scalar invariants of any

dyadic differs from the corresponding invariants of the transformed dyadic.[1]

By virtue of the identity $\Phi_k = \Phi_k I_k$, it appears that

(37)        $\Phi_{k\,s} = (\Phi_k \, I_k)_s = \Phi_k \underset{\times}{\times} I_{k\,s} = \Phi_k \underset{\times}{\times} I_{n-k}.$

This may be taken as the definition of $\Phi_{k\,s}$ in place of (32) and analogous equations, and it offers a ready interpretation of the scalar invariants of $\Phi$ according to the matricular form (9) in which the antecedents and consequents are reciprocal sets. In this case $\Phi_s$ is merely the sum of all the coefficients in the main diagonal; $\Phi_{2s}$ is the sum of all two-rowed minors of the matrix which have two terms of the main diagonal as their main diagonal; $\Phi_{3s}$ is the sum of all three-rowed minors which have the three terms of their main diagonal selected from the terms of the main diagonal of the matrix; and so on until finally $\Phi_n$ is the determinant of the matrix. The values of these sums would be unchanged if the matrix underwent a transformation of coordinates. The importance of these invariants to the theory of matrices and to the mathematical theories of elasticity is well known. It would be possible indefinitely to multiply the interpretation of the theory of dyadics in the theory of matrices by reference to the expression of the dyadics in the form (9) where the antecedents and consequents are reciprocal sets: but this would not be worth while.

11. *The identical equation.*—It was shown in article 6 that any dyadic satisfies an equation of degree not greater than $n^2$, and from this fact was deduced the existence of an equation of least degree. Consider the relation (28′) of article 9 as applied to the dyadic $\Phi - g\,I$, where $\Phi$ is any dyadic and $g$ is any scalar.

$$(\Phi - g\,I)_n \, I = (\Phi - g\,I)\,(\Phi - g\,I)_{n-1,\,c}$$

The left-hand side may be expanded by the binomial theorem (25) and simplified by the relations (37); the right-hand side may also be expanded by the binomial theorem and then multiplied out. The result is

$$[\Phi_n - g\,\Phi_{n-1,\,s} + g^2\,\Phi_{n-2,\,s} - \ldots + \overset{n-1}{(-1)}\,g^{n-1}\,\Phi_s + \overset{n}{(-1)}\,g^n]\,I = \Pi(g),$$

where $\Pi(g)$ is a polynomial of degree $n$ in $g$ with dyadic coefficients. The relation is an identity in $g$. By the same reasoning which shows that two identical algebraic polynomials with scalar

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[1] The relation of a dyadic to its family of transformed dyadics $\Psi \Phi \Psi^{-1}$, where $\Psi$ is any complete dyadic, was apparently left unmentioned by Gibbs. Perhaps this was due merely to the brevity of his course on multiple algebra.

coefficients must have equal coefficients, it may be seen that two identical polynomials with dyadic coefficients must have identical coefficients. Therefore not only may any scalar be put in the place of $g$, but any vector or dyadic may be put in its place without disturbing the identity. If $\Phi$ be substituted for $g$, the righthand side vanishes. Hence

$$(38) \qquad \Phi^n - \Phi_s\,\Phi^{n-1} + \Phi_{2s}\,\Phi^{n-2} - \ldots + (-1)^{n-1}\,\Phi_{n-1,\,s}\,\Phi$$
$$+ (-1)^n\,\Phi_n\,I = 0.$$

This is the identical equation which any dyadic must satisfy. It is sometimes called the Hamilton-Cayley equation.

The actual equation of the coefficients of the different powers of $g$ gives the dyadic equations

$$(39) \qquad \Phi\;\Phi_{n-1,\,c} = \Phi_n\,I$$
$$\Phi_{n-1,\,c} + (\Phi_{n-2} \overset{\times}{\times} I)_c = \Phi_{n-1,\,s}\,I,$$
$$(\Phi_{n-2} \overset{\times}{\times} I)_c + \Phi\,(\Phi_{n-3} \overset{\times}{\times} I_2)_c = \Phi_{n-2,\,s}\,I$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$(\Phi_2 \overset{\times}{\times} I_{n-3})_c + \Phi\,(\Phi \overset{\times}{\times} I_{n-2})_c = \Phi_{2s}\,I$$
$$(\Phi \overset{\times}{\times} I_{n-2})_c + \Phi = \Phi_s\,I$$

If the relation $(\Phi - g\,I)_n\,I_2 = (\Phi - g\,I)_2\;(\Phi - g\,I_{n-2,\,c})$ had been used, the same identical equation of the matrix would have been found, but the equations obtained from comparing coefficients would have been

$$(39') \qquad \Phi \overset{\times}{\times} I_{n-2,\,c} + \Phi_2\,(\Phi_{n-3} \overset{\times}{\times} I)_c = \Phi_{n-1,\,s}\,I_2$$
$$\Phi_{n-2,\,c} + \Phi \overset{\times}{\times} I\,(\Phi_{n-3} \overset{\times}{\times} I)_c + \Phi_3\,(\Phi_{n-4} \overset{\times}{\times} I_2)_c = \Phi_{n-2,\,s}\,I_2$$
$$\cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot \quad \cdot$$
$$(\Phi_2 \overset{\times}{\times} I_{n-4})_c + \Phi \overset{\times}{\times} I\,(\Phi \overset{\times}{\times} I_{n-3})_c + \Phi_2 = \Phi_{2\,s}\,I_2$$
$$(\Phi \overset{\times}{\times} I_{n-3})_c + \Phi \overset{\times}{\times} I = \Phi_s\,I_2$$

Consider the scalar equation, called the characteristic equation,

$$(40) \qquad x_n - \Phi_s\,x^{n-1} + \Phi_{2s}\,x^{n-2} - \ldots + (-1)^{n-1}\,\Phi_{n-1,\,s}\,x$$
$$+ (-1)^n\,\Phi_n = 0,$$

and suppose the roots are $a, b, c, \ldots$ with the multiplicities $p, q, r, \ldots$. The identical equation (38) may then be factored into

$$(38') \qquad (\Phi - a\,I)^p\,(\Phi - b\,I)^q\,(\Phi - c\,I)^r \ldots = 0.$$

As the scalar equation may be regarded as the expansion of $(\Phi - x\,I)_n$, it appears that each of the factors $\Phi - a\,I$, $\Phi - b\,I$, $\Phi - c\,I$, $\ldots$ must have at least one degree of nullity, that is, there must be at least one element $\alpha$ such that $(\Phi - a\,I)\,\alpha = 0$ or $\Phi\,\alpha = a\,\alpha$, etc. The roots $a, b, c, \ldots$ of the scalar equation are called the latent roots

of the dyadic. There is at least one element fixed except as to magnitude for each latent root of $\Phi$.

If any element is a fixed element (except for magnitude will be understood) corresponding to the latent root $a$, no product of the form

(41)                    $\Pi (\Phi) = (\Phi - bI)^q (\Phi - c I)^r \ldots$

can annihilate the element. For by direct substitution it is seen that

$$\Pi \alpha = \alpha (a - b)^q (a - c)^r \ldots$$

In like manner, if $\beta$ is an element which satisfies the equation $(\Phi - a I)^p \beta = 0$ but does not satisfy the equation $(\Phi - a I)^{p-1} \beta = 0$, then $(\Phi - a I)^{p-1} \beta$ is a fixed element corresponding to the root $a$, and no product of the above type (41) can annihilate it. It follows, therefore, that factors of the type $(\Phi - a I)^p$, $(\Phi - c I)^q$, $\ldots$ are entirely independent in their nullities, and the product of such terms has the same nullity as the sum of the nullities of the factors. It appears also that the equation of lowest degree must contain each of the factors of the type $\Phi - a I$, $\Phi - b I$, $\ldots$ at least once, or there would be some elements which would not be annihilated by the product. The equation of least degree may therefore be written as

(42)        $A (\Phi) = (\Phi - a I)^{p'} (\Phi - b I)^{q'} (\Phi - c I)^{r'} \ldots = 0,$

$$p' \leqq p, \; q' \leqq q, \; r' \leqq r, \ldots,$$

where none of the exponents vanish and the degree of the equation is not greater than $n$.

12.   *The reduction of a dyadic.*—With his usual desire for general hypotheses, Gibbs made no use of the Hamilton-Cayley equation and the resulting fact that the degree of the equation of least degree is not greater than $n$ when he came to reduce the dyadic to a canonical form. He based his work on the existence of an equation of least degree as proved in (16) of article 6. Suppose this equation were factored by the methods indicated in that article. Let the equation be

(43)          $A (\Phi) = (\Phi - a I)^p (\Phi - b I)^q (\Phi - c I)^r \ldots,$

and let                    $p + q + r + \ldots = m.$

Further let

(44)   $\Phi - a I = \Psi, \; \Phi - b I = \Psi + (a - b) I, \; \Phi - c I = \Psi + (a - c) I, \ldots$

Then

(45)   $(\Phi - b I)^q (\Phi - c I)^r \ldots = A I + B \Psi + C \Psi^2 + \ldots + H \Psi^{m-p},$

where

$$A = (a - b)^q (a - c)^r \ldots \neq 0.$$

Divide $A I + B \Psi + C \Psi^2 + \ldots + H \Psi^{m-p}$ into $I$ by the ordinary

algorithm and carry the division up to and including the power $\Psi^{p-1}$ in the quotient. Then

(46) $$\frac{I}{AI + B\,\Psi + \ldots + H\,\Psi^{m-p}} = A'I + B'\Psi + \ldots + E'\,\Psi^{p-1}$$
$$+ \frac{\Psi^p\,P(\Psi)}{AI + B\,\Psi + \ldots + H\,\Psi^{m-p}}$$

where $P(\Psi)$ is a polynomial of degree $m-p-1$ in $\Psi$. Set

(47) $$I_a = (AI + B\,\Psi + \ldots + H\,\Psi^{m-p})\,(A'I + B'\,\Psi \ldots + E'\,\Psi^{p-1})$$
$$= I - \Psi^p\,P(\Psi).$$

In like manner compute $I_b$, $I_c$, .. corresponding to the values $b$. $c$. . . . .

The dyadic $I_a$ does not contain $\Psi^p$ as a factor; for it is $I - \Psi^p\,P(\Psi)$. It does, however, contain $(\Psi - b\,I)^q\,(\Phi - c\,I)^r \ldots$, which represents the other factors of the equation of least degree. Hence $I_a\,\Psi^p$ contains the equation of least degree and vanishes. Hence

$$I_a{}^2 = I_a\,[I - \Psi^p\,P(\Psi)] = I_a - I_a\,\Psi^p\,P(\Psi) = I_a.$$

The product $I_a\,I_b$ contains in $I_a$ all the factors of the equation of least degree except $(\Phi - a\,I)^q$ and in $I_b$ it contains those. Hence $I_a\,I_b = 0$. Thus

(48) $$I_a{}^2 = I_a,\ \ I_b{}^2 = I_b,\ I_c{}^2 = I_c,\ \ .,$$
$$I_a\,I_b = 0,\ I_a\,I_c = 0,\ I_b\,I_c = 0,\ \ldots.$$

Let $\Omega$ be the sum

(49) $$\Omega = I_a + I_b + I_c + \ldots.$$

It follows that

$$\Omega^2 = \Omega\ \ \text{or}\ \ (\Omega - I)\,\Omega = 0.$$

This expression is a polynomial in $\Phi$ and is equal to zero. It must contain the equation of least degree as shown in article 6. But $\Omega$ contains no factor of this equation, because any factor such as $\Phi - a\,I$ is contained in all the $I$'s except $I_a$. Hence all the factors of the equation of least degree must be contained in $\Omega - I$. As $\Omega - I$ is of degree $m-1$ in $\Phi$, which is less than that of the equation of least degree. the only possibility is

(50) $$\Omega - I \equiv 0\ \ \text{and}\ \ \Omega = I.$$

Suppose $I_a$, $I_b$, $I_c$, ... written as the sum of the fewest possible number of dyads, so that

(51) $$I_a = \alpha\,|\bar{\alpha} + \beta\,|\bar{\beta} + \gamma\,|\bar{\gamma} + \ldots,$$
$$I_b = \lambda\,|\bar{\lambda} + \mu\,|\bar{\mu} + \nu\,|\bar{\nu} + \ldots,$$
$$\cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot$$

Then if $I_a{}^2$ be compared with $I_a$.

$$I_a{}^2 = (\bar{\alpha}\,\alpha)\,\alpha\,|\bar{\alpha} + (\bar{\alpha}\,\beta)\,\alpha\,|\bar{\beta} + (\bar{\alpha}\,\gamma)\,\alpha\,|\bar{\gamma} + \ldots$$
$$+ (\bar{\beta}\,\beta)\,\beta\,|\bar{\beta} + (\bar{\beta}\,\alpha)\,\beta\,|\bar{\alpha} + (\bar{\beta}\,\gamma)\,\beta\,|\bar{\gamma} + \ldots$$
$$+ \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot$$

it follows that
$$\bar{\alpha}\,\alpha = \bar{\beta}\,\beta = \bar{\gamma}\,\gamma = \ldots = 1, \quad \bar{\alpha}\,\beta = \bar{\beta}\,\alpha = \bar{\alpha}\,\gamma = \bar{\gamma}\,\alpha = \bar{\beta}\,\gamma = \ldots = 0.$$
Furthermore the equation $I_a\,I_b = 0$ gives
$$\bar{\alpha}\,\lambda = \bar{\lambda}\,\alpha = \bar{\alpha}\,\mu = \bar{\mu}\,\alpha = \bar{\beta}\,\lambda = \ldots = 0.$$
And if any linear relation existed between the antecedents of the different $I$'s such as
$$\sigma = a\,\alpha + b\,\beta + l\,\lambda + m\,\mu + \ldots + \ldots = 0,$$
there would result the equation
$$I_a\,\sigma = a\,\alpha + b\,\beta + \ldots = 0,$$
which contradicts the hypothesis that $I_a$ is expressed as the sum of the fewest possible number of dyads. And similarly in the case of the consequents. Hence the total number of dyads in all the $I$'s cannot exceed $n$; and on the other hand, as their sum is the idemfactor, it cannot be less than $n$. Hence the sets
$$\alpha, \ \beta, \ \gamma, \ \ldots, \ \lambda, \ \mu, \ \nu, \ \ldots, \ \ldots \quad \text{and} \quad \bar{\alpha}, \ \bar{\beta}, \ \bar{\gamma}, \ \ldots, \ \bar{\lambda}, \ \bar{\mu}, \ \bar{\nu}, \ \ldots, \ \ldots$$
are reciprocal, that is
$$(51') \qquad\qquad I_a = \alpha\,|\,\alpha' + \beta\,|\,\beta' + \gamma\,|\,\gamma' + \ldots$$
$$I_b = \lambda\,|\,\lambda' + \mu\,|\,\mu' + \nu\,|\,\nu' + \ldots$$

Next consider the expression
$$(52) \qquad \Phi = \Phi I = \Phi\,(I_a + I_b + I_c + \ldots) = \Phi_a + \Phi_b + \Phi_c + \ldots$$
The dyadics $\Phi_a, \Phi_b, \Phi_c, \ldots$ have the property
$$(52') \qquad\qquad \Phi_a\,\Phi_b = \Phi_a\,\Phi_c = \Phi_b\,\Phi_c = \ldots = 0,$$
owing to the presence of the factors $I_a, I_b, I_l, \ldots$. The dyadic $\Phi$ has now been resolved into the sum of as many dyadics as there are latent roots. These are all homologous with one another and with the original dyadic. The equations
$$(52'') \qquad\qquad \Phi_a = I_a\,\Phi I_a, \quad \Phi_b = I_b\,\Phi I_b, \ \ldots$$
which follow from this fact, shows that $\Phi_a, \Phi_b, \ldots$ have the same antecedents and consequents as $I_a, I_b, \ldots$. Hence if $\Phi$ be expressed in the form (9) where the antecedents and consequents are reciprocal sets, it follows that $\Phi$ will consist of a series of matrices strung along the main diagonal and equal in number to the number of latent roots. The further discussion of $\Phi$ may be restricted to the treatment of these individual dyadics $\Phi_a, \Phi_b, \ldots$. The question of the reduction of a dyadic to standard form has been reduced to the single case in which the dyadic has only one latent root.

Gibbs then reduced dyadics with only one latent root to a matricular form in which the terms underneath the main diagonal disappear. This reduction, however, is not complete, and consequently a modified form of it will be given. It may be pointed out that the above reduction of $\Phi$ to a sum of independent dyadics is in no way dependent on the completeness of $\Phi$. The result is equally

valid for incomplete dyadics. If the dyadic $\Phi$ be arranged in matric-
ular form with no terms beneath the main diagonal, the existence
of the Hamilton-Cayley equation is evident. Finally it may be noted
that the reduction yields the same form for all of the transformed
dyadics of $\Phi$. For if $\Phi$ satisfies the equation

$$A(\Phi) = \Phi^m + A_1 \Phi^{m-1} + \ldots + A_{m-1} \Phi + A_m I = 0,$$

then

$$(54) \quad A(\Psi\Phi\Psi^{-1}) = (\Psi\Phi\Psi^{-1})^m + A_1(\Psi\Phi\Psi^{-1})^{m-1} + \ldots A_{m-1} \Psi\Phi\Psi^{-1}$$
$$+ A_m I$$
$$= \Psi\Phi^m \Psi^{-1} + A_1\Psi\Phi^{m-1}\Phi^{-1} + \ldots + A_{m-1} \Psi\Phi\Psi^{-1}$$
$$+ A_m I$$
$$= \Psi A(\Phi) \Psi^{-1} = 0.$$

It is obvious that this proof could have been given just as well in
article 6, and that in particular the equation of least degree is
identical for all the transformed dyadics $\Psi\Phi\Psi^{-1}$. The fact that
the scalar invariants of $\Psi\Phi\Psi^{-1}$, as shown in article 10, are identical
with those of $\Phi$ shows that the Hamilton-Cayley equations are the
same in both cases. The remaining steps to fill in for the purpose
of establishing the identity of the reduction of $\Phi$ and $\Psi\Phi\Psi^{-1}$ are
too obvious to need detail.

13. *The canonical form of a dyadic.*—The equation of least degree
gives the relation

$$(54) \quad (\Phi - a I)^p I_a = 0 \text{ or } (\Phi - a I)^p I_a{}^p = 0 \text{ or } (\Phi a - a I_a)^p = 0$$

For brevity let

$$(55) \qquad\qquad \Phi_a - a I_a = Z.$$

The further classification and reduction of dyadics therefore depends
on the classification and reduction of nilpotent dyadics. Consider
the successive powers

$$(56) \qquad\qquad Z, Z^2, Z^3, \ldots, Z^{p-1}, Z^p = 0.$$

These have increasing nullities, but the change of nullity between
two successive powers never increases. This may be expressed as

$$(57) \quad Z_k^{p-1} \neq 0, \ Z_{k+k_1}^{p-2} \neq 0, \ \ldots \ Z^2{}_{k+k_1 + \ldots + k_{p-3}} \neq 0,$$
$$Z_{k+k_1 + \ldots + k_{p-2}} \neq 0.$$

where, by the theorem at the end of article 5,

$$(58) \qquad k \leqq k_1 \leqq k_2 \leqq \ldots \leqq k_{p-2} \leqq k_{p-1}$$

and

$$k + k_1 + k_2 + \ldots + k_{p-2} + k_{p-1} = m,$$

if $m$ be the multiplicity of the root $a$, where it is understood that
the next higher double powers of each dyadic must vanish. The
subscripts therefore denote the number of independent dyads in the
dyadics. It remains to show that, with the aid of these relations,

$\Phi_a$ may be put in a form which is such that any other dyadic which has the same form may be written as the transformed of the given dyadic.

Consider the dyadic $Z^{p-1}$ to be represented as the sum of $k$ dyads, that is,

(59)    $$Z^{p-1} = \alpha_1 | \bar{\alpha}_1 + \alpha_2 | \bar{a}_2 + \ldots \alpha_k | \bar{a}_k$$

Then each antecedent will be a fixed element of $\Phi_a$. For

$$ZZ^{p-1} = 0 \quad \text{or} \quad \Phi_a = a\, \alpha_1, \ \Phi_a = a\, \alpha_2, \ \ldots, \ \Phi_a = a\, \alpha_k.$$

By the reasoning of article 5 it appears that the ensemble of elements linearly derivable from the antecedents of $\Phi^m$ must be contained in the ensemble of elements linearly derivable from the antecedents of $\Phi^{m-1}$. Let the dyadic $Z^{p-2}$ be expressed in terms of the following antecedents: $1^0$ the elements $\alpha_1, \alpha_2, \ldots, \alpha_k$, $2^0$ any other independent fixed elements of $\Phi_a$ which may be necessary, $3^0$ as many other elements $\beta_1, \beta_2, \ldots$ as may be necessary. Then $Z^{p-2}$ has the form

(59')    $$Z^{p-2} = \alpha_1|() + \alpha_2|() + \ldots + \alpha_k|()$$
$$+ \beta_1|() + \beta_2|() + \ldots$$
$$+ \alpha_{k+1}|() + \ldots + \alpha_{k+l}|(),$$

where it is assumed that no linear relations exist among $\alpha_1, \alpha_2, \ldots, \alpha_k, \alpha_{k+1}, \ldots$ or among $\beta_1, \beta_2, \ldots$

The number of $\beta$'s cannot be fewer than $k$: for $ZZ^{p-2}$ must be equal to $Z^{p-1}$. This establishes the equations

(60)    $$Z\beta_i = L_i(\alpha_1, \alpha_2, \ldots, \alpha_k)$$

where $L_i$ denotes a linear expression in the $\alpha$'s. If the number of $\beta$'s were greater than $k$, it would be possible to eliminate the $\alpha$'s and obtain an equation of the form

$$Z(a_1\beta_1 + a_2\beta_2 + \ldots) = 0.$$

That is, some linear combination of the $\beta$'s would have to be a fixed element of $\Phi_a$. This is impossible, because all the available independent fixed elements have been expressly assumed as antecedents of $Z^{p-2}$ before the $\beta$'s were chosen. Hence the number of $\beta$'s is exactly equal to $k$. As the $\beta$'s are independent, the determinant of equations (60) cannot vanish, and it is possible to find a new set of $\beta$'s such that

(61)    $$Z\beta_1 = \alpha_1, \ Z\beta_2 = \alpha_2, \ldots, \ Z\beta_k = \alpha_k.$$

As the number of dyads in $Z^{p-2}$ is $k + k_1$, the number $l = k_1 - k$. In like manner $Z^{p-3}$ may be written as

(59'')   $$Z^{p-3} = \alpha_1|() + \alpha_2|() + \ldots + \alpha_k|() + \alpha_{k+1}|() + \ldots + \alpha_{k_1}|()$$
$$+ \ldots + \beta_1|() + \beta_2|() + \ldots + \beta_k|()$$
$$+ \gamma_1|() + \gamma_2|() + \gamma_3|() + \ldots$$

and it may be shown that there are exactly $k_1$ of the $\beta$'s and that they may satisfy the relations

(61') $\qquad Z\gamma_1 = \beta_1,\ Z\gamma_2 = \beta_2,\quad .\ ,\ Z\gamma_l = \beta_l.$

$\qquad\qquad Z\gamma_{l+1} = \alpha_{l+1},\qquad ,\ Z\gamma_l = \alpha_l$

Moreover, $l_1 = k_2 - k_1$. And so on.

In this way the expression for $Z$ becomes

(62) $\ Z = \alpha_1|() + \alpha_2|() + \ .\ .\ + \alpha_l\ (\ ) + \alpha_{l+1}\ (\ ) + \ldots + \alpha_{l_1}\ (\ )$

$\qquad\qquad\qquad + \ .\ \ .\ + \alpha_{lp-2}|()$

$\qquad + \beta_1|() + \beta_2|() + \ldots + \beta_l\ (\ )$

$\qquad + \gamma_1|() + \gamma_2|() + \ldots + \gamma_k\ (\ ) + \gamma_{k+1}|() + \ldots + \gamma_{l_1}\ (\ )$

$\qquad + \ .\ \ .\ \ .\ \ .\ -\ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .$

and all the antecedents are independent. Let $I_a$ be expressed as

(63) $\ I_a = \alpha_1\ \alpha'_1 + \ldots + \alpha_{lp-2}|\alpha'_{kp-2} + \beta_1|\beta_1' + \ldots + \gamma_1|\gamma_1' + \ldots$

$\qquad\qquad + \alpha_{lp-1}|\alpha'_{kp-1}$

The relation $Z\alpha_i = 0$ shows that the product of $\alpha_i$ and any consequent of $Z$ is zero. The relation $Z\beta_i = \alpha_i$ shows that the product of $\beta_i$ and any consequent of $Z$ except that in $\alpha_i|(\ )$ is zero whereas that is 1. The relations for $Z\gamma, \ldots$ show similar facts. From the properties of reciprocals, it follows that $\Phi_a$ may be written in the form

(64) $\Phi_a = \overset{k}{\underset{1}{\Sigma}} \Big\{ \begin{array}{l} a\,\alpha_1|\,\alpha'_1 + a\,\beta_1|\,\beta'_1 + a\,\gamma_1|\,\gamma'_1 + \ldots \text{ to } p \qquad \text{terms} \\ \qquad\qquad + \alpha_1|\beta'_1 + \beta_1|\gamma'_1 + \ldots \text{ to } p\text{—1 terms} \end{array}$

$\qquad + \overset{k_1}{\underset{k+1}{\Sigma}} \Big\{ \begin{array}{l} a\,\alpha_{k+1}|\,\alpha'_{k+1} + a\,\gamma_{k+1}\,|\,\gamma'_{k+1} + \ldots \text{ to } p\text{—1 terms} \\ \qquad\qquad + \alpha_{k+1}|\gamma'_{k+1} + \ldots \text{ to } p\text{—2 terms} \end{array}$

$\qquad + \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .\ \ .$

$\qquad + a\,\alpha_{kp-2}|\,\alpha'_{kp-2} + \ldots + a\,\alpha_{kp-1}|\,\alpha'_{kp-1}.$

This is the form obtained by elementary divisors. The terms in the second rows of the sums $\Sigma$ are called the shearing terms.

Any other dyadic $\Omega_a$ which satisfied the equations (57) could be reduced to the same form, the only difference being that the antecedents and consequents would be different. If however $\Psi^{-1}$ be the dyadic which carries these antecedents into the antecedents of $\Phi_a$, there results the relation

(65) $\qquad\qquad\qquad \Omega_a = \Psi\,\Phi_a\,\Psi^{-1}.$

To revert to the case of the general dyadic with any number of latent roots, the theorem may be stated that: A dyadic $\Phi$ and its transformed dyadics $\Psi\,\Phi\,\Psi^{-1}$, are entirely determined by the scalar invariants

$$\Phi_s,\ \Phi_{2s},\ \Phi_{3s},\ \ldots,\ \Phi_{n-1,s},\ \Phi_n$$

and by the sets of invariant numbers

$$k,\ k_1,\ \ldots,\ k_{p-2},\ k_{p-1},$$

which correspond to each of the latent roots. It is hardly neces-
sary at this point to indicate the relation of this result to the
theory of elementary divisors. Another matter which will be passed
without examination is the reduction of a real dyadic to a real canon-
ical form. This is not of importance to the work that follows and
it was not treated in any detail by Gibbs. All that is essential in
his treatment of dyadics, as given in his course on multiple algebra,
has now been set forth.

## *Square Roots of the Idemfactor.*

14. *Involutory strains.*—If a strain represented by $\Phi$ be involutory,
its square is the identical transformation and analytically
(66)            $\Phi^2 = I$,      $(\Phi - I)(\Phi + I) = 0$.
Any dyadic which satisfies this equation may be called a square
root of the idemfactor. The algebraic theory of these square roots and
the geometric theory of involutory strains correspond, and each may
be used to study the other.[1]   Equation (66) is clearly of lowest
degree, and the latent roots are $+1$ and $-1$. As the individual
factors enter the equation of lowest degree only to the first power,
the reduction is

$$\Phi_{(+1)} = I_{(+1)} = \alpha_1 | \alpha'_1 + \alpha_2 | \alpha'_2 + \dots \alpha_k | \alpha'_k,$$
$$\Phi_{(-1)} = I_{(-1)} = \alpha_{k+1} | \alpha'_{k+1} + \dots + \alpha_n | \alpha'_n.$$

Hence

(67)            $\Phi = I_{(+1)} - I_{(-1)} = \sum_1^k \alpha_i | \alpha'_i - \sum_{k+1}^n \alpha_i | \alpha'_i$

There are $n + 1$ different types of these roots according as $\Phi$ con-
tains $0, 1, 2, \dots, n-1$, or $n$ negative signs. The first and last are

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[1] The relation of involutory strains to the group of unimodular strains
in the simple case where $n = 3$ has been treated in detail by me in an
article entitled Oblique reflections and unimodular strains, Transactions
of the American Mathematical Society, volume 8, pp. 270–298, 1907.
Reference to the case of three dimensions will be to this article. A
number of references to the literature of involutory transformations may
be found there or in my article Involutory transformations in the pro-
jective group and in its subgroups, The Annals of Mathematics, second
series, volume 8, pp. 77–86, 1907, where only the most general questions
are discussed.

respectively the idemfactor and its negative; and the other types
occur in pairs, namely, 1 and $n-1$, 2 and $n-2$, ... which differ
only by the factor $-1$. The number of square roots of type $k$ is
$\infty 2k (n-k)$. Another form in which $\Phi$ may be expressed is

$$(68) \qquad \Phi = \pm (I - 2 \sum_1^l a_i \mid \bar{a}_i), \qquad k = E\left(\frac{n}{2}\right),$$

where $E\left(\frac{n}{2}\right)$ is the integral part of $\frac{n}{2}$ and where the relations

$$(68') \qquad \bar{a}_i \, a_i = 1, \qquad \bar{a}_i \, a_j = 0$$

hold. For some purposes this form is more convenient. It should
be remarked that what is important is not the individual antecedents
and the individual consequents, but the spaces

$$R_k \, (a_1, a_3, \ldots, a_k) \qquad R_{n-k} \, (\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_k)$$

of $k$ and of $n-k$ dimensions which are determined by them. It is
clear that the spaces $R_l$ and $R_{n-l}$ are invariant under the trans-
formation $\Phi$; the former having all vectors identically fixed and
the latter having all vectors reversed in direction or vice versa,
according as the $-$ or the $+$ sign is taken with the parenthesis.

To consider the transformation of vectors in general, it will be
best to resolve the vectors along the two fixed spaces. Then the
component along the space identically fixed will remain fixed, and
the other component will be reversed in direction. It is clear that
if either of the fixed spaces be taken with all the dimensions of the
other fixed space except one, the result be a space of $n-1$ di-
mensions which will be fixed. The volume of an $n$-dimensional region
is not changed in magnitude or in sign by the even types $0, 2, \ldots$;
and is changed only in sign by the odd types $1, 3 \ldots$  As the
transformations may evidently be regarded as a generalisation of
reflection, namely a reflection through the space $R_k \, (a_1, a_2, \ldots a_k)$
parallel to the space $R_{n-k} \, (\bar{a}_1, \bar{a}_2, \ldots, \bar{a}_k)$ or vice versa, according
as the $-$ or $+$ sign is used, the designation 'oblique reflection' or
merely 'reflection' will be applied to the geometric counterpart of
the square roots of the idemfactor. In case the volume does not
change sign the reflection will be called proper, in other cases it
will be called improper. And these terms will be used to apply to
dyadics in general; if $\Phi_n > 0$, the dyadic is a proper dyadic, and
if $\Phi_n < 0$, it is improper.

If two square roots of the idemfactor are to be homologous,
they must be commutative. It is a general theorem in transfor-
mations that the necessary and sufficient condition that the product
of two involutory transformations be commutative, is that it shall
itself be involutory. Hence two square roots of $I$ will be hom-

ologous when and only when their product is a square root of $I$. As the involutory strains of types 0 and $n$ are respectively $+I$ and $-I$, they may be excluded as trivial when referring to the product of two. As the product is $I$ when and only when the two roots are identical, that case may also be laid aside. In the case of two dimensions the only involutory transformation is $\alpha \,|\, \alpha' - \beta \,|\, \beta'$. The determinant is negative and hence the determinant of the product is positive. The product is therefore $-I$, and it is seen that the line through which the reflection takes place in one is the line parallel to which it takes place in the other, and vice versa. In three dimensions there is a line and a plane entering into the characterisation of any involutory transformation, and unless the product of two is to be $I$, it is necessary and sufficient that the line of one reflection lie in the plane of the other and vice versa if the product is to be commutative.

Consider next the case of $n$ dimensions. Let $\Phi$ denote an involutory transformation and let $\Omega$ by any transformation which is commutative with it. Then

$$\Phi\,\Omega = \Omega\,\Phi \quad \text{or} \quad \Omega\,\Phi\,\Omega^{-1} = \Phi.$$

And $\quad \Phi = \alpha_n \,|\, \alpha'_n + \alpha_{n-1} \,|\, \alpha'_{n-1} + \ldots + \alpha_{k-1} \,|\, \alpha'_{k+1} - \alpha_k \,|\, \alpha'_k$
$$- \ldots - \alpha_1 \,|\, \alpha'_1.$$

If $\Omega$ carries $\alpha_1, \alpha_2, \ldots, \alpha_n$ into $\beta_1, \beta_2, \ldots, \beta_n$,
$$\Omega = \beta_1 \,|\, \alpha'_1 + \beta_2 \,|\, \alpha'_2 + \ldots + \beta_n \,|\, \alpha'_n$$

and $\quad \Omega\,\Phi\,\Omega^{-1} = \beta_n \,|\, \beta'_n + \beta_{n-1} \,|\, \beta'_{n-1} + \ldots + \beta_{k+1} \,|\, \beta'_{k+1} - \beta_k \,|\, \beta'_k$
$$- \ldots - \beta_1 \,|\, \beta'_1.$$

If this is to be identical with $\Phi$, the spaces $R_k$ $(\beta_1, \beta_2, \ldots, \beta_k)$ and $R_k$ $(\alpha_1, \alpha_2, \ldots, \alpha_k)$ must coincide, and also the spaces $R_{n-k}$ $(\beta_{k+1}, \ldots, \beta_n)$ and $R_{n-k}$ $(\alpha_{k+1}, \ldots, \alpha_n)$. Now if it be involutory, the transformation between the $\beta$'s and $\alpha$'s in $R_k$ must be involutory; and so must the transformation between the $\beta$'s and $\alpha$'s in $R_{n-k}$. If $\gamma_1, \gamma_2, \ldots, \gamma_k$ and $\gamma_{k+1}, \ldots, \gamma_n$ be the fixed elements of the involutory transformation $\Omega$, it is seen that they all lie in the fixed spaces $R_k$ and $R_{n-k}$. A different way of stating the result is this. Let

$$\Phi = \pm \left(I - 2 \sum_1^k \alpha_i \,|\, \bar{\alpha}_i\right) \quad \text{and} \quad \Psi = \pm \left(I - 2 \sum_1^l \beta_i \,|\, \bar{\beta}_i\right), \quad k, l \leqq E\!\left(\frac{n}{2}\right),$$

with the spaces $R_k$, $R_{n-k}$ and $S_l$, $S_{n-l}$. Suppose $R_k$ and $S_l$ intersect in $T_m$. Then if the product $\Phi\,\Psi = \Psi\,\Phi$ is involutory, $R_k$ intersects $S_{n-l}$ in $R'_{k-m}$ and $S_l$ intersects $R_{n-r}$ in $S'_{l-m}$ and the space $V_{k+l-2m}$ compounded of $R'_{k-m}$ and $S'_{l-m}$ is fixed in the product. Furthermore $R_{n-k}$ and $S_{n-l}$ will have in common a space $T_{n-k-l+m}$ which compounded with $T_m$ gives $V_{n-k-l+2m}$ as a fixed space of

the product. Which of the spaces $V'_{i+l-1}$ $_n$ and $V''_{----}$ is identically fixed and which is involutorily fixed depends on the sign of $\Phi$ $\Psi$. If the sign is $+$, $V_{n-l-l+2m}$ is identically fixed. The problem of determining the conditions under which two square roots of $I$ are homologous may therefore be considered as solved.

15. *The product of two involutory transformations.*—Next consider the product of any two involutory transformation, $\Omega = \Psi \Phi$, where

(69)
$$\Phi = \pm \left(I - 2 \overset{k}{\underset{1}{\Sigma}} a_i | \bar{a}_i\right) \qquad k \leqq E\left(\frac{n}{2}\right)$$

$$\Psi = \pm \left(I - 2 \overset{l}{\underset{1}{\Sigma}} \beta_i | \bar{\beta}_i\right) \qquad l \leqq E\left(\frac{n}{2}\right)$$

As $\Phi$ and $\Psi$ are their own reciprocals, $\Omega^{-1} = \Phi \Psi$ by (14). Hence by (35)

(70) $\qquad \Omega^{-1}_{k\ s} = (\Phi_k \Psi_k)_s = (\Phi_k \Psi_k)_s = \Omega_{k\ s}.$

On substitution from the relations (30), there results

(70') $\qquad \Omega_{k s} = \Omega_n \Omega_{n-k,\ s}, \qquad \Omega_n = \pm 1.$

There arise, then, four different cases of the scalar or characteristic equation (40):

(71) $x^n - \Omega_s x_{n-1} + \Omega_{2s} x^{n-2} - \ldots - \Omega_{1s} x^2 + \Omega_s x - 1 = 0,$ n odd, $\Omega_n > 0,$

$\qquad x^n - \Omega_s x^{n-1} + \Omega_{2s} x^{n-2} - \ldots + \Omega_{1s} x^2 - \Omega_s x + 1 = 0,$ n even, $\Omega_n > 0,$

$\qquad x^n - \Omega_s x^{n-1} + \Omega_{2s} x^{n-2} - \ldots + \Omega_{1s} x^2 - \Omega_s x + 1 = 0,$ n odd, $\Omega_n < 0,$

$\qquad x^n - \Omega_s x^{n-1} + \Omega_{2s} x^{n-2} - \ldots - \Omega_{2s} x^2 + \Omega_s x - 1 = 0,$ n even, $\Omega_n < 0,$

according as $n$ is odd or even and $\Omega$ proper or improper. The first three of these equations are reciprocal equations, the last is not, unless $\Omega_{\frac{n}{2}\ s} = 0$. Thus, if a dyadic can be written as the product of two square roots of the idemfactor, the scalar equation is reciprocal except in the case that $n$ is even and the determinant of the dyadic is negative. This case is treated later.

If the number of dimensions is odd, the determinant of $-I$ is negative. Hence the third case in the above list may be reduced to the first case by making the simple change of $\Omega$ to $-\Omega$. Moreover, if the question of interest were to decide whether, given a reciprocal scalar equation, every dyadic which satisfied it were resoluble into two reflections, it would be sufficient to answer the question for dyadics of positive determinant, in case $n$ is odd, inasmuch as $-I$ is commutative with any dyadic. In the fourth case, it would be possible to replace $\Omega$ by $\Omega$ $(I - a | \bar{a})$ or by $\Omega$ times any reflection of determinant $-1$. But here nothing is gained, because the product $\Omega$ $(I - a | \bar{a})$ may not satisfy an equation of the

second type. In fact, when $n = 2$, a dyadic which satisfies an
equation of the type $x^2 + a x - 1 = 0$ is not in general resoluble
into two reflections. The case of the fourth type must be examined
more in detail.

Consider the product of two reflections in an even number of
dimensions and let the determinant of the product be negative. It
is evident that the two reflections cannot be the same. In fact if
the types are $k$ and $l$, it is necessary and sufficient that $k + l$ be
odd in order that the determinant of the product be negative. The
product may then be written in the form

$$\Omega = \pm (I - 2 \sum_1^k \beta_i | \bar{\beta}_i) \ (I - 2 \sum_1^l \alpha_i | \bar{\alpha}_i), \qquad l + k < n.$$

The spaces $\bar{\beta}_1, \ \bar{\beta}_2, \ \ldots, \ \bar{\beta}_k, \ \bar{\alpha}_1, \ \bar{\alpha}_2, \ \ldots, \ \bar{\alpha}_l$ of the consequents are
therefore together greater than $n$ and must intersect. The trans-
formation of vectors in this space of intersection is either identical
or is such as to reserve the direction of each vector without in-
troducing any other change. In the former case $+1$ and in the
latter case $-1$ is a root of the scalar equation. On substituting
either of these values in the fourth equation of (71), if is seen that
$\Omega_{\frac{n}{2}S} = 0$. In other words it appears from special considerations

that the equation of the fourth type is also reciprocal if $\Omega$ is the
product of two reflections. As there is this additional condition in
this case, the question might arise whether there were not also
additional conditions in other cases.

This question may be phrased as follows: Given any reciprocal
equation

(72) $\qquad x_n - a_1 x^{n-1} + a_2 x^{n-2} - \ldots \pm \frac{1}{a_2} x^2 \mp \frac{1}{a_1} x \pm 1 = 0$

of degree $n$, can a dyadic $\Omega$ be found such that

(73) $\quad \Omega_s = a_1, \quad \Omega_{2\,s} = a_2, \ \ldots, \quad \Omega_{n-2;\,s} = \pm \frac{1}{a_2}, \quad \Omega_{n-1,\,s} = \pm \frac{1}{a_1},$

$$\Omega_n = \pm 1$$

and such that $\Omega$ may be written as the product of two square roots
of $I$? Suppose that the roots of (72) with their respective multip-
licities are

$\qquad r_1, \ r_1^{-1}, \ m_1; \ r_2, \ r_2^{-1}, \ m_2; \ r_3, \ r_3^{-1}, \ m_3; \ \ldots; \ r_h, \ r_h^{-1}, \ m_h;$

The dyadic which has these roots may be written as

(74) $\quad \Omega = r_1 \, \alpha_1 | \alpha'_1 + r_1 \, \alpha_2 | \alpha'_2 + \ldots r_1 \, \alpha_{m_1} | \alpha'_{m_1} + r_2 \, \alpha_{m_1 + 1} | \alpha'_{m_1 + 1}$

$\qquad\qquad + r_2 \, \alpha_{m_1 + 2} | \alpha'_{m_1 + 2} + \ldots .$

$\qquad + r_1^{-1} \beta_1 | \beta'_1 + r_1 \beta_2 | \beta'_2 + \ldots + r_1^{-1} \beta_{m_1} | \beta'_{m_1} + r_2^{-1} \beta_{m_1 + 1} | \beta'_{m_1 + 1}$

$\qquad\qquad + r_2^{-1} \beta_{m_1 + 2} | \beta'_{m_1 + 2} + \ldots .$

$$+ a_1 | a'_2 + a_2 | a'_3 + \ldots + a'_{m_1+1} a'_{m_1+2} - \ldots$$
$$+ \beta_1 | \beta_2 + \beta_2 | \beta'_3 + \ldots + \beta_{m_1+1} \beta'_{m_1+2} - \ldots$$

where the shearing terms in the last two rows may or may not occur; and such a dyadic will evidently satisfy the relations (73). It remains to ascertain whether $\Omega$ is resoluble as desired.

The answer is negative. For suppose that $\Omega = \Psi \Phi$, where $\Phi$ and $\Psi$ are involutory. It has been seen in article 12 that the equation of lowest degree is the same for the set of dyadics which are the transformeds of a given dyadic. Now if $\Omega$ satisfies the equation

(75) $\quad \Omega^p - b_1 \Omega^{p-1} + b_2 \Omega^{p-2} - \ldots \pm b_{p-2} \Omega^2 \mp b_{p-1} \Omega \pm 1 = 0. \quad p < n,$

of least degree, so will $\Phi \Omega \Phi^{-1}$. But as $\Phi$ and $\Psi$ are involutory,

$$\Phi \Omega \Phi^{-1} = \Phi \Psi = \Omega^{-1},$$

and

(75′) $\quad (\Omega^{-1})^p - b_1 (\Omega^{-1})^{p-1} + b_2 (\Omega^{-1})^{p \cdot 2} - \ldots \pm b_{p-2} (\Omega^{-1})^2 \mp$
$$b_{p-1} \Omega^{-1} \pm 1 = 0.$$

Hence, to extend the use of the term reciprocal to equations in dyadics, it may be stated that if a dyadic is the product of two square roots of the idemfactor, its equation of lowest degree is reciprocal. This is stating more than equations (72): for the dyadic (74) would not in general have an equation of lowest degree which was reciprocal. If the equation of lowest degree is reciprocal, the factors $\Omega - a I$, $\Omega - \frac{1}{a} I$ which correspond to a pair of reciprocal roots $a$, $\frac{1}{a}$ of the scalar equation must enter to the same degree. Moreover, from the results of article 13 it is seen that the invariant numbers $k$, $k_1$, $k_2$, $\ldots$, $k_{p-1}$ are the same for a dyadic and its transformed dyadics. It is therefore clear that the invariant numbers which correspond to two roots $a$, $\frac{1}{a}$ must be equal in case the dyadic is the product of two square roots of $I$. The question now is whether these conditions are sufficient for such a resolution.

Consider the spaces $R(a_1, a_2, \ldots, a_{m_1}, \beta_1, \beta_2, \ldots, \beta_{m_1})$ and $S(a_{m_1+1}, \ldots, \beta_{m_1+1} \ldots)$, made up of the antecedents which correspond to any root and its reciprocal and of all other antecedents. These spaces are fixed and moreover the space $R$ and the space $S$ are independent and together contain $n$ independent directions. The transformation in two such spaces will determine the transformation in all space. But the transformation in each of these two spaces is such that its scalar equation would also be reciprocal. If now the transformation in these spaces of dimension less than $n$

can be resolved into two reflections, the transformation in $n$-dimensional space may be so resolved by merely combining the elements through which the transformation takes place in the two spaces $R$, $S$ and the elements along which it takes place. Thus the question has been reduced to the same question for a fewer number of dimensions provided that there are two independent fixed spaces $R$ and $S$ in $\Omega$.

There remains to consider only the cases where there is just one pair of reciprocal roots $a, \dfrac{1}{a}$ or one root which is either $+1$ or $-1$. The first of these arises when $n$ is even and the type of both reflections is $\dfrac{n}{2}$ with the $n$ consequents of the two reflections and the $n$ antecedents each independent. However, if there is only one pair of reciprocal roots, the dyadic may be written in the form

$$(76) \quad \Omega = a\,\alpha_1|\alpha'_1,\; a\,\alpha_2|\alpha'_2 + \ldots + a\,\alpha_n|\alpha'_n + a^{-1}\beta_1|\beta'_1 + a^{-1}\beta_2|\beta_2'$$

$$+ \ldots + a^{-1}\beta_{\frac{n}{2}}|\beta'_{\frac{n}{2}}$$

$$+ \alpha_1|\alpha'_2 \qquad + \alpha_2|\alpha'_2 + \ldots \qquad + \beta_1|\beta'_1 + \beta_2|\beta'_2 \ldots,$$

where it must be assumed that the shearing terms which occur in the second row are equal in number for both roots and are similarly distributed. Moreover it may be assumed that none of them are lacking, namely, that their number is $n-2$: for otherwise the reasoning just given for different pairs of roots would apply. The transformation may be written in oblique coordinates as

$$(76') \qquad x'_1 = ax_1,\; x'_2 = a\,x_2 + x, \ldots, x'_{\frac{n}{2}} = a\,x_{\frac{n}{2}} + a\,x_{\frac{n}{2}-1},$$

$$y'_1 = \frac{1}{a}y_1,\; y'_2 = \frac{1}{a}y_2 + y_1, \ldots, y'_{\frac{n}{2}} = \frac{1}{a}y_{\frac{n}{2}} + y_{\frac{n}{2}-1}.$$

This transformation leaves a quadratic form invariant. For consider the terms

$$(77)\; A_{11}\; x_1 y_1 + \qquad A_{21}\, x_2 y_1 + \qquad A_{31}\, x_3 y_1 + \ldots \qquad + A_{\frac{n}{2}-1\,\frac{n}{2}}\, x_n y_1$$

$$+ \quad A_{12}\; x_1 y_3 + \ldots \ldots \ldots \ldots \ldots \ldots + A_{\frac{n}{2}-2\,\frac{n}{2}-1}\; x_n\; y_1 + 0\cdot$$

$$+ \quad \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots$$

$$+ A_{1,\frac{n}{2}-3}\, x_1 y_n{}_{\frac{n}{2}-3} + A_{2,\frac{n}{2}-3}\, x_2 y_n{}_{\frac{n}{2}-3} + A_{3,\frac{n}{2}-5}\, x_3 y_n{}_{\frac{n}{2}-3} + A_{4,\frac{n}{2}-3}\, x_4 y_n{}_{\frac{n}{2}-3} \ldots 0$$

$$+ A_{1,\frac{n}{2}-2}\, x_1 y_n{}_{\frac{n}{2}-2} + A_{2,\frac{n}{2}-2}\, x_2 y_n{}_{\frac{n}{2}-2} + A_{3,\frac{n}{2}-2}\, x_3 y_n{}_{\frac{n}{2}-2} \qquad 0 \;\ldots\; \ldots 0$$

$$+ A_{1,\frac{n}{2}-1}\, x_1 y_n{}_{\frac{n}{2}-1} + A_{2,\frac{n}{2}-1}\, x_2 y_n{}_{\frac{n}{2}-1} \qquad 0 \quad \ldots \ldots \ldots \ldots \ldots 0$$

$$+ A_{1,\frac{n}{2}}\; x_1 y_n{}_{\frac{n}{2}} \qquad 0 \qquad 0 \quad \ldots \ldots \ldots \ldots \ldots 0.$$

Any term $A_{ij}$ $x_i$ $y_j$ may arise only from the possible combinations of
$$x'_i = a\,x_i + x_{i-1}, \quad x'_{i+1} = ax_{i+1} + x_i$$
and $\quad y'_j = \dfrac{1}{a}y_j + y_{j-1}, \quad y'_{j+1} = \dfrac{1}{a}y_{j+1} + y_j.$

If $A_{ij}$ $x_i$ $y_j$ is to be an invariant term, the relation

(78) $\qquad A_{ij} + \dfrac{1}{a}\,A_{i+1,\,j} + a\,A_{i,\,j+1} + A_{i+1,\,j+1} = A_{ij}$

must hold. If this be applied to any of the zero terms, it is seen that they yield nothing. If it be applied to the terms in the main diagonal it is seen that they an invariant. If it be applied to any of the terms in the diagonal next above the main diagonal, there is established a set of conditions imposed upon the terms of the main diagonal, namely,

$$A_{1,\,\frac{n}{2}} : A_{2,\,\frac{n}{2}-1} : A_{3,\,\frac{n}{2}-2} : A_{4,\,\frac{n}{2}-3} : \ldots = 1 : -a^2 : a^4 : -a^6 : \ldots$$

If it be applied to the terms in the next diagonal line, these arises a condition to be imposed on the coefficients in the diagonal next to the main diagonal, and so on. These conditions are such that they may obviously be solved for the ratios of the coefficients in the successive diagonal lines. The result in case of six variables $x_1$, $x_2$, $x_3$, $y_1$, $y_2$, $y_3$ gives a quadratic form of the type (stars indicate the possible presence of terms)

|       | $x_1$ | $x_2$ | $x_3$ | $y_1$ | $y_2$  | $y_3$ |
|-------|-------|-------|-------|-------|--------|-------|
| $x_1$ | 0     | 0     | 0     | *     | *      | 1     |
| $x_2$ | 0     | 0     | 0     | *     | $-a^2$ | 0     |
| $x_3$ | 0     | 0     | 0     | $a^4$ | 0      | 0     |
| $y_1$ | *     | *     | $a^4$ | 0     | 0      | 0     |
| $y_2$ | *     | $-a^2$| 0     | 0     | 0      | 0     |
| $y_3$ | 1     | 0     | 0     | 0     | 0      | 0     |

of which the determinant is clearly not zero; and a similar form may be written down for any even number of variables. Now, Smith[1] has shown that the transformation of a quadratic form with itself may always be resolved into the product of two involutory transformations. Hence the dyadic (76) may be factored into the product of two square roots of $I$.

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[1] P. F. Smith, On the linear transformations of a quadratic form into itself, Transactions of the American Mathematical Society, volume 6, pp. 1–16 (1905). The theorem here referred to is found on p. 13. The more detailed exposition of the relations between collineations and strains is taken up in our next article 16.

In case there is only one root $+1$, it may be assumed that the dyadic takes the form

(79)   $\alpha|\alpha' + \alpha|\beta + \beta|\beta' + \beta|\gamma' + \gamma|\gamma'' + \gamma|\delta' + \delta|\delta' + \delta|\varepsilon' + \varepsilon|\varepsilon' + \ldots,$

in which all the shearing terms occur: for if any of them were absent, a reduction to two spaces of lower dimensions could be effected as in the case of two pairs of roots. This may be factored. In case $n = 2$,

(80)   $\alpha|\alpha' + \alpha|\beta' + \beta|\beta' = (\alpha|\alpha' - \beta|\beta')\,(\alpha|\alpha' + \alpha|\beta' - \beta|\beta')$

where each of the factors is a square root of $I$. In case $n = 3$,

(80')   $\alpha\ \alpha' + \alpha|\beta' + \beta|\beta' + \beta|\gamma' + \gamma|\gamma' =$

$(\alpha|\alpha' - \beta\ \beta' + \beta|\gamma' + \gamma|\gamma')\,(\alpha|\alpha' + \alpha|\beta' - \beta|\beta' + \gamma|\gamma')$

with similar remarks. Again in case $n = 4$, the factors are

(80'')   $\alpha|\alpha' + \alpha|\beta' + \beta|\beta' + \beta|\gamma' + \gamma|\gamma' + \gamma|\delta' + \delta|\delta' = (\alpha|\alpha' - \beta|\beta' +$

$\beta\gamma' + \gamma|\gamma' - \delta|\delta' - 2\gamma\ \delta' - \beta|\delta')\,(\alpha|\alpha + \alpha|\beta' - \beta|\beta' + \gamma|\gamma' - \gamma|\delta - \delta|\delta').$

If the root were $-1$, the factors would be respectively

$-\alpha|\alpha' + \alpha|\beta' - \beta|\beta' = (\alpha|\alpha' - \beta|\beta')(-\alpha|\alpha' + \alpha|\beta' + \beta|\beta'),$

(80''')   $-\alpha|\alpha' + \alpha|\beta' - \beta|\beta' + \beta|\gamma' - \gamma|\gamma'$

$= (\alpha|\alpha' - \beta|\beta' - \beta|\gamma' + \gamma|\gamma')(-\alpha|\alpha' + \alpha|\beta' + \beta|\beta' - \gamma|\gamma'),$

$-\alpha\ \alpha' + \alpha|\beta' - \beta\ \beta' + \beta|\gamma' - \gamma|\gamma' + \gamma|\delta' - \delta|\delta' = (\alpha|\alpha' - \beta|\beta' - \beta|\gamma'$

$+ \gamma\ \gamma' - \delta\ \delta' + 2\gamma|\delta' - \beta\ \delta')(-\alpha|\alpha' + \alpha\ \beta' + \beta|\beta' - \gamma|\gamma' - \gamma|\delta' + \delta|\delta').$

Although this method of factoring could be carried on to higher dimensions, it is better to proceed in another way, which at the same time will indicate how the factors may be obtained if they are not evident. Consider, for example, the case of seven dimensions, where

(79')   $\alpha|\alpha' + \alpha|\beta' + \beta|\beta' + \beta|\gamma' + \gamma|\gamma' + \gamma|\delta' + \delta|\delta' + \delta|\varepsilon' + \varepsilon|\varepsilon'$

$+ \varepsilon|\zeta' + \zeta|\zeta' + \zeta|\eta' + \eta|\eta'$

and note that the two expressions

(81)   $\alpha|\alpha' + \alpha\ \beta' - \beta\ \beta' + \gamma|\gamma' + \gamma|\delta' - \delta|\delta' + \varepsilon|\varepsilon' + \varepsilon|\zeta'$

$- \zeta|\zeta' + \eta|\eta'$

$\alpha|\alpha' - \beta\ \beta' + \beta|\gamma' + \gamma|\gamma' - \delta|\delta' + \delta|\varepsilon' + \varepsilon|\varepsilon' - \zeta|\zeta' + \zeta|\eta' + \eta|\eta'$

are obviously square roots of I and that their product has the form

(79'')   $\alpha|\alpha' + \alpha|\beta' + \beta|\beta' + \beta\ \gamma' + \gamma|\gamma' + \gamma|\delta' + \delta|\delta' + \delta|\varepsilon' + \varepsilon|\varepsilon'$

$+ \varepsilon|\zeta' + \zeta|\zeta' + \zeta|\eta' + \eta|\eta' + \beta|\delta' + \delta|\zeta'.$

This fails to be identical with (79') owing to the extra terms $\beta|\delta' + \delta\ \zeta'$. Nevertheless it belongs to the same type of dyadic as that. In fact it is true that when a dyadic has been reduced to the form (64), which in matricular expression means that the only terms occurring are those of the main diagonal and some (the shearing terms) along the diagonal next above it, then the addition of any terms in the half-square whose diagonal is constructed of the $p$ terms

$\alpha_i\ \alpha'_i + \beta_i|\beta'_i + \gamma_i|\gamma'_i + \ldots,\ 1 \leqq i \leqq k$, or of the $p-1$ terms

$\alpha_i \mid \alpha'_i + \beta_i \mid \beta'_i + \gamma_i \mid \gamma'_i + \ldots ,\; k + 1 \quad i \quad k_1,$ and so forth, does not effect the nullities of $Z$, $Z^2$, $Z^3$, $\ldots$ and hence does not alter the type of the dyadic. Hence if the transformation which carries (79'') into (79') be obtained and applied to the factors (81), the factors will take the desired form.

From the preceding analysis it is seen that in every case either $\Omega$ may be factored or may be referred to similar transformations in a less number of dimensions. As the proof that any dyadic which satisfies a reciprocal equation has been given when $n = 2$ and when $n = 3$,[1] it follows that: The necessary and sufficient condition that a dyadic $\Phi$ be factorable into two square roots of the idemfactor (or geometrically, that a strain be resoluble into two oblique reflections) is that the scalar invariants

$$\Phi_s,\; \Phi_{2s},\; \ldots ,\; \Phi_{n-1,\,s},\; \Phi_n = \pm 1$$

be such that the scalar equation is reciprocal and that the sets of invariant numbers

$$k,\; k_1,\; \ldots ,\; k_{p-2},\; k_{p-1}$$

which correspond to any root and its reciprocal be equal. This is the generalisation of the result I obtained for the case $n = 3$.

16. *Relations between strains and collineations.*—If a strain in $n$ dimensions be written as a matrix by chosing the antecedents and consequents as reciprocal systems, so that

(82)
$$\begin{aligned}
\Omega = &\; c_{11}\, \alpha_1 \mid \alpha'_{12} + c_1\, \alpha_1 \mid \alpha'_2 + \ldots + c_{1n}\, \alpha_1 \mid \alpha'_n \\
&+ c_{21}\, \alpha_2 \mid \alpha'_1 + c_{22}\, \alpha_2 \mid \alpha'_2 + \ldots + c_{2n}\, \alpha_2 \mid \alpha'_n \\
&+ \ldots \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\
&+ c_{n1}\, \alpha_n \mid \alpha'_2 + c_{n2}\, \alpha_n \mid \alpha'_2 + \ldots + c_{nn}\, \alpha_n \mid \alpha'_n
\end{aligned}$$

and

$$\varrho = x_1\, \alpha_1 + x_2\, \alpha_2 + \ldots + x_n\, \alpha_n$$

the transformation $\varrho' = \Omega\, \varrho$ in oblique coordinates becomes

(82')
$$\begin{aligned}
x'_1 &= c_{11}\, x_1 + c_{12}\, x_2 + \ldots + c_{1n}\, x_n \\
x'_2 &= c_{21}\, x_1 + c_{21}\, x_2 + \ldots + c_{2n}\, x_n \\
&\; \ldots \ldots \ldots \ldots \ldots \ldots \ldots \\
x'_n &= c_{n1}\, x_1 + c_{n2}\, x_2 + \ldots + c_{nn}\, x_n
\end{aligned}$$

---

[1] It should be noted that in these cases the additional condition that the invariant numbers be equal for reciprocal roots is fulfilled necessarily as the equation of lowest degree completely characterizes a dyadic when $n < 4$. The treatment for $n = 3$ is given in the first reference of p. 32): the treatment for $n = 2$ may be regarded as a special case of that or as a special case of the investigation I gave in A generalized conception of area : applications to collineations in the plane—The Annals of Mathematics, second series, volume 5, pp. 29—45 (1903).

The ratios $x_1:x_2\ldots:x_n$ of these coordinates may be regarded as
as homogeneous coordinates in a space of $n-1$ dimensions. In
particular that space may be taken as the space at infinity in the
original space of $n$ dimensions. As the coefficients in (82) can be
arbitrary, the transformation (82′) is the general projective trans-
formation in $n-1$ dimensions. The correspondence between the
two is not one to one: for all the equations in (82′) may be multi-
plied by a constant. In particular the constant may be so chosen
that the determinant of (82′), which is supposed not to vanish, may
be $\pm 1$.[1]   Thus the correspondence may be considered to be be-
tween unimodular strains and the collineations.

In this correspondence any projective reflection in the $(n-1)$-
dimensional space at infinity becomes an oblique reflection of the
types here considered by merely passing spaces through the fixed
spaces of the projective reflection and through the origin, and con-
versely, any reflection in the spaces $R_k$, $R_{n-k}$ of the $n$-dimensional
space becomes a projective reflection in the plane at infinity and
with the intersections of that plane and $R_k$, $R_{n-k}$ as its fixed spaces.
In the projective reflection the distinction between the reflections
of types 0, $n$ or 1, $n-1$ or 2, $n-2$ or ... entirely disappears: there
is nothing corresponding to reversal of direction, as only the ratios
of the coordinates are considered. Moreover the Hamilton-Cayley
equation of the matrix of the coefficients in a projective trans-
formation may be written

(83)      $\Omega^n - \Omega_s\,\Omega^{n-1} + \Omega_{2s}\,\Omega_{n-2} - \ldots \pm \Omega_{n-2,\,s}\,\Omega^2 \mp \Omega_{n-1,\,s}\,\Omega$
$$\pm \Omega_n\,I - 0,$$

without any factors arising from the factor of proportionality which
may effect the coordinates: for that factor enters into $\Omega_{k\,s}$ to the
power $k$ and into $\Omega_{n-k}$ to the power $n-k$, and hence may be can-
celed out. If the projective transformation may be resolved into
the product of two projective reflections the equation for $\dfrac{\Omega}{\sqrt[n]{\mid \Omega_n \mid}}$
must be reciprocal and the invariant numbers corresponding to a
pair of reciprocal roots of the scalar equation must be equal.

The connection with Smith's work already referred to is inter-
esting. If a quadratic form in the $n$ homogeneous variables in the
plane at infinity is invariant, under any projective transformation
of the variables, the same quadratic form must be invariant under

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[1] The distinction between $+1$ and $-1$ may be disregarded except for
questions of reality.

the corresponding strain. The interpretation of the form in this case gives a quadratic cone issuing from the origin and cutting the plane at infinity in the quadratic locus represented by the form in the $n$ homogeneous variables. Smith has shown that any transformation with an invariant quadratic form may be resolved into two reflections. From this it is evident that the matrix of any such transformation must satisfy a reciprocal Hamilton-Cayley equation and that the invariant numbers corresponding to a pair of reciprocal roots must be equal. It may be noted that it is not true to say that any strain which leaves a quadratic cone issuing from the origin invariant is resoluble into two oblique reflections; it is necessary to add that the strain is unimodular or that the form which represents the cone is invariant.

The question naturally arises whether every projective transformation which is compounded of two reflections always has an invariant quadratric locus, that is, whether the conditions stated for resolubility into two reflections are both necessary and sufficient for a transformation with a non-degenerate quadratic form in $n$ homogeneous variables. The answer is negative. To show whether any transformation resoluble into two reflection leaves a non-degerate quadric form invariant, it is merely necessary to examine the different cases that may arise. Consider the transformation written in the reduced form (76'). Let $a$ and $\dfrac{1}{a}$ be a pair of roots corresponding to no shearing. As far as they are concerned the transformation may be written as

$$x' = a x_1, \; x'_2 = a x_2, \; \ldots, \; y'_1 = \frac{1}{a} y_1, \; y'_2 = \frac{1}{a} y_2, \; \ldots$$

and the quadratic terms

|       | $x_1$ | $x_2$ | $\cdots$ | $y_1$ | $y_2$ | $\cdots$ |
|-------|-------|-------|----------|-------|-------|----------|
| $x_1$ | 0     | 0     | 0        | 1     | 0     | 0        |
| $x_2$ | 0     | 0     | 0        | 0     | 1     | 0        |
| $\cdots$ | 0  | 0     | 0        | 0     | 0     | 1        |
| $y_1$ | 1     | 0     | 0        | 0     | 0     | 0        |
| $y_2$ | 0     | 1     | 0        | 0     | 0     | 0        |
| $\cdots$ | 0  | 0     | 1        | 0     | 0     | 0        |

of non-vanishing determinant are invariant. If there are shearing terms, the quadratic terms

|        | $x_1$ | $x_2$ | $\ldots$ | $y_1$ | $y_2$ | $\ldots$ |
|--------|-------|-------|----------|-------|-------|----------|
| $x_1$  | 0     | 0     | 0        | *     | *     | *        |
| $x_2$  | 0     | 0     | 0        | *     | *     | 0        |
| $\ldots$ | 0   | 0     | 0        | *     | 0     | 0        |
| $y_1$  | *     | *     | *        | 0     | 0     | 0        |
| $y_2$  | *     | *     | 0        | 0     | 0     | 0        |
| $\ldots$ | *   | 0     | 0        | 0     | 0     | 0        |

as seen above are invariant. If a root is $+1$ without shearing terms the invariant terms of the second degree are

|        | $x_1$ | $x_2$ | $\ldots$ |
|--------|-------|-------|----------|
| $x_1$  | 1     | 0     | 0        |
| $x_2$  | 0     | 1     | 0        |
| $\ldots$ | 0   | 0     | 1        |

and the determinant does not vanish; and similarly in case of the root $-1$. If there are shearing terms corresponding to $+1$, the transformation may be written

$$x'_1 = x_1, \; x'_2 = x_2 + x_1, \; x'_3 = x_3 + x_2, \; \ldots \,,$$

and are invariant quadratic terms are

|        | $x_1$ | $x_2$ | $x_3$ | $\ldots$ |
|--------|-------|-------|-------|----------|
| $x_1$  | 1     | $-\frac{1}{2}$ | $-1$ | 0 |
| $x_2$  | $-\frac{1}{2}$ | 1 | 0 | 0 |
| $x_3$  | $-1$  | 0     | 0     | 0        |
| $\ldots$ | 0   | 0     | 0     | 0        |

The determinant vanishes if $n > 3$, and similarly for the case of a root $-1$. If these sets of quadratic terms corresponding to the various roots with or without shearing terms be arranged along the main diagonal of a matrix of order $n$, and if all the other spaces be filled with zeros, the result is a quadratic form in $n$ variables which is invariant and which certainly has a non-vanishing determinant, unless $+1$ or $-1$ is a root with as many as three consecutive shearing terms.

It therefore appears that there are linear transformations in more than three variables which are compounded of two reflections and which leave no quadratic surface (with non-vanishing determinant) invariant. In other words, the converse of Smith's theorem is not always, although it is generally, true. The simplest example of the failure of the converse is in the collineations of three dimensions.

The collineation

(84) $\quad \varrho\, x'_1 \quad x_1 + x_2, \quad \varrho'\, x_2 \quad x_2 + x_{..}, \quad \varrho\, x'_{..} \quad x_{..} + x_4, \quad \varrho\, x'_4 \quad x_1.$

which corresponds to the strain

$$\Omega == \alpha\, \alpha' + \alpha\, \beta' + \beta\, \beta' + \beta\, \gamma' + \gamma\, \gamma' + \gamma\, \delta' + \delta\, \delta'$$

which has been factored into two reflections which in turn corre-
spond to

$T: \varrho\, x'_1 = x_1 + x_2, \quad \varrho\, x'_2 \,--\, -x_2, \qquad \varrho\, x'_3 \,-\, x_3 - x_4, \quad \varrho\, x'_4 = x_1.$

$S: \varrho\, x'_1 = x_1, \quad \varrho\, x'_2 -\, -x_2 + x_3 - x_4, \quad \varrho\, x'_3 == x_3 - 2\, x_4, \quad \varrho\, x'_4 = -x_4,$

leaves no non-degenerate quadratic surface $Q$ $(x_1,\ x_2,\ x_3,\ x_4) == 0$
invariant. This is clearly seen from the analysis or from the following
simple geometric reasoning. The collineation (84) has one and
only one fixed plane $p$. This plane must be tangent to the quadric
$Q$: for if it cut the quadric in a true conic, the conic being trans-
formed into itself must have one fixed point, and the plane tangent
to $Q$ at this point would also be fixed. The fixed plane $p$, which
is tangent to the conic quadric $Q$ must intersect the quadric in two
coincident straight lines or generators: for the collineation (84) can-
not have two distinct fixed lines. But the only quadrics which
can have a double line in common with a plane are the cones or
other more degenerate quadrics. Hence the theory of collineations
compounded of two reflections is not quite identical with the theory
of collineations which leave invariant a non-degenerate quadric but
includes it.

## On the Resolution of Strains into Reflections.

17. *The product of a unimodular strain by the simplest reflection.*
Consider a dyadic $\Omega$ where $\Omega_n = \pm 1$, and a reflection $I - 2\,\sigma\,|\,\bar\sigma$ of
type 1 where the relation $\bar\sigma\,\sigma = 1$ holds. The scalar invariants
$X_{2s},\ X_{3s},\ \ldots,\ X_{n-1,\ s}$ of the product

(85) $\qquad\qquad X = \Omega\,(I - 2\,\sigma\,|\,\bar\sigma),$

are determined by the expressions

(86) $\qquad X_{k\,s} = [\Omega_k\,(I - 2\,\sigma\,|\,\bar\sigma)_k\,]_s \qquad k = 2,\ 3,\ \ldots,\ n-1$

If $(I - 2\,\sigma\,|\,\bar\sigma)_k$ be expanded by the binomial theorem, there are only
two terms in the expansions, namely,

(87) $\qquad\qquad (I - 2\,\sigma\,|\,\bar\sigma)_k = I_k - 2\,I_{k-1}\,\overset{\times}{\times}\,\sigma\,|\,\sigma.$

Hence the scalar invariants take the form

(86') $\quad \Omega_k\,_s - 2\,(\Omega_k\,I_{k-1}\,\overset{\times}{\times}\,\sigma\,|\,\bar\sigma)\,_s = \Omega_k\,_s - 2\,\Omega_k\,_o\,\overset{\times}{\times}\,(I_{k-1}\,\overset{\times}{\times}\,\sigma\,|\,\bar\sigma)$

It becomes necessary to investigate the expressions $\Omega_k\,_o\,\overset{\times}{\times}\,(I_{k-1}\,\overset{\times}{\times}\,\sigma\,|\,\bar\sigma)$
more in detail. It is clear that if the scalar quantity $\Omega_k\,_o\,\overset{\times}{\times}\,(I_{k-1}\,\overset{\times}{\times}\,\sigma\,|\,\bar\sigma)$

be expanded, each term of the expansion will contain a factor of the form $\bar{\sigma}\,\alpha$, a factor of the form $\bar{\beta}\,\sigma$, and no other factors which contains either $\sigma$ or $\bar{\sigma}$. In other words, it will be possible to write

(88) $\Omega_k \cdot \overset{\times}{\wedge} (I_{k-1} \overset{\zeta}{\wedge} \sigma | \bar{\sigma}) \quad \Sigma (\bar{\sigma}\,\alpha) \ldots (\bar{\beta}\,\sigma) \quad \bar{\sigma}\,(\Sigma\,\alpha(..)|(..)\,\bar{\beta})\,\sigma \quad \bar{\sigma}\,\Xi^{(h)}\sigma$

where $\Xi^{(h)}$ is some dyadic with antecedents of one dimension and consequents of $n-1$ dimensions. The form of this dyadic will depend only on the dyadic $\Omega$ and not at all on its particular mode of representation. That is to say, the dyadics $\Xi^{(2)}$, $\Xi^{(3)}$, ..., $\Xi^{(n-1)}$ are invariant dyadics associated with $\Omega$.

At first it will be best to treat $\Xi^{(2)}$. Let $\Omega$ be written as the sum

$$\Omega = \alpha|\bar{\alpha} + \beta|\bar{\beta} + \gamma|\bar{\gamma} + \ldots$$

of any number of dyads. Then $\Omega_2$ will be of the form

$$\Omega_2 = \alpha\beta|\bar{\alpha}\bar{\beta} + \alpha\gamma|\bar{\alpha}\bar{\gamma} + \beta\gamma|\bar{\beta}\bar{\gamma} + \ldots$$

Let the idemfactor be $I = \lambda\lambda' + \mu\mu' + \nu\nu' + \ldots$, and consider the value of any term

(89) $\qquad (\bar{\alpha}\bar{\beta}|\alpha\beta) \overset{\times}{\underset{\times}{}} (\lambda\lambda' \overset{\times}{\underset{\times}{}} \sigma|\bar{\sigma}) = (\bar{\alpha}\bar{\beta}\lambda\sigma)(\alpha\beta\lambda'\bar{\sigma})$

A product like $\bar{\alpha}\bar{\beta}\lambda\sigma$ or $\alpha\beta\lambda'\bar{\sigma}$ is called a mixed product in distinction to the pure progressive or regressive products. The only formula which will be required here is

(90)

$$\lambda\bar{\mu}\bar{\nu}\ldots\alpha\beta\gamma\ldots = \begin{vmatrix} \bar{\lambda}\,\alpha & \bar{\mu}\,\alpha & \bar{\nu}\,\alpha & \ldots \\ \bar{\lambda}\,\beta & \bar{\mu}\,\beta & \bar{\nu}\,\beta & \ldots \\ \bar{\lambda}\,\gamma & \bar{\mu}\,\gamma & \bar{\nu}\,\gamma & \ldots \\ \ldots & \ldots & \ldots & \ldots \end{vmatrix}$$

which expresses the value of the scalar which results from multiplying any number (less than $n$) of vectors into the same number of spaces of $n-1$ dimensions.[1]

From the application of this formula to the case in hand there results

$$(\bar{\alpha}\bar{\beta}\lambda\sigma)(\bar{\lambda}\bar{\sigma}\alpha\beta) = \begin{vmatrix} \bar{\alpha}\,\lambda & \bar{\alpha}\,\sigma \\ \bar{\beta}\,\lambda & \bar{\beta}\,\sigma \end{vmatrix} \begin{vmatrix} \lambda'\,\alpha & \lambda'\,\beta \\ \bar{\sigma}\,\alpha & \bar{\sigma}\,\beta \end{vmatrix}$$

$$= (\bar{\alpha}\,\lambda)(\lambda'\,\alpha)(\bar{\beta}\,\sigma)(\bar{\sigma}\,\beta) - (\bar{\alpha}\,\lambda)(\bar{\beta}\,\sigma)(\bar{\sigma}\,\alpha)(\lambda'\,\beta)$$
$$- (\bar{\beta}\lambda)(\bar{\alpha}\,\sigma)(\lambda'\,\alpha)(\bar{\sigma}\,\beta) + (\bar{\beta}\,\lambda)(\bar{\alpha}\,\sigma)(\bar{\sigma}\,\alpha)(\lambda'\,\beta)$$
$$= \sigma[(\bar{\alpha}\lambda\lambda'\alpha)\beta|\bar{\beta} - (\bar{\alpha}\lambda\lambda'\beta)\alpha|\bar{\beta} - (\bar{\beta}\lambda\lambda'\alpha)\beta\bar{\alpha} + (\bar{\beta}\lambda\lambda'\beta)\alpha|\bar{\alpha}].$$

There is a similar expression for the other terms $\mu\mu'$, $\nu\nu'$, ... of $I$. These may then be added together and simplified by the relations $\alpha I\alpha = \bar{\alpha}\alpha$ and so forth. The result is that the contribution of $\alpha\beta|\bar{\alpha}\bar{\beta}$ to $\Xi^{(2)}$ is

$$(\bar{\alpha}\,\alpha)\beta|\bar{\beta} - (\bar{\alpha}\,\beta)\alpha|\bar{\beta} - (\bar{\beta}\,\alpha)\beta|\bar{\alpha} + (\bar{\beta}\,\beta)\alpha|\bar{\alpha}.$$

Hence finally

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[1] See footnote to p. 9.

(91)  $\Xi^{2)}$   $\Sigma\left[(\bar\beta\,\beta)\,\alpha|\bar\alpha + (\bar\alpha\,\alpha)\,\beta\,\bar\beta - (\bar\alpha\,\beta)\,\alpha|\bar\beta - (\bar\alpha\,\beta)\,\beta|\bar\alpha\right],$

where the summation covers all pairs of antecedents $\alpha,\ \beta,\ \gamma,\ \ldots$ of $\Omega$. The form of this expression evidently bears out the statement that the expression is independent of the manner in which $\Omega$ is written: for if $\alpha$ is replaced by $\alpha_1 + \alpha_2$, the sum of the terms due to $\alpha_1\beta|\bar\alpha\bar\beta$ and $\alpha_2\beta|\bar\alpha\bar\beta$ is the same as those due to $\alpha\beta|\bar\alpha\bar\beta$, and so is it for similar changes in any of the elements that enter into $\Omega$.

To pass on to $\Xi^{(3)}$ consider the typical product

$$(\bar\alpha\,\bar\beta\,\bar\gamma\,\lambda\,\mu\,\sigma)\ (\lambda'\,\mu'\,\bar\sigma\,\alpha\,\beta\,\gamma)\ -\ \bar\sigma\begin{vmatrix}\gamma'\,\alpha & \mu'\,\alpha & \alpha \\ \gamma'\,\beta & \mu'\,\beta & \beta \\ \lambda'\,\gamma & \mu'\,\gamma & \gamma\end{vmatrix}\begin{vmatrix}\bar\alpha\,\gamma & \bar\alpha\,\mu & \bar\alpha \\ \bar\beta\,\lambda & \bar\beta\,\mu & \bar\beta \\ \bar\gamma\,\lambda & \bar\gamma\,\mu & \bar\gamma\end{vmatrix}\sigma$$

The coefficient of the dyad $\alpha|\bar\alpha$ is

$$\begin{vmatrix}\lambda'\,\beta & \mu'\,\beta \\ \lambda'\,\gamma & \mu'\,\gamma\end{vmatrix}\begin{vmatrix}\bar\beta\,\lambda & \bar\beta\,\mu \\ \bar\gamma\,\lambda & \bar\gamma\,\mu\end{vmatrix} = (\bar\beta\,\bar\gamma\,\lambda\,\mu)\ (\lambda'\,\mu'\,\beta\,\gamma) = \bar\beta\,\bar\gamma\,(\lambda\mu|\lambda'\mu')\,\beta\gamma.$$

If this be summed over all pairs of different antecedents of $I$, the result is the term $(\bar\beta\,\bar\gamma\,\beta\,\gamma)\ \alpha|\bar\alpha$ of $\Xi^{(3)}$. In like manner the coefficients of $\alpha|\bar\beta,\ \alpha|\bar\gamma,\ \beta|\bar\gamma,\ \ldots$ may be found. In fact the rule for writing down the desired term is merely to pick out the two elements from the set $\bar\alpha\bar\beta\bar\gamma\alpha\beta\gamma$, leaving the other four to form the scalar coefficients with the proper sign (which may be determined according to the rule for expanding the determinants above) — thus

$$-(\bar\alpha\,\bar\gamma\,\alpha\,\beta)\,\gamma|\bar\beta, \qquad +(\alpha\,\beta\,\alpha\,\beta)\,\gamma|\bar\gamma, \qquad -(\bar\beta\,\bar\gamma\,\alpha\,\gamma)\,\beta|\bar\alpha.$$

The extension to $\Xi^{(k)}$ is now immediate. There is no need of going into the details.

If $\Omega$ is expressed in matricular form as

$$\Omega = \Sigma\,a_{ij}\,\alpha_i\,|\,\alpha'_j, \quad \Omega_2 = \tfrac{1}{2}\,\Sigma\,a_{ij}\,a_{lm}\,\alpha_i\,\alpha_l\,|\,\alpha'_j\,\lambda'_m,\ \ldots$$

it is possible to calculate $\Xi^{(2)},\ \Xi^{(3)},\ \ldots$ also in matricular form. From (9)

$$\Xi^{(2)} = \tfrac{1}{2}\,\Sigma\,a_{ij}\,a_{lm}\,[(\alpha'_m\,\alpha_l)\,\alpha_i\,|\,\alpha'_j + (\alpha'_j\,\alpha_i)\,\alpha_l\,|\,\alpha'_m - (\alpha'_j\,\alpha_l)\,\alpha_i\,|\,\alpha'_m$$
$$- (\alpha'_m\,\alpha_i)\,\alpha_l\,|\,\alpha'_j]$$
$$= \Sigma\,a_{ij}\,a_{lm}\,(\alpha'_m\,\alpha_l)\,\alpha_i\,|\,\alpha'_j - \Sigma\,a_{ij}\,a_{lm}\,(\alpha'_j\,\alpha_l)\,\alpha_i\,|\,\alpha'_m$$

It should be noted that $i \neq l$ and $j \neq m$, and that terms in the first sum vanish unless $l = m$, in which case $\alpha'_m\,\alpha_l = 1$; and similarly in the second sum $j = l$. Hence the coefficient of any dyad $\alpha_i\,|\,\alpha'_j$ in $\Xi^{(2)}$ is

$$\left(a_{ij}\,\underset{l}{\Sigma'}\,a_{ll} - \underset{l}{\Sigma'}\,a_{il}\,a_{lj}\right)\,\alpha_i\,|\,\alpha'_j$$

where the accent on the $\Sigma$ indicates that those terms for which the variable index is equal to either of the fixed indices are to be omitted. These sums may be written in the form

(91')  $\Xi^{(2)} = \underset{i,\,j}{\Sigma}\,\underset{l}{\Sigma'}\begin{vmatrix}a_{ij} & a_{il} \\ a_{lj} & a_{ll}\end{vmatrix}\alpha_i\,|\,\alpha'_j.\quad (l-i)\ (l-j) \neq 0.$

Analogously to (91) the formula for $\Xi^{(3)}$ is

(92)    $\Xi^{(3)} = \Sigma |(\bar{\beta}\,\bar{\gamma}\,\beta\,\gamma)\,\alpha|\bar{\alpha} \;|\; (\bar{\alpha}\,\bar{\gamma}\,\alpha\,\gamma)\,\beta|\beta \;|\; (\alpha\,\beta\,\alpha\,\beta)\,\gamma|\gamma$
$- (\bar{\alpha}\,\bar{\gamma}\,\beta\,\gamma)\,\alpha|\beta \;|- (\alpha\,\beta\,\beta\,\gamma)\,\alpha|\gamma \quad (\beta\,\gamma\,\alpha\,\gamma)\,\beta|\alpha$
$- (\bar{\alpha}\,\bar{\beta}\,\alpha\,\gamma)\,\beta|\gamma + (\bar{\beta}\,\bar{\gamma}\,\alpha\,\beta)\,\gamma|\alpha \; - (\alpha\,\gamma\,\alpha\,\beta)\,\gamma|\beta|.$

And

$$\Omega_3 = \frac{1}{6}\,\Sigma\,a_{i\,j}\,a_{l\,m}\,a_{p\,q}\,\alpha_i\,\alpha_l\,\alpha_p|\,\alpha'_j\,\alpha'_m\,\alpha'_q, \quad \begin{matrix} i,l,p \text{ all unequal} \\ j,m,q \text{ all unequal} \end{matrix}$$

After making the necessary substitutions and reductions, it appears that

(92')
$$\Xi^{(3)} = -\,\Sigma_{i,j}\,\Sigma''_{l,m}\,\begin{vmatrix} a_{i\,j} & a_{i\,l} & a_{i\,m} \\ a_{l\,j} & a_{l\,l} & a_{l\,m} \\ a_{m\,j} & a_{m\,l} & a_{m\,m} \end{vmatrix}\,\alpha_i\,|\,\alpha'_j$$

where the double accent on $\Sigma$ means that $l$ and $m$ cannot be equal among themselves nor equal to $i$ or $j$. Moreover the equal results obtained from $l = a$, $m = b$ and from $l = b$, $m = a$ have been accounted for. The formula for $\Xi^{(k)}$ is the obvious generalisation of the results for $\Xi^{(2)}$ and $\Xi^{(3)}$. The result for $\Xi^{(k)}$ may be stated in words: To find the coefficients of $\alpha_i\,|\,\alpha'_j$ in $\Xi^{(k)}$, form a determinant of the $k$th order from the matrix of $\Omega$ by taking as the main diagonal $a_{i\,j}$ and any combination (not permutation) of $k-1$ of the elements in the main diagonal of $\Omega$ excluding $a_{i\,i}$, $a_{j\,j}$ and add the determinants of all possible combinations.

With the aid of these dyadics it is possible to express the invariants of the product of a dyadic and $I-2\,\sigma|\bar{\sigma}$. Let

(85)                        $X = \Omega\,(I-2\,\sigma\,|\,\bar{\sigma})$

Then

(93)    $X_3 = \Omega_3 - 2\,\bar{\sigma}\,\Omega\,\sigma, \; X_{k\,8} = \Omega_{k\,8} - 2\,\bar{\sigma}\,\Xi^{(k)}\,\sigma, \; k = 2, 3, \ldots, n-1,$
$X_n = -\,\Omega_n$

A similar result could be obtained for the product

(94)                    $X' = \Omega\,(I-2\,\sigma\,\bar{\sigma} - 2\,\tau|\bar{\tau})$

Here however, the expansion by the binomial theorem is

$(I-2\,\sigma|\bar{\sigma} - 2\,\tau|\bar{\tau})_k = I_k - 2\,I_{k-1}\,\underset{\wedge}{\cdot}\,(\sigma_i\bar{\sigma} + \tau|\bar{\tau}) + 4\,I_{k-2}\,\vdots\,\sigma\tau|\bar{\sigma}\bar{\tau};$

and hence

(94')    $X'_{k\,8} = \Omega_{k\,8} - 2\,\bar{\sigma}\,\Xi^{(k)}\,\sigma - 2\,\bar{\tau}\,\Xi^{(k)}\,\tau + 4\,\Omega_{k\,8}\underset{\times}{\times}(I_{k-2}\underset{\times}{\times}\sigma\tau|\bar{\sigma}\bar{\tau})$

The term $\Omega_{k\,8}\underset{\times}{\times}(I_{k-2}\underset{\times}{\times}\sigma\tau|\bar{\sigma}\bar{\tau})$ could be treated as $\Omega_k\,\underset{\times}{\cdot}\,(I_{k-1}\,\underset{\wedge}{\cdot}\,\sigma|\bar{\tau})$ was treated; and the invariant dyadic which resulted would be of the second type, that is, the antecedents would be of the form $\alpha\,\beta$ and the consequents of the form $\bar{\alpha}\,\bar{\beta}$. In like manner for products with more complex square roots of $I$, would yield invariant dyadics of higher types. The study of these dyadics will not be taken up at this time.

The converse problem is interesting, namely, given a dyadic $\Omega$ to find what dyadics may result from the product $\Omega\,(I-2\,\sigma|\bar{\sigma})$ by a suitable choice of the reflection $I-2\,\sigma|\bar{\sigma}$. Consider the scalar invariants of the product. The determinant of the product must be the negative of $\Omega_n$. Suppose it be desired to make the other scalar invariants take assigned arbitrary values. This amounts to the solution of the equations

(96) $\quad\bar{\sigma}\,\Omega\,\sigma = l \qquad\qquad \bar{\sigma}\,(\Omega-lI)\,\sigma = 0$

$\qquad \bar{\sigma}\,\Xi^{(2)}\,\sigma = m \quad$ or $\quad \bar{\sigma}\,(\Xi^{(2)}-mI)\,\sigma = 0 \qquad n-1$ equations

$\qquad \bar{\sigma}\,\Xi^{(3)}\,\sigma = p \qquad\qquad \bar{\sigma}\,(\Xi^{(3)}-pI)\,\sigma = 0$

. . . . . . . . . . . . . . . .

under the condition $\bar{\sigma}\,\sigma = \neq 0$. In other words it amounts to finding a space $\bar{\sigma}$ which shall contain $(\Omega-lI)\,\sigma$, $(\Xi^{(2)}-mI)\,\sigma$, . . . but not contain $\sigma$. Suppose the roots of $\Omega$ are distinct and for simplicity let $n=4$. Then

$$\Omega = a\,\alpha|\alpha' + b\,\beta|\beta' + c\,\gamma|\gamma' + d\,\delta|\delta'$$

(96) $\Xi^{(2)} = a\,(b+c+d)\,\alpha|\alpha' + b\,(c+d+a)\,\beta|\beta' + c\,(d+a+b)\,\gamma|\gamma'$

$\qquad\qquad + d\,(a+b+c)\,\delta|\delta'$

$\Xi^{(3)} = a\,(bc+bd+cd)\,\alpha|\alpha' + b\,(cd+ca+da)\,\beta|\beta'$

$\qquad\qquad + c\,(da+db+ab)\,\gamma|\gamma' + d\,(ab+ac+bc)\,\delta|\delta'$

$$\sigma = x\,\alpha + y\,\beta + y\gamma + w\,\delta$$

The three vectors $(\Omega-lI)\,\sigma$, $(\Xi^{(2)}-mI)\,\sigma$, $(\Xi^{(3)}-pI)\,\sigma$ are easily written down and the desired $\bar{\sigma}$ may be passed through them unless the condition

(97) $\begin{vmatrix} (a-l)\,x & [a\,(b+c+d)-m]\,x & [a\,(bc+bd+cd)-p]\,x & x \\ (b-l)\,y & [b\,(c+d+a)-m]\,y & [b\,(cd+ca+da)-p]\,y & y \\ (c-l)\,z & [c\,(d+a+b)-m]\,z & [c\,(da+db+ab)-p]\,z & z \\ (d-l)\,w & [d\,(a+b+c)-m]\,w & [d\,(ab+ac+bc)-p]\,w & w \end{vmatrix} = 0$

which expresses the fact that they lie in a 3-dimensional space with $\sigma$, holds. This may be reduced to the simpler form

(97') $\begin{vmatrix} a & a\,(b+c+d) & a\,(bc+bd+cd) & 1 \\ b & b\,(c+b+a) & b\,(cd+ca+da) & 1 \\ c & c\,(d+a+b) & c\,(da+db+ab) & 1 \\ d & d\,(a+b+c) & d\,(ab+ac+bc) & 1 \end{vmatrix} = P_6\,(a,b,c,d) = 0$

This is a polynomial of degree 6 in $a$, $b$, $c$, $d$. It is obvious that the polynomial will vanish if any two of the roots are equal. Hence

(97'') $\quad P_6\,(a,b,c,d) = k\,(a-b)\,(a-c)\,(a-d)\,(b-c)\,(b-d)\,(c-d)$,

and according to the supposition that the roots are distinct $P_6 \neq 0$.

From this it follows that if the roots of $\Omega$ are distinct, a reflection, $I-2\,\sigma|\bar{\sigma}$ of type $I$ may be found which will make the product $\Omega\,(I-2\,\sigma|\bar{\sigma})$ take such a form as to have any desired scalar invariants, with the exception of the determinant which is $-\Omega_n$.

Moreover it appears that the choice of $\sigma$ is arbitrary except that it shall not lie in any of the invariant spaces of $n-1$ dimensions. The result is stated for $n$ dimensions because the proof is the same as for four. In particular these scalar invariants may be chosen so as to make the Hamilton-Cayley equation of the product a reciprocal equation with distinct roots, and hence the product is resoluble into two reflections. Geometrically this means that a reflection of the first type may be found in $\infty^{n-1}$ ways such that the product of the reflection and any collineation (or any unimodular proper or improper strain) which has distinct roots is resoluble into two reflections. In other words, $\Omega$ in this case is always resoluble into three reflections, or analytically may be regarded as the product of three square roots of the idemfactor.

18. *Some special cases of the product.*—The theorem which has just been established for the general case where the roots of $\Omega$ are distinct may be extended. It is clear that $\Omega$ may be such that the product $\Omega \, (I-2 \, \sigma | \bar{\sigma})$ cannot have arbitrary scalar invariants: for if $\Omega$ were a reflection the product would have to have such invariants as to make a reciprocal equation. It may, however, be shown that: If the Hamilton-Cayley equation of a dyadic $\Omega$ is the equation of lowest degree, the choice of a reflection $I-2 \, \sigma | \bar{\sigma}$ may be made in $\infty^n$ [1] ways so that the scalar invariants of the product $\Omega \, (I-2 \, \sigma | \bar{\sigma})$ are arbitrary with the exception of the determinant which is $-\Omega_n$. It should be noted that the condition that the Hamilton-Cayley equation be identical with the equation of lowest degree is equivalent to saying that in the canonical form (64) to which the dyadic may be reduced all the shearing terms corresponding to equal roots must be present. If $n$ $\quad$ 4 the possible cases are

(98) $\quad \begin{aligned} \Omega = \ & a \, \alpha \alpha' \\ & + a \, \beta \beta' + \alpha \beta' \\ & + c \, \gamma \gamma' \\ & + d \, \delta \delta' \end{aligned}$ $\qquad \begin{aligned} \Omega = \ & -a \, \alpha \alpha' \\ & + a \, \beta \beta' + \alpha \beta' \\ & + a \, \gamma \gamma' + \beta \gamma' \\ & + d \, \delta \delta' \end{aligned}$ $\qquad \begin{aligned} \Omega = \ & a \, \alpha \alpha' \\ & + a \, \beta \beta' + \alpha \beta' \\ & + a \, \gamma \gamma' + \beta \gamma' \\ & + a \, \delta \delta' + \gamma \delta' \end{aligned}$

$\qquad\qquad \begin{aligned} \Omega = \ & a \, \alpha \alpha' \\ & + a \, \beta \beta' + \alpha \beta' \\ & + c \, \gamma \gamma' \\ & + c \, \delta \delta' + \gamma \delta' \end{aligned}$ $\qquad$ (The vertical bar has been omitted for brevity.)

As the proof of theorem in general involves great detail, and at the same time general reasoning on tolerably varied and involved formulas, it will be well to carry the computation through in these cases, after which the general cases will offer no particular difficulty. The expressions for $\Xi^{(2)}$ and $\Xi^{(3)}$ in each of the cases above are

$\Xi^{(2)} = a(a+c+d)\,\alpha\alpha'$
$\qquad + a(a+c+d)\beta\beta' + (c+d)\alpha\beta'$
$\qquad + c(a+a+d)\gamma\gamma'$
$\qquad + d(a+a+c)\,\delta\delta'$

$\Xi^{(3)} = a(ac+ad+cd)\alpha\alpha$
$\qquad + a(ac+ad+cd)\beta\beta' + cd\,\alpha\beta'$
$\qquad + c(ad+ad+aa)\gamma\gamma'$
$\qquad + d(ac+ac+aa)\delta\delta'$

$\Xi^{(2)}\,a(2a+d)\,\alpha\alpha'$
$\quad a(2a+d)\,\beta\beta' + (a+d)\alpha\beta'$
$\quad a(2a+d)\,\gamma\gamma' + (a+d)\beta\gamma' - 1\,\alpha\gamma'$
$\quad d(2a+a)\,\delta\delta'$

$\Xi^{(3)} = a(a^2+2ad)\alpha\alpha'$
$\qquad + a(a^2+2ad)\beta\beta' + ad\,\alpha\beta'$
$\qquad + a(a^2+2ad)\gamma\gamma' + ad\,\beta\gamma' - d\alpha\gamma'$
$\qquad + d(a^2+2a^2)\,\delta\delta'$

$\Xi^{(2)} = a(3a)\,\alpha\alpha'$
$\qquad + a(3a)\,\beta\beta' + 2a\,\alpha\beta' - \alpha\gamma'$
$\qquad + a(3a)\,\gamma\gamma' + 2a\beta\gamma' - \beta\delta'$
$\qquad + a(3a)\,\delta\delta' + 2a\gamma\delta'$

$\Xi^{(3)} = a(3a^2)\,\alpha\alpha'$
$\qquad + a(3a^2)\,\beta\beta' + a^2\,\alpha\beta'$
$\qquad + a(3a^2)\,\gamma\gamma' + a^2\beta\gamma' - a\alpha\gamma'$
$\qquad + a(3a^2)\,\delta\delta' + a^2\gamma\delta' - a\gamma\delta' + a\delta'$

$\Xi^{(2)} = a(a+2c)\,\alpha\alpha'$
$\qquad + a(a+2c)\,\beta\beta' + 2c\,\alpha\beta'$
$\qquad + c(c+2a)\,\gamma\gamma'$
$\qquad + c(c+2a)\,\delta\delta' + 2a\gamma\delta'$

$\Xi^{(3)} = a(2ac+c^2)\,\alpha\alpha'$
$\qquad + a(2ac+c^2)\,\beta\beta' + c^2\alpha\beta'$
$\qquad + c(2ac+a^2)\,\gamma\gamma'$
$\qquad + c(2ac+a^2)\,\delta\delta' + a^2\gamma\delta'$

In the first case the condition that the solution desired be impossible is

$$\begin{vmatrix} (a-l)x+y[a(a+c+d)-m]x+(c+d)y[d(ac+ad+cd)-p]x+cdy & x \\ (a-l)y & [a(a+c+d)-m]y & [a(ac+ad+cd)-p]y & y \\ (c-l)z & [c(a+a+d)-m]z & [c(ad+ad+aa)-p]z & z \\ (d-l)w & [d(a+a+c)-m]w & [d(ac+ac+aa)-p]w & w \end{vmatrix} = 0$$

Here the last column may be multiplied by $l$, $m$, $p$, and subtracted from the first, second, third columns respectively, and thus the $l$, $m$, $p$ disappear; the $w$, $z$, $y$ of the last three lines may be canceled out; the second line may be multiplied by $x$ and substracted from the first and then the $y$ in that line disappears. Finally the second line may be subtracted from each of the last two. The condition reduces to

(99′)
$$\begin{vmatrix} 1 & c+d & cd & 0 \\ a & a(a+c+d) & a(ac+ad+cd) & 1 \\ c-a & (c-a)(a+d) & (c-a)ad & 0 \\ d-a & (d-a)(a+c) & (d-a)cd & 0 \end{vmatrix} = 0.$$

Here it still is clear that even if the factors $c-a$ and $d-a$ are stricken out the expression will vanish if $a$ equals $c$ or $d$ or if $c$ and $d$ are equal. As the expression is only of degree five, it must be of the form

(99″)
$$P_5\,(a,\ c,\ d) \equiv k\,(c-a)^2\,(d-a)^2\,(c-d).$$

Hence $P_5$ does not vanish unless two of the roots $a$, $c$, $d$ are equal.

The second case is between the first and third. In this case it turns out that the polynomial to which the determinant reduces is

$$P_3 (a, d) \quad k (d - a)^3$$

and cannot vanish unless $a$ and $d$ are equal. In the third case the determinant is

$$(100) \quad \begin{vmatrix} ax+y & 3a^2x + 2ay - z & 3a^3x + a^2y - az + w & x \\ ax + z & 3a^2y + 2az - w & 3a^3y + a^2z - aw & y \\ az + w & 3a^2z + 2aw & 3a^3z + a^2w & z \\ aw & 3aw & 3a^3w & w \end{vmatrix} = 0,$$

where the $l$, $m$, $p$ have been omitted as they obviously go out by the same reasoning as before. Here the $w$ factors out of the last row, which may then be multiplied by $x$, $y$, $z$ and subtracted respectively from the first, second, third rows. The $w$ then factors out of the third row, which may be multiplied by $y$, $z$ and subtracted from the rows above. Now the factor $w$ drops out of the second row, which may be multiplied by $z$ and subtracted from the top row whereupon the $y$ drops out. The condition is reduced to

$$(100') \quad \begin{vmatrix} 0 & 0 & 1 & 0 \\ 0 & -1 & -a & 0 \\ 1 & 2a & a^2 & 0 \\ a & 3a^2 & 3a^3 & 1 \end{vmatrix} = 1 = 0,$$

which is clearly unfulfilled.

The last case is instructive, because it illustrates the dependence and independence of different repeated roots. The condition is

$$(101) \quad \begin{vmatrix} ax+y & a(a+2c)x - c^2y & a(2ac+c^2)x + c^2y & x \\ ay & a(a+2c)y & a(2ac+c^2)y & y \\ cx+w & c(c+2a)z + a^2w & c(2ac+a^2)z + a^2w & z \\ cw & c(c+2a)w & c(2ac+a^2)w & w \end{vmatrix} = 0$$

The $y$, $w$ of the second and third lines go out, and a reduction similar to that given before removes the $x$, $z$. It is this possibility to get rid of the coefficients in the expression for $\sigma$, which shows that these coefficients may have any values other than 0. The condition reduces to

$$(101') \quad \begin{vmatrix} 1 & -2c & c^2 & 0 \\ a & a(a+2c) & a(2ac+c^2) & 1 \\ 1 & -2a & a^2 & 0 \\ c & c(c+2a) & c(2ac+a^2) & 1 \end{vmatrix} = 0$$

which amounts to merely

$$(101'') \qquad P_4 (a, c) = k (a-c)^4$$

and cannot vanish.

The foregoing cases are typical of all that can arise. With regard to the case where $n$ has any value the following remarks will suffice. Suppose that $\Omega$ has $k$ repeated roots $a$. Construct the half square upon the portion of the main diagonal of $\Omega$ correspond-

ing to these roots. The shearing terms will be $k-1$ in number and will be situated in the next diagonal line. In constructing $\varXi^{(2)}$ there will be shearing terms in the corresponding line. Moreover the next diagonal line will contain terms to the full number $k-2$: but there will be no other terms in the half square. In construction $\varXi^{(3)}$ there will be terms in these two lines and also in the line third removed from the main diagonal. And so on until $\varXi^{(k-1)}$ has a term in the corner of the half-square. Such is the case for every set of repeated roots. All the other terms will be lacking in $\varXi^{(2)}$, $\varXi^{(3)}$, . . . . These results are all obvious consequences of the determinantal definition of the coefficients in $\varXi^{(2)}$, $\varXi^{(3)}$, . . . . The determinant which must not vanish if the solution of the problem is possible will reduce to the discriminant of the roots of $\varOmega$, where however the differences which correspond to repetitions of the same root have disappeared. This is the only change: for the differences which correspond to different repeated roots occur as many times as the product of the multiplicities of those roots.

The proof of these general theorems is carried out by mathematical induction. It is merely necessary to show that, on the assumption that the results are true for any given distribution of roots, they still remain true when the number of roots and the number of dimensions is increased by one, whether by adding a root equal to one already existing or different from all those present. In any given case the proof is very simple; but on the assumption that there are $k$ roots of multiplicities $m_1$, $m_2$, . . . , $m_k$ the notation becomes very cumbersome. As there is no other difficulty than this, it seems hardly worth while to insert the general proof at this point. The geometrical consequences of the theorem of this article are: That any collineation or strain of which the Hamilton-Cayley equation is the equation of lowest degree may be converted by multiplication with a reflection $1-\sigma|\bar\sigma$, which may be chosen in $\infty^{n-1}$ ways, into a collineation or strain which has roots arbitrary except that their product must be the negative of the product of the roots of the given collineation or strain. In particular these roots may be chosen in such a way that the resulting collineation or (unimodular) strain may be resolved into two reflections.

19. *On the product of a strain and a reflection.*—Although it is evident that no reflection of type 1, nor any reflection of any type can be found which will make the scalar invariants of the product of any given unimodular strain and that reflection arbitrary, and

that therefore the method adopted in the last two articles for showing that such a strain may be resolved into three reflections must break down in some of the special cases (for instance when the given strain is itself a reflection), the theorem that any strain is resoluble into three properly chosen reflections is strongly suggested. If $\Omega$ be a strain and $\Phi$ a reflection, the necessary and sufficient condition that $\Omega \Phi$ be resoluble into two reflections is seen from article 15 to be that the Hamilton-Cayley equation of $\Omega \Phi$ shall be reciprocal, and that the invariant numbers which correspond to a pair of reciprocal roots of the scalar equation shall be equal. The first part of this condition is not hard to state and, in the simplest cases, to examine. The last part of the condition apparently requires very detailed consideration.

For the present purposes the fourfold division of the problem, according as $n$ is odd or even and $\Omega_n$ is $+1$ or $-1$, may be somewhat abridged by the use of (70). If $X = \Omega \Phi$, the conditions become

$$(102) \qquad X_{ks} = X_{ks}^{-1}, \quad k < E\left(\frac{n}{2}\right), \quad X_{\frac{n}{2}s} = 0$$

with the supplementary condition necessary only when $n$ is even and the determinant of the product $X_n = -1$, and with the further condition that the invariant numbers which correspond to any pair of reciprocal roots of $X$ must be equal. These are the necessary and sufficient conditions that $\Omega$ be resoluble into three reflections. In case $\Phi$ happens to be of type 1, these conditions reduce to

$$(102') \qquad \sigma\left[\Omega - \Omega^{-1} - \tfrac{1}{2}\left(\Omega_s - \Omega_s^{-1}\right)I\right] = 0,$$

$$\sigma\left[\Xi^{(k)} - \Xi^{(-k)} - \tfrac{1}{2}\left(\Omega_{ks} - \Omega_{ks}^{-1}\right)I\right]\sigma = 0, \quad 1 < k < E\left(\frac{n}{2}\right),$$

and if $n$ is even and $X_n = -1$, $\quad \bar{\sigma}\left[\Xi^{\left(\frac{n}{2}\right)} - \tfrac{1}{2}\Omega_{\frac{n}{2}s}I\right]\sigma = 0$,

where $\Xi^{(-k)}$ has been written as an abbreviation for the $k$th invariant dyadic $\Xi$ associated with $\Omega^{-1}$. Thus there are $E\left(\frac{n}{2}\right)$ equations to be satisfied in case $n$ is odd, or in case $n$ is even and $X_n = -1$; but if $n$ is even and $X_n = +1$ there are only $E\left(\frac{n}{2}\right) - 1$ equations to be fulfilled. In all cases they must be satisfied subject to the restriction $\sigma\sigma = \neq 0$. If $\Phi$ were of type 2, or higher up to type $E\left(\frac{n}{2}\right)$, the conditions which would be analogous to (102') might be expressed in terms of the invariant dyadics of higher class referred to in article 17 but not investigated.

The connection of the conditions (102') with work which has already been accomplished is this. In case $n - 2$, the only reflection is of type 1 and if $\Omega_n = -1$, $X_n = +1$, there is no condition to be fulfilled. Hence the transformation $\Omega$ may always be written as the product of three reflections. If $\Omega_n = +1$, the Hamilton-Cayley equation of $\Omega$ is necessarily reciprocal and $\Omega$ is resoluble into two reflections. These results are well known; and as far as collineations are concerned there is no distinction between the two cases except as regards reality. If $n = 3$, there is only the one reflection, which is of type one, (except for its negative) and only one condition — which may always be satisfied.[1] Hence in three dimensions the resolution into three reflections is always possible. In case $n - 4$ and $\Phi$ is of type 1 and $\Omega_n = -1$, there is still only one condition (102') to be satisfied, and it can clearly be satisfied: but another difficulty arises owing to the fact that if $a$, $\dfrac{1}{a}$ are double roots of the product $X$, it may conceivably arise that for all reflections $\Phi$ which satisfy the condition there may be a shearing term for one of the roots and none for the other, so that the supplementary condition concerning the invariant numbers would not be fulfilled. In a delicate question of this sort a count of constants is of no value; a detailed investigation of the product $X$ is required. Whereas if $\Omega_n = +1$, there are two conditions (102') to be satisfied simultaneously, and in view of the developments of article 47 it is by no means evident that this may always be accomplished. If $\Phi$ is of type 2 and $\Omega_n = -1$, there are again two conditions (102) to satisfy, not to mention the conditions imposed by the invariant numbers, and again it is not obvious that they can be met. If however $\Omega_n = +1$, Smith's theorem previously cited, and arising out of the special fact that a collineation in four homogeneous variables may be regarded as a collineation in six variables with an invariant quadratic form, may be adduced to show that all the conditions (102) may be satisfied. If $n > 4$, the difficulties signalised for the first three cases when $n = 4$, are further emphasized.

To show that these difficulties are not only conceivable but actually arise, it is worth while to treat the simplest case. Suppose
(103)  $\Omega = a\,\alpha\,|\,\alpha' + a\,\beta\,|\,\beta' + a\,\gamma\,|\,\gamma' + a^{-3}\,\delta\,|\,\delta'$, $\Omega_n = +1$, $\Phi = I - 2\,\bar{\sigma}\,|\,\sigma$
Here there are two conditions so satisfy, namely
(104)  $\bar{\sigma}\,[\Omega - \Omega^{-1} - \tfrac{1}{4}(\Omega_s - \Omega_s^{-1})\,I]\,\sigma = 0$,  $\bar{\sigma}\,[\Xi^{(2)} - \tfrac{1}{4}\,\Omega_{2*}\,I]\,\sigma = 0$.

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[1] This is precisely the condition of my theorem 23, p. 295, of my communication to the Transactions cited in the footnote on page 32.

As $\Phi$ possesses six degrees of freedom, a count of constants would indicate that the two conditions could be satisfied. The actual computation is conducted as follows.[1]

$$\Omega^{-1} = a^{-1}\,\alpha\,|\,\alpha' + a^{-1}\,\beta\,|\,\beta' + a\,\gamma\,|\,\gamma' + a^3\,\delta\,|\,\delta'$$

$$\Omega_s = 3\,a + a^{-3}, \quad \Omega_s^{-1} = 3\,a^{-1} + a^3, \quad \Omega_{2\,s} = \Omega_{2\,s}^{-1} - 3a^2 + 3\,a^{-2},$$

$$\Xi^{(2)} = a\,(2a + a^{-3})\,\alpha\,|\,\alpha' + a\,(2a + a^{-3})\,\beta\,|\,\beta' + a\,(2a + a^{-3})\,\gamma\,|\,\gamma' + 3a^{-2}\,\delta\,|\,\delta',$$

$$\Xi^{(2)} - \tfrac{1}{2}\,\Omega_2\,{}_s\,I = \tfrac{1}{2}\,(a^2 - a^{-2})\,[\alpha\,|\,\alpha' + \beta\,|\,\beta' + \gamma\,|\,\gamma' - 3\,\delta\,|\,\delta']$$

$$\Omega - \Omega^{-1} - \tfrac{1}{2}\,(\Omega_s - \Omega_s^{-1})\,I - \tfrac{1}{2}\,(a - a^{-1})\,[(a + a^{-1})^2\,(\alpha\,|\,\alpha' + \beta\,|\,\beta' + \gamma\,|\,\gamma') \\ - (a^2 + 4 + a^{-2})\,\delta\,|\,\delta']$$

From these expressions. it is found that $|\Omega - \Omega^{-1} - \tfrac{1}{2}\,(\Omega_s - \Omega_s^{-1})\,I\,|\,\sigma$ and $[\Xi^{(2)} - \tfrac{1}{2}\,\Omega_2\,{}_s\,I]\sigma$ are $\tfrac{1}{2}\,(a - a^{-1})\,[A\,x\,\alpha + A\,y\,\beta + A\,z\,\gamma - A'w\,\delta)$ and $\tfrac{1}{2}\,(a^2 - a^{-2})\,[x\,\alpha + y\,\beta + z\,\gamma - 3\,w\,\delta]$ where $A = (a + a^{-1})^2$, $A' = a^2 + 4 + a^{-2}$, $\sigma = x\,\alpha + y\,\beta + z\,\gamma + w\,\delta$. Neither of these vectors vanishes identically unless $a - a^{-1} = 0$ or $a^2 - a^{-2} = 0$, that is, unless $a$ is a square root or fourth root of unity. From the form of the vectors it is clear that they cannot be collinear unless

$$\begin{vmatrix} A & A' \\ 1 & 3 \end{vmatrix} = \begin{vmatrix} a^2 + 2\,a^{-2} & a^2 + 4 + a^{-2} \\ 1 & 3 \end{vmatrix} = 2\,(a^2 + 1 + a^{-2}) = 0 \text{ or } w = 0.$$

In the first case $a$ must be a sixth root of unity, and in the second the two vectors are both parallel to $\sigma$. As the constant $a$ in $\Omega$ may be arbitrary, the first case can be excluded, and the second violates the condition $\bar{\sigma}\sigma = |= 0$. Hence it may be assumed that the two vectors are independent and determine a plane through which any $\bar{\sigma}$ must pass. But this plane clearly contains $\sigma$ inasmuch as any three rowed determinant from the matrix

$$\begin{Vmatrix} Ax & Ay & Az & A'w \\ x & y & z & 3\,w \\ x & y & z & w \end{Vmatrix}$$

vanishes. Hence again the condition $\bar{\sigma}\sigma = |= 0$ is violated, and it is evident that despite the six degrees of freedom, no reflection $\Phi = I - 2\,\bar{\sigma}\,|\,\sigma$ can be chosen such that the conditions (104) may be fulfilled, and $\Omega\Phi$ may (possibly) be resoluble with two reflections. It is necessary to try a different type of reflection. As a matter of fact Smith's theorem happens to be applicable to this particular case.

The detailed discussion of the various difficulties which arise in the different special cases must be postponed to a later time. There is one question which will be suggested and left as an easy exercise in the use of the dyadics $\Xi$. It is geometrically apparent that, if

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[1] It may be noted that in the fourth line down $X_{2\,s}^{-1} = \Xi^{(2)} - \tfrac{1}{2}\,\Omega_{2\,s}^{-1}\,I$ would be the negative of the given value as it should be.

$\Omega$ is itself resoluble into two reflections, then it must be possible to find a reflection $\Phi$ of type 1 such that the product $\Omega \Phi$ is still resoluble into two reflections, and consequently the conditions (102') must be capable of fulfilment in this case.

The results of the second part of this paper may be summarised as follows:

1⁰. The determination of the square roots of the idemfactor by means of the properties of the equation of least degree.

2⁰. The determination of the necessary and sufficient conditions that a dyadic be resoluble into the product of two square roots of the idemfactor.

3⁰. The correlation of these results with the theory of reflections in connection with unimodular strains and with collineations. And in particular, the fact that not all products of two reflections leave a non-degenerate quadric invariant.

4⁰. The introduction of invariant dyadics $\Xi$ and their application to the problem of finding the scalar invariants of the product $\Omega \Phi$.

5⁰. The fact that in case the Hamilton-Cayley equation of $\Omega \Phi$ is the equation of lowest degree, there may be found a $\Phi = I - 2 \bar{\sigma} | \sigma$ such that the scalar invariants of the product are arbitrary. The corollary that in such cases, if $\Omega_n = \pm 1$, $\Omega$ be written as the product of three square roots of the idemfactor with the appropriate interpretation in the theory of reflections.

6⁰. The determination of the necessary and sufficient conditions that a dyadic be resoluble into the product of three square roots of the. idemfactor, with an example to show that it is not always possible to take a square root of type 1 as the first of the three

Massachusetts Institute of Technology,
Boston, Mass., December, 1907.

# The Morphology of Ruppia Maritima

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## PLATES I -XV.

# INTRODUCTION

Of late years there has been manifested a tendency toward a systematic investigation of the internal structure of plant species, as contrasted with the more superficial description of external characters by systematists. For although, as stated by Campbell (1897), a description of external characters "is usually quite sufficient for the mere identification of a plant, and for determining its relation to nearly allied forms, it is quite inadequate for settling questions of relationship between more remote groups, and especially those of obscure affinities."

This movement has been directed especially to the study of the origin and development of the sexual generation in Spermatophytes and the problems of embryology in this group, resulting in an overwhelming mass of literature on these subjects.

But, in the meantime, it is commendable that some botanists have also directed their attention to a study of the internal structure of the vegetative organs; for it is only by a complete account of the development and structure of the whole plant, together with its life history, that we can hope to acquire sufficient knowledge for the solution of one of the most interesting and vitally important problems in all plant morphology—i. e., the interrelationship of the various plant groups.

Some of the recent works of Campbell (1897, 1898) are most noteworthy in presenting in this way a connected account of several species, especially certain little understood monocotyledons.

The present work, extending over a period of five years, was undertaken with a similar purpose; and seeks to comprehend, as far as possible, a connected account of the development and structure of the plant organs, together with the life history, of one of the simpler monocotyledons. *Ruppia maritima.*

The Potamogetonaceae of Ascherson (1889) to which the genus Ruppia belongs, comprise an interesting family of remarkably simple plants. As to whether this simple structure represents a primitive or a reduced condition is an extremely important question, but in the present state of our knowledge a clear, unassailable verdict on either side is impossible. A discussion of this point will, however, be postponed for the present.

A few remarks may be in order here as to the literature which relates to the subject of this paper, and which I have found most helpful in its preparation. Ascherson (1889), in Engler and Prantl's Natürliche Pflanzenfamilien, cites the most important references bearing on the Potamogetonaceae up to the date of his contribution. Among these, the papers on Althenia by Prillieux (1864), Cymodocea by Bornet (1864), and Zostera by Grönland (1851) are valuable for their contributions to the morphology of these allied plants. In this respect, also, the works of Irmisch (1851 and 1858) are remarkable for thĕ accuracy with which they describe the external morphological relations of *R. rostellata* and other Potamogetonaceae. Of later works, which are especially useful in a comparative morphological study of the vegetative organs, are, among others, those of Sauvageau (1891, II), Campbell (1897) and Goebel (1898). As to ecological considerations pertaining to water plants, the work of Schenck (1886) deserves especial mention. In the preparation of the parts on the reproductive organs, embryo, &c., the works cited are too numerous to mention here, and the reader is referred to the bibliography (p. 165).

If we except a rather doubtful reference of Hofmeister, (1861, Figures 1–7, Pl. II, and see p. 148 of this paper), the only investigation of *Ruppia maritima* which has ever been published, so far as I can ascertain, is that of Roze (1894), who describes chiefly the conditions of fertilization, but also presents a valuable historical review of the whole genus. Others, it is true, such as Miss Scott (1906) and Chrysler (1907), have dealt with special features of the plant, in the course of their comparative morphological investigations.

*Ruppia rostellata*, on the other hand, if we piece together the results of various observers, has been pretty thoroughly worked out. First Irmisch (1851 and 1858) described carefully its external morphological characters; next, Wille (1883) studied the development of the embryo, and quite recently Murbeck (1902) published an admirable paper on the reproductive organs and embryo.

The genus Ruppia is aquatic, characterized by its long, linear, grass-like leaves with basal sheaths, and grows in brackish and salt water (but probably never salt water of normal ocean strength, see p. 124), in creeks and bays along the coast and in the neighborhood of inland salt springs, throughout all parts of the temperate and tropical zones. Except at low tide it is completely submerged until the period of flowering, when it produces its flowers a short distance (1–3 cm.) above the surface of the water. The flowers, always two to each peduncle, are borne one above the other, on

opposite sides of the rhachis, the latter being homologous to a spadix, but not at all fleshy. The flowers are naked and consist of two practically sessile anthers, each with its two large sacs or thecae separate and arranged transversely on the rhachis, making a diamond-shaped arrangement, in the center of which is a group of pistils, always four in number in the specimens I have examined (Pl. IX, fig. 49).[1]   After the shedding of the pollen, the pistils if fertilized develop a stipe or pedicel of considerable length (Pl. IV, fig. 18).   The peduncle, or floral axis, however, elongates whether fertilization is consummated or not.   After fertilization the fruit is drawn below the surface of the water, the elongated peduncle usually coiling up to aid in this process (see p. 85).

In company with most of the submerged plants, Ruppia is perennial. The majority of the leaves and stems die at the approach of cold weather, leaving the living rootstock buried in the mud. Some green leaves, however, remain, connected with the rootstock, and lie at the bottom of the ditch or pool, so that it is possible to collect all of the vegetative organs all winter.

One locality, indeed, is worthy of note, where *Ruppia maritima* flourishes all winter.   Here, possibly on account of springs, the water is tempered somewhat, and I have gathered Ruppia in a green, vigorous condition, when the surface of the ditch was covered with ice over an inch in thickness. Although there are doubtless springs here, the water nevertheless contains a large percentage of salt.   It is quite possible that there are many other similar locations where Ruppia grows all through the winter.

In no case, however, have I found the bulbous winter buds, which have been noted in the Potamogetons by Irmisch (1858).

As indicated by Britton and Brown (1896 vol. I, p. 79), considerable variation exists in *Ruppia maritima.*   Thus, one form which I have noticed especially, is of a more slender habit, with narrower leaves and stems than those of the ordinary individual.   Its branching is ultimately quite irregular, although the system accords with that of the normal form.   This form grows in quiet pools.

There is, moreover, still considerable uncertainty as to the number of existing species of Ruppia (Ascherson, 1889; Sauvageau, 1891, II, p. 209; Roze, 1894, p. 479). The question is such a large one that it cannot be discussed here, and would require, besides, a careful study of herbarium material from different parts of the world. I will only state that as far as I can learn from a study of systematic

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[1] But see Roze, 1894, p. 479.

descriptions and figures (Griffith, 1851, I–II; Irmisch, 1858; Hille-brand, 1888; Ascherson. 1889; Sauvageau, 1891, II; Hooker, 1894; Roze, 1894; Britton, 1907), four species may be safely recognized; namely, *R. maritima* L., *R. rostellata* Koch, *R. brachypus* Gay, and *R. occidentalis* Wats.

All of the material for the present work was collected in tidal ditches in Fort Hale Park, near New Haven, Conn. Here it grows in abundance, forming large tufts with its grass-like leaves, which follow easily the direction of the current. At extreme low tide a large portion of the plant floats, but at high tide it is completely submerged. The period of flowering commences in June and continues until the severe frosts in autumn, up to which time, even in October, I have gathered flowers in good condition.

The material was killed either in chromacetic acid, Flemming's solution, or Keyser's fluid. The first seemed the best, although the others gave good results. Several stains were used: for general work haematoxylin and erythrosin; for cytological study in the development of the male and female gametophytes, &c., and the embryo, the triple stain was used with excellent results. For the latter investigations Heidenhain's iron haematoxylin was also found to be good. In all, about 20,000 microtome sections were cut of the different parts of the plant. For all of the microtechnique Chamberlain's "Methods in Plant Histology" was found invaluable, and in all preparations the directions given therein were closely followed.

I would like to avail myself of this opportunity to express my gratitude to Professor Alexander W. Evans, Eaton Professor of Botany in Yale University, for the great interest he has taken in the work and for the invaluable criticism and suggestions which he has been ever ready to give. I desire also to thank Professor W. R. Coe, Professor J. W. Toumey, and Dr. A. L. Dean, of Yale University, for their kindness in offering suggestions, material, &c.

# THE MORPHOLOGY OF THE VEGETATIVE ORGANS

## STEM

### A. General Characters

Under the heading of "Stem" may be classed several parts of the plant, all of which are cauline in their morphology and origin, but differ to some extent in their functions and relations to the other parts of the plant. The horizontal axis, which lies prostrate on the soil, bearing the roots and upright shoots, may be termed the *rootstock*;[1] the axis of the upright shoot may for convenience be designated the *stem*; the parts which bear the flowers and fruit we may speak of respectively as *peduncle, rhachis,* and *stipe*.

In general, all these parts are slender, terete, and of a whitish color, although the stem at times assumes a greenish tinge due to a small content of chlorophyll.

I shall consider first the branching and anatomy of the stem and rootstock; next, the development of the stem structures which are connected with the production of flowers and fruit, comparing their anatomy with that of the first two structures.

### B. Branching

1. Branching in *Ruppia maritima*.

Two principal systems of branching occur in Ruppia : one in stem and rootstock and connected with the ordinary growth, which we may call the vegetative branch system; and the other in the stem only, and associated with the production of flowers. which we may therefore designate the inflorescent branch system.

a. Vegetative Branch System.

The vegetative branch system is a distichous monopodium, the branches being borne alternately on opposite sides of the axis and in the same plane. In the rootstock, indeed, except near its growing point, this initial arrangement is generally later much obscured, due to the upward growth of many of the lateral branches (Pl. VII,

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[1] As will readily be seen—a point which will be brought out more clearly later (p. 82)—the rootstock does not differ from the stem from a morphological point of view, either externally or internally; for any stem, by becoming horizontal and producing roots at the nodes, assumes the character of what I have termed the rootstock.

fig. 36). In the case of the stem, however, the natural arrangement often remains apparent (Pl. I, fig. 2; Pl. II, fig. 6).

On account of this distichous system, it is possible to cut a longitudinal section through all of the branches and the stem at the same time, especially if the region of the growing point is selected, as is represented in Pl. I, fig. 1 (cf. Pl. VI, fig. 25).

A study of this figure of a vigorously developing stem apex will show clearly the order of development and the orientation of the branches. Since the origin of the branches is inseparably connected with the leaf development, it will be necessary to include in a description of the figure some reference to leaf development.

The apical growing point $G\,P$ has given off in alternate succession the primary leaves $L^I$, $L^{II}$, $L^{III}$, $L^{IV}$, &c. A secondary growing point arises in the axil of each of these leaves, which develops leaves in the same manner. The first leaf, however, is a scale leaf, which will be discussed at length later. (See p. 99.)

Since $L^I$, $L^{II}$, $L^{III}$, $L^{IV}$, &c. are developed in acropetal succession and hence show equal gradations in age, a study of their successive axillary structures presents a clear idea of the manner of development of the branch. The youngest leaves, $L^{VIII}$ and $L^{VII}$ are hardly more than protuberances from opposite sides of the growing point, and as yet reveal no structures in their axils. At $L^{VI}$ we first see a small axillary growing point—the beginning of a branch. In the axil of $L^V$ the rudimentary scale leaf, $v\,s\,l$, appears on the left of the growing point $g\,p^{(V)}$. At the base of $L^{IV}$ we find the scale leaf $(v\,s\,l)$ in its normal position, and also on the left of the growing point point the first vegetative leaf of the branch, $l_1^{(IV)}$. The axils of $L^{III}$ and $L^{II}$ show successively advanced stages until in $L^I$ appear four leaves of the branch besides the scale leaf;—$l_1^{(I)}$, $l_2^{(I)}$, $l_3^{(I)}$ and $l_4^{(I)}$. Moreover in the axils of $l_1^{(I)}$ and $l_2^{(I)}$ appear young growing points to form branches of a second order.

Such a growing point under ordinary conditions will develop in a regular way for some time, the leaves and axillary shoots appearing in acropetal succession and the nodes of increasing length basipetally, according to their age. This stage is represented in Pl. I, fig. 2, a sketch from a living specimen.

Sooner or later, however, in the course of the development of the system, its regularity becomes more and more modified to suit environmental relations, a condition which is of course true of any branch system. Pl. II, fig. 6 is a drawing from nature of the upper portion of a plant whose vegetative branch system has undergone

considerable modification in this way. The period of blooming is at hand, and although no flowers have yet appeared, we find flower buds at the ends of the lateral branches as well as at the termination of the main shoot itself.

Some of the modifications here exhibited are the varying orientation of nodes and internodes—an arrangement evidently brought about to obtain the most advantageous light, space, &c.;—the irregularity in length of internodes, those at the base of the plant being not necessarily the longest; and the number of leaves borne on the branches previous to the development of flowers, for since the latter must bloom above the surface of the water, the lowest branches must necessarily be longer and hence bear more leaves.

In its development of flower buds, the case is a good illustration of the manner of change from the vegetative branching to the inflorescent type, as described below.

b. Inflorescent Branch System.

Irmisch (1851) has accurately described the inflorescent type of branching in *R. rostellata*, but for the sake of completeness and confirmation in *R. maritima*, I will review the more important points.

When the upright or upward growing branches arrive at or near the surface of the water, their apices terminate in inflorescences, as shown diagrammatically in Text-fig. 1, consisting of a slender peduncle and a rhachis homologous to a spadix, the latter bearing always two flowers.
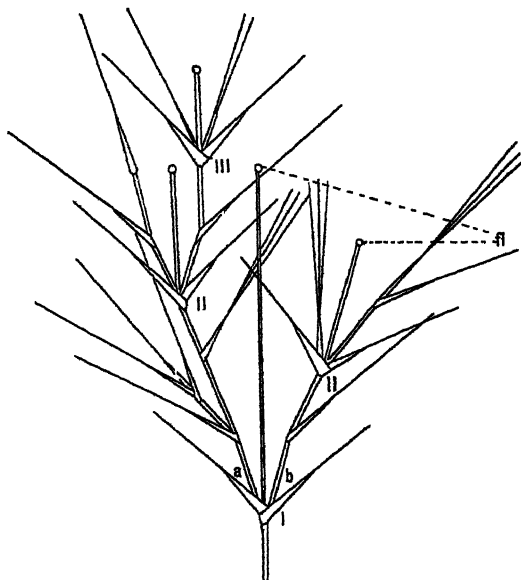


Figure 1.—Diagrammatic representation of inflorescent branch system; *a* and *b*, stronger and weaker shoots respectively; *I*, *II*, and *III*, successive generations of sympodial development; *fl*, flower.

From now on the inflorescent type of branching prevails. For from the axil of each of the two nearly opposite leaves subtending the peduncle (see p. 97 ff.) arises a secondary branch (Text-

fig. 1, *a* and *b*), which continues the growth of the stem. Of these
branches, that in the axil of the upper leaf (Text-fig. 1, *a*) is invariably
of stronger growth, exceeding considerably the branch on the other
side of the floral axis. These axillary branches, after forming a few
leaves, terminate in flowers again, and the process of branch form-
ation is repeated. In this way, since the main axis is continued
by the stronger branch, the system in the region of the flowers is
sympodial. In most instances, however, the weaker axillary branch
has a fairly vigorous development, as in Text-fig. 1, in which case
the branching approximates a false dichotomy, and a characteristic
fan-like form is thus often produced.

In Text-fig. 1 the branch *a* develops two ordinary leaves before
it terminates in a flower. This brings the upper of the pair of sub-
floral leaves at the right side instead of the left, as at 1. Such
an arrangement as this is not as common as is the continual pro-
duction of the stronger branches on the same side of the stem,
but in this way also the fan-like form may be attained.

## 2. Comparative Study of the Branching of the Potamogetonaceae.

In Zannichellia, Campbell (1897, pp. 38 ff.) finds that the apex of
the stem divides into two equal parts, one of which develops into
the female inflorescence, while the other continues as the main
axis of the stem. Farther on he says "the inflorescence is the
result of the dichotomy of the main shoot, whose other member
continues the growth of the axis." So that, although a sympodium
is the ultimate result, it is attained by a dichotomy of the growing
point, one of the branches becoming a male or female flower.
Schumann's (1892) interpretation had been somewhat different from
this, maintaining that the female inflorescence terminated the main
axis and that the growth of the shoot was continued by an axillary
branch arising in the upper of the two subtending leaves. The
position of these leaves much resembles that of the subfloral leaves
of Ruppia, so that if Schumann's idea were correct, the inflorescent
branch system of Zannichellia would be a sympodium similar in
origin and development to that of Ruppia.

Campbell's figures, however, show a vigorous, simultaneous de-
velopment of the primordia of both flower and main axis, which
lend to his theory a great deal of weight. In Ruppia the primordium
of the axillary shoot arises much later than that of the flower, so
that such a dichotomy as Campbell claims is here impossible.
(See Pl. IX, figs. 54—56.)

Irmisch 1858) investigated the vegetative branching of Zannichellia and found it to be a pure sympodium as regards its horizontal axis. This develops beyond its second leaf into an upright shoot, which ultimately bears the flowers; at its base, however, from the axil of the second leaf, arises a branch which prolongs the horizontal axis. This again, after two leaves are given off, develops into an upright shoot and the horizontal axis is again repeated from a branch in the axil of the second leaf. This process goes on indefinitely, so that the horizontal axis is therefore merely a succession of generations of axillary branches and hence a pure sympodium.

The branching of Naias is peculiar, according to Campbell's (1897) interpretation. The leaves are developed approximately in pairs—one slightly below the other and larger. From the axil of this lower one a primordium arises which by dichotomy produces a flower and a lateral branch. The branch bears at its base a single leaf. The upper leaf of the original pair is sterile, so that the stem of Naias has its leaves apparently in whorls of three, with a branch and flower arising from each whorl. Apparently the apex of the main stem never terminates its growth, but goes on producing its pairs of leaves—one sterile and one fertile—so that the system is very unlike that of Ruppia.

Cymodocea, an entirely submerged marine genus, distinguished by its long ribbon-like leaves, very simple flowers and filamentous pollen, is placed by Ascherson (1889) next in order to Ruppia. In *Cymodocea nodosa*, Bornet (1864, pp. 15 ff.) describes types of branching which conform to those of Ruppia. The vegetative branching is monopodial, and a longitudinal section of the bud (l..c. Pl. IV, fig " resembles very closely that of Ruppia. The flowers are terminal and the growth in length is continued by a lateral bud. It appears, however, that this bud is not necessarily one of the two buds nearest the flower.

In Zostera the rootstock is also monopodial, the inflorescent system being sympodial (Sauvageau, 1891, 1).

Phyllospadix closely resembles Zostera (Dudley, 1893).

In Althenia (Prillieux, 1864), on the other hand, a genus resembling Zannichellia and found in Europe, Africa and Australia, the vegetative branching is sympodial, much as in Zannichellia. The growth of the rootstock is continued by successive branches from the horizontal axis which in each case itself becomes vertical and bears the flowers. The flowers are unisexual—the male terminating the upright axis or stem. In the axils of the two leaves subtending the male flower arise secondary branches, which bear male or

female flowers, but more often the latter. From these, branches
of the third order arise, and in this manner a complex sympodial
development is the rule.

Irmisch (1858) found that the vegetative branching of *Potamogeton
lucens, natans, crispus, obtusifolius,* and *pectinatus* was purely sym-
podial as to the rootstock, just as it is in Zannichellia and
Althenia. It is very probable that the other Potamogetons have
the same system. As to the floral system the branching is sym-
podial.

Summary. The branching of Ruppia is of two main types:—a
sympodial system occurring in the region of the flowers, and a
monopodial system present in all other parts of the plant.

Zannichellia, Althenia, and Potamogeton have, however, a sym-
podial system in the case of the rootstock, but a purely monopodial
development like that of Ruppia occurs in Cymodocea, Phyllospadix,
and Zostera. With the exception of Zannichellia, where Campbell
finds a true dichotomy, all these genera have a similar inflorescent
branch system—a single or double sympodium formed from branches
in the axils of the two characteristic subfloral leaves, or at any rate
from a lateral bud as in Cymodocea.

# C. Anatomical Structure

## 1. Growing Point.

In a good median section the growing point of the stem reveals
the three divisions of primary tissue more or less clearly marked :'
the whole is covered by a layer of dermatogen; beneath this lies
the periblem, composed of usually one layer and surrounding the
three or four layers of plerome cells.

Text-fig. 2, besides demonstrating these meristematic divisions,
shows an interesting very early stage in the development of the
youngest leaf. Here the periblem on the right has undergone
several divisions preparatory to the formation of the youngest leaf
primordium, which is destined to appear at this region, opposite
the next youngest leaf, $L^{II}$.

On the whole, the arrangement of cells and young lateral or-
gans is very similar to that figured by Douliot (1890) for *Cym-
odocea aequorea.*

## 2. Stem Structures in the Vegetative Region.

### a. Upright Stem.

In its internal structure the stem is remarkable in many respects, but chiefly because of the reduction and consolidation of the vascular system into what may be termed a single axial vascular bundle, if we except the two minute bundles situated in the cortex. The morphogenesis of this reduced structure, as well as the structure of the whole stem as regards its adaptability to its environment, will be considered more in detail later.

The epidermis surrounds a large zone of cortical parenchyma cells with a ring of lacunae in their midst; these cortical cells adjoin an endodermis, which encloses the axial vascular area (Pl. I, figs. 4–5; Pl. III, figs. 7, 8 and 9). The four parts —epidermis, cortex, endodermis and vascular system will now be described in order.

### 1. Epidermis.

The epidermal cells are small in comparison with the cortical parenchyma cells and much smaller than the epidermal cells of the root (Pl. I, fig. 5). Their walls are thin, and yet thicker than the walls of



Figure 2.—Longitudinal section through growing point of stem, showing origin of youngest leaf primordium in the periblem: *d*, dermatogen ; *pb*, periblem ; *pl*, plerome ; *LI*, *LII*, older leaves. × 210.

the interior cells, the free wall being slightly thicker than the others. Occasionally these cells contain a small amount of chlorophyll.

As might be expected from a comparison with other submerged plants already investigated (Schenck, 1886), no stomata occur throughout the plant.

In the epidermis, rather regularly distributed over the stem, but especially abundant in the region of the nodes, are secretion cells, which are, however, more numerous in the leaf, and will be described more in detail there (see p. 90). These cells occur scattered here and there also in the cortex, and some may be seen in the axial vascular bundle.

### (2) Cortex.

The typical cortical parenchyma cells are large in comparison with the cells of the epidermis, endodermis and vascular tissue; they are

rounded so that intercellular spaces often occur, and elongated in the direction of the length of the stem ⟨ Pl. I, fig. 5 ; Pl. III, fig. 7 ; Text-fig. 3⟩. The majority of the cortical parenchyma cells show in an oblique view parallel horizontal stripes or bands on their side walls. A longitudinal section proves that this appearance is due not to bands of thickening, but to slight undulations (Text-fig. 3). These are not necessarily regular in size or distance apart; they may or may not occur all through the cell, and certain cells are apparently entirely without them. Their function is not clear. Similar undulations were observed by Caspary (1858) in several aquatic plants, notably in *Zannichellia palustris*. Prillieux (1864) has also noted them in the stem of *Althenia filiformis* and in the roots of epiphytic orchids.

Figure 3.—Longitudinal section of a portion of cortical parenchyma cells, showing undulations in the side walls. × 210.

Another point of general interest in the stem cortex is the occurrence of starch grains. These, as shown in Pl. I, fig. 5, are quite numerous. Their size evidently increases in the vicinity of the axial and cortical bundles and everywhere at the nodes. Irmisch (1858, p. 35) has found starch abundant in the Potamogetons and has described its occurrence and appearance in considerable detail.

In the cortex appear also the secretion cells noted in the epidermis, that is, they are apparently of the same nature, behaving in general in a similar way. They do not contain starch, as do most of the cortical cells, but are filled with fine granular contents and are especially abundant at the nodes. Bornet (1864, pp. 40–41) has noted cells evidently quite similar in the cortex and vascular tissue of Cymodocea, filled with a " liquide oléagineux, un peu teinte de jaune, tout-à-fait semblable à celui que renferment les cellules épidermiques des anthères."

In describing the cortex we may for convenience divide it into three portions: (a) an outer zone of cells, compactly arranged and bounded on the outside by the epidermis; (b) a central area. with a large portion of its territory occupied by a ring of lacunae or air spaces; and (c) an interior zone of cells limited on the inner side by the endodermis (Pl. I, fig. 5).

(a) *Outer cortex.*

The outer portion of the cortex consists of two or three layers of cells (Pl. I, fig. 5 *oc*; Pl. III, fig. 7). The cells of the outermost

layer are much larger than those of the epidermis, but partake somewhat of the nature of the latter in their slightly thickened walls. Toward the outer part of this cortical region intercellular spaces rarely occur: instead the triangular prismatic areas which the spaces would occupy are filled with thickening and probably supplement the firmness imparted to the stem by the epidermis.

It is in this outer portion of the cortex that two small vascular bundles occur, which may be termed cortical bundles (Pl. I, figs. 4, 5. *cb*). An account of these will be given in connection with the vascular system.

Toward the interior of this zone the cells increase in size, and intercellular spaces begin to appear.

(b) *Middle cortex.*

The most striking feature of the middle area of the cortex is the ring of large air spaces or lacunae (Pl. I, figs. 4, 5, *mc* and *la*; Pl. III, fig. 7). These originate schizogenously by a splitting apart of the walls of adjacent cells when these are in a very young stage, and subsequently they become much enlarged. A ring of from twelve to eighteen is disposed in a fairly symmetrical way in this central part of the cortex. As a rule somewhat elongated radially, they are separated from each other laterally by a single column of cells. In length up and down the stem they extend from node to node, no diaphragms being stretched across as in the leaf lacunae. Communication through the node is maintained by means of small intercellular spaces.

Between the lacunae and the small intercellular spaces occurring near the inner and outer limits of the cortex lie many air spaces of intermediate size, so that although the large lacunae appear quite distinct on account of their large size and fairly regular arrangement, yet gradations exist between them and the small triangular intercellular spaces (Pl. I, fig. 5).

(c) *Inner cortex.*

The interior zone of the cortex, terminating with the endodermis (Pl. I, fig. 5, *end*), is a good deal similar to the outer zone. The cells and intercellular spaces gradually decrease in size, and the latter also in frequency, toward the endodermis, while the cell walls gradually increase in thickness.

A striking peculiarity of the cells of this inner cortical zone is their chlorophyll content, which is very noticeable in freehand sections from fresh stems. It was definitely ascertained that the chlorophyll grains belong to these cells and were not drawn inward from the epidermis by the razor.

(3) Endodermis.

The endodermis is a fairly regular layer of cells surrounding the central vascular region (Pl. I, fig. 3). On the application of concentrated sulphuric acid all the walls of the endodermal cells become beautifully undulated, the radial walls showing a stronger suberization. In many cases the ring of cells outside of the endodermis is also quite strongly suberized and therefore withstands to a considerable extent the action of the sulphuric acid (Text-fig. 4). No U-shaped stripes of thickening appear on the walls, such as were seen by Schenck (1886) in *Potamogeton pectinatus* and other species. Although a banded appearance is produced in a slightly oblique view, a longitudinal section shows that it is due merely to the slightly undulating walls, essentially as has



Figure 4.—Cross section of portion of endodermis and next outer ring of cells treated with concentrated sulphuric acid, showing slightly thickened radial walls. × 400.

been shown in the cortical cells. The radial walls, however, are slightly thickened uniformly.

(4) Vascular System.

One of the most interesting features of the stem is its remarkably simple vascular system. This comprises two small cortical bundles and a larger central bundle (Pl. I, fig. 4; Pl. III, fig. 7). I shall describe first the course of these bundles, and then turn to a description of their anatomical structure.

(a) *Course of Vascular Bundles.*

The course taken by the vascular bundles in the stem is very simple. The main central bundle travels in the center of the stem, and at each node sends off two branches in the manner figured (Pl. V, fig. 17)—a large branch to the lateral member which almost invariably occurs at each node, and a smaller contribution to its subtending leaf. This is repeated at each succeeding node until the floral region is reached. Here the main bundle continues to its termination in the rhachis, but gives off on each side a couple of branches to the lower and upper subfloral leaves with their axillary members in turn. This is simply a repetition of the process occurring at the nodes, except that here, between the lower and upper subfloral leaves, the node is practically obliterated (Pl. V, fig. 18). This arrangement, at least in the vegetative region, is much like that of *Potamogeton pectinatus*, described and figured by De Bary (1884, p. 273).

The course taken by the cortical bundles, although simple, is at t quite as evident. The cortical bundles are entirely independent of the central bundle, never connecting with it, so far as I have been able to ascertain. Their position can be best shown by the use of figures selected from a series of cross sections. A cross section in the region of a node, just below the point where the leaf is given off, is represented in Pl. V, fig. 19. Here the cortical bundles are situated in the outer cortex, not quite diametrically opposite each other. This figure also shows the axial bundle somewhat dilated radially. Pl. V, fig. 20 shows the edges of the leaf sheaths appearing at the cleft at the upper side of the figure. The cortical bundles occupy about the same position as before, but the axial bundle is commencing to give off its two branches—to the leaf and axillary branch. In Pl. V, fig. 21 the leaf sheaths are still more separated from the stem; the two large leaf and branch bundles are now distinct from the axial stem bundle; the cortical bundles have moved outward slightly, and a new cortical bundle appears nearer the central bundle. Since the section is cut slightly obliquely, and the right side represents a portion higher up in the stem, only one of the pair of new cortical bundles thus appears. Pl. V, fig. 22 shows both, however, and also represents the former pair of cortical bundles as moving gradually into the still further separated leaf. Pl. V, fig. 23 shows the leaf with its lateral (cortical) bundles, separate from the stem, and fig. 24 does the same for the branch which it subtends.

It is clear, therefore, that the cortical bundles of the stem furnish the lateral nerves of the leaf. It will be seen also that not only do the cortical bundles have no connection with the axial bundle, but also that a new pair arises at each node to pass into the leaf at the node next higher up. After a careful examination, I find absolutely no connection between these successive pairs of cortical bundles.

Although the figures and description of De Bary (1884, p. 274) for *Potamogeton crispus* seemed to present an arrangement similar to the above, the work of Chrysler (1907), which appeared while this paper was being prepared for publication, corrects De Bary's account but confirms my investigation of *Ruppia maritima*. Chrysler's conclusion (l. c. p. 171) is undoubtedly correct that " this condition has in all probability been derived by reduction from that in which the cortical bundles joined the central cylinder at the next node below the insertion of the leaf to which they belong."

The cortical bundles of *Zostera marina* (De Bary, 1884, p. 275;

Sauvageau, 1891 I ; Chrysler, 1907) are similar to those of Ruppia. but are continuous up and down the cortex and send a branch to the central cylinder at the nodes.

(b) *Structure of Vascular Bundles.*

Anatomically the cortical bundles in a young stage consist of small, closely packed cells, which are all tracheids or tracheae (Pl. III. fig. 8). No sieve tubes can be distinguished (Text-fig. 5). At a later stage (Pl. I, fig. 5; Text-fig. 6) these small elements have for the
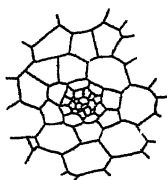


Figure 5.—Cross section of cortical bundle, before dissolution of its vascular elements. × 210.
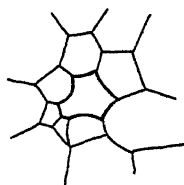
Figure 6.—Cross section of cortical bundle, after dissolution of its vascular elements. × 300.

most part been resorbed, so that only a passageway remains, except at the nodes, where they remain intact.

In structure, the central cylinder is composed of a central xylem region surrounded by a zone of phloem. The xylem is in a young stage represented by a group of tracheae at the center. which, however, very early in the development of the stem, become pulled apart and disorganized except at the nodes, where they are conspicuous in the mature stage. The resulting space, then, is a lysigenous cavity. and does not represent an enlarged cell (Pl. I, fig. 3). The thickenings of the tracheae
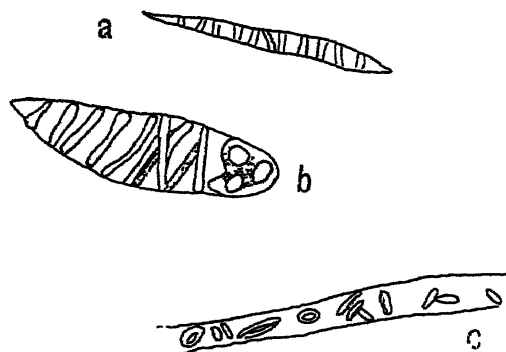


Figure 7.—Tracheae and tracheids from axial vascular bundle of stem: a. tracheid with annular thickenings ; b. tracheid with annular, spiral. and pitted thickenings; c. disorganized trachea with thickenings lying in the cavity. × 1350.

are mostly annular, although spiral and other forms may occur (Text-fig. 7). Occasionally these thickenings may be found lying loose

in the central lysigenous opening. In sections of the stem it appeared as if tracheids were also present, especially at and near the nodes. possessing the same characters as to their thickenings as the tracheae. but showing cross walls (Text-fig. 7 *a. b*).

Surrounding the central cavity are thin-walled xylem parenchyma Pl. I, fig. 3). Just outside of these lies the phloem region, the boundary between the two being impossible to determine on account of the apparently identical characters of the xylem and phloem parenchyma.

In the phloem zone are distributed the sieve tubes, with their companion cells, in an irregular ring in much the same way as in Zannichellia (Schenck, 1886). As a rule, the sieve tubes are larger than the surrounding cells, and on this account and because of their arrangement in a ring. each with its companion cell, they stand out quite clearly even in freehand sections. Between them and the endodermis lie one or occasionally two layers of parenchyma cells (Pl. I. fig. 3).

One noteworthy feature of the phloem region is that in the vicinity of the node. i. e. immediately before and after the branches are given off from the central cylinder, it is localized into four distinct symmetrically situated portions (Text-fig. 8), although this is never evident in the internodal region. Such an arrangement would be expected when one considers the true nature of this fibrovascular area. as described below, and compares it with certain Potamogetons which reveal this character more distinctly (Schenck, 1886. pp. 40—41 and figures 35, 36 and 38), and with *Zostera marina* (Chrysler, 1907, p. 172 and fig. 29).



Figure 8.—Photomicrograph of cross section of stem, &c., similar to Pl.III, fig. 8, but cut at a higher plane and near a node; showing localization of phloem into four regions in central cylinder of stem. × 35.

In his work already cited, Schenck (1886, pp. 27 ff.) reviews in a most admirable manner the various interpretations of the concentric type of bundle as it exists in water plants. He concludes with Sanio and Russow that it represents phylogenetically not a single

bundle, but the product of the gradual centripetal union of many stem bundles, and is therefore not comparable to the concentric bundle of the majority of the Gleicheniaceae, for example. This is true both of mono- and dicotyledonous water plants.

In support of this theory Schenck shows how there exists at the present time in different aquatic and amphibious species, every gradation from the typical bundle arrangement common to land plants to the simplest structure as exemplified in the submerged aquatics. He says, " Im Laufe der phylogenetischen Entwicklung rückten infolge immer weiter gehender Anpassung der Structur an die Lebensweise der Pflanze unter Wasser diese Leitbündel bei gleichzeitiger Reduction des Xylems nach der Axe zusammen zu einem gemeinsamen Strang. in welchem die Xylemteile nach und nach zu einem einzigen axilen Körper verschmolzen, während die Phloemteile, ihre normale Lage nach außen beibehaltend, zu einer Ringzone sich vereinigten.''

According to this interpretation therefore, Ruppia is descended from a form with four distinct main vascular bundles, traces of which are still apparent in the four phloem regions at the nodes; for here, as is generally admitted, ancestral characters are wont to appear.

Summary. The stem is composed of a thin walled epidermis; a loosely constructed cortex, containing as its principal feature a ring of radially arranged lacunae, and also two small bundles ending in the cortex—fibrovascular connections with the leaf sheaths; and a well defined endodermis, surrounding a very simple axial vascular region of a concentric structure, the appearance of which at the nodes indicates its reduction from four vascular bundles.

b. Rootstock.

The branching of the rootstock, or horizontal axis, as already stated, conforms strictly to the monopodial type, although torsion and the decay of the leaves often renders this obscure.

In its internal structure, as would be expected, it agrees with the stem (Pl. III, fig. 9). No points of difference were noted, except that sometimes the cells are somewhat larger than is customary in the upright axis, and often contain a large amount of starch.

The main differences between this and its morphological equivalent, the upright axis, are its horizontal position and the occurrence of roots at the nodes

3. Stem Structures in the Floral Region.

Three forms coming under the category of stem are found in connection with the production of flowers and fruit: (a) the peduncle, or the stalk of the inflorescence, (using this term in its narrower sense); (b) the rhachis or axis of the inflorescence. which is joined to the apex of the peduncle and bears the sessile flowers; (c) the stipe. a stalk which acquires its full development subsequent to the fertilization of the ovule and bears the mature fruit at some distance from the rhachis.

These three parts will be considered in order.

a. Peduncle.

(1) General Characters.

At the time of blooming the peduncle is short, about 2.5 cm., in length. and is raised nearly its whole length vertically above the surface of the water. About the time that the pollen is shed and the anthers fall off, the peduncle becomes horizontal, floating on the surface of the water. From now on it commences to elongate, presumably by simple stretching of its cells, as no meristematic regions were found to prove the contrary. It elongates until in some cases it attains great length. One specimen I measured was 18.2 cm. long. Elongation, takes place whether fruit is matured or not, and the length is apparently just as great in either case. Indeed,

the specimen just mentioned bore no fruit. The average length of the peduncle when no fruit was matured was 5.39 cm. The average length in 80 specimens when ovules were matured was 4.9 cm. On the whole, therefore, the average length of the extended peduncle is about 5 cm. or about twice that at the time of flowering.

(2) Anatomical structure.

In general, the anatomical structure of the ped-



Figure 9.—Cross section of peduncle, showing at *x* the apparently double epidermal cells; *ep*, epidermis; *la*, lacunae; *c*, cortex; *axb*, axial bundle. × 185.

uncle is very similar to that of the leafy stem (Text-fig. 9; Pl. VI, fig. 34). The following points, however, are noteworthy :—

1. As would be expected, no cortical leaf trace bundles are present.

2. The central vascular area is very similar to that of the stem. The tracheae in the center are pretty well disorganized, which is natural when one calls to mind the considerable extension of the peduncle.

3. The lacunae are more numerous and much larger in proportion to the diameter of the peduncle than in the stem (Text-fig. 9; Pl. VI, fig. 34).

4. In comparison with the stem the epidermis has slightly thicker walls, and small intercellular spaces may occur between it and the subjacent layer, an appearance which one never sees in cross sections of the stem (Text-fig. 10).

5. Occasionally some of the epidermal cells have a peculiar halved appearance, as if a periclinal wall divided an ordinary epidermal cell into two parts. This is due probably not to a division in the epidermal cell, but to the fact that a small subepidermal cell is wedged up close to a small epidermal cell (Text-fig. 10).
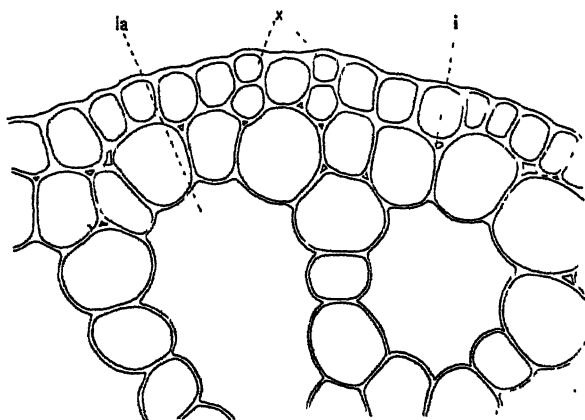


Figure 10.—Portion of cross section of peduncle: *x*, apparently double epidermal cells: *la*, lacunae: *i*, intercellular spaces. × 300.

On the whole, the main differences between this structure and that of the ordinary stem are its lack of cortical bundles and its lighter construction. The latter is explained when one recalls the fact that the peduncle for a great period of its existence floats, and hence its specific gravity must be small.

3 Coiling of the peduncle.

Sooner or later the peduncle becomes spirally coi'ed. drawing the fruit. if any is matured. below the surface of the water. Some individuals exhibit this characteristic more markedly than others. Often. in fact, the peduncle is fairly straight. In case the coiling takes place, it is due to the cells on that side of the peduncle which will form the outer side of the spiral becoming longer than those on the inner side. This difference in length was ascertained in several specimens by actual measurement of the cells. Such curvatures as this occurring in plants are most often, as Strasburger (1908, p. 269) has explained. due in like manner to unequal growth.

b) Rhachis.

Here the internal structure is practically the same as in the peduncle. No appreciable elongation takes place as in the peduncle, and. therefore, the tracheae are not disorganized to such an extent. At the point of attachment of the stamens the rhachis is practically triangular in outline (Pl. III, fig. 11). The single axial vascular bundle sends a branch to each stamen and to each of the four pistils of a flower (Pl. III, fig. 12). This axial vascular region terminates with the branch sent to the uppermost stamen (Pl. III, fig. 12), the most distal member of the inflorescence, and does not therefore have a blind ending at the end of the rhachis.

The termination of the rhachis is simply a rounded knob (Pl. III, fig. 12).

All through the rhachis, in the interior as well as in the epidermal tissue, the " secretion " cells are abundant.

c) Stipe.

Although the pistils are termed sessile, and correctly so from a macroscopic standpoint, early in their development, before fertilization (Pl. III, fig. 12), considerable tissue is formed between the ovule and the rhachis by a region which remains meristematic for some time. At length the cell division ceases, and the cells stretch out, forming a stalk of considerable length, having at its end the mature fruit (Pl. IV, fig. 13). This stipal development is correlative; when no fruit is developed, the stipe is also wanting.

Pl. IV, fig. 14, which shows the secretion cells in the epidermis of the stipe, represents a stage when the cells are still quite young and short.

In its internal structure the stipe reveals no marked differences from the stem (Pl. IV, fig. 15). As in the peduncle, the cortical bundles are lacking. The lacunae are less pronounced than in the peduncle. The apparently halved epidermal cells occasionally appear,

and in places the subepidermal layer is thick-walled and constructed much like the epidermis (Pl. IV, fig. 16). As a whole, the structure shows greater strength than the peduncle.

4. Brief Comparative Study of the Stem Anatomy of the Potamogetonaceae.

Zannichellia (Campbell, 1897) and Althenia (Prillieux, 1864) perhaps most closely resemble Ruppia in their cauline structure, although they possess no cortical bundless. *Potamogeton pectinatus* (Schenck, 1886) also, which externally resembles Ruppia so closely, internally reveals a very similar stem anatomy. Moreover, the Potamogetons possess cortical bundles lacking in Zannichellia and Althenia. Cymodocea (Bornet, 1864), appears to differ mainly in having a thicker stem and a ring of small cortical bundles with lacunae between these and the axial vascular area.

In its vascular system Zostera (Sauvageau, 1891. I.) shows close relationship to Ruppia. *Z. capricorni* and *Z. nana* have one cortical bundle on opposite sides of the stem, as in Ruppia (Sauvageau, 1891). *Zostera marina* (Chrysler, 1907. p. 172, fig. 29) is also similar. In other species these bundles are more numerous. The bundles run free the whole length of the internode, unite with the central area at the node and send out branches which become the lateral nerves of the leaves. The central vascular area appears composed of four fibrovascular strands, the phloem parts isolated and separated, and the xylem bundles united into an axial strand.

LEAF

## A. Enumeration of Kinds of Leaves

Of Strasburger's (1908, p. 26) four categories of leaves, three occur in Ruppia:—the scale leaves, small specialized structures at the base of every branch (Pl. II, fig. 6, *vsl*; Text-figs. 19—20, *vsl, fsl*); secondly, the ordinary long, narrow foliage leaves; and finally, the subfloral leaves, belonging properly to the category of bracts, a pair of which is borne at the base of the peduncle (Text-figs. 19—20, *up. sfl, low. sfl*). Since the flower is naked, there are no floral leaves.

## B. Arrangement

An investigation of the position of the leaves throughout the plant reveals the following:

1. The general leaf arrangement is distichous (Pl. I ng 2; Pl. II fig. 6, as in other members of the Potamogetonaceae so far as known, with the exception of *Potamogeton natans* and *Potamogeton lucens*, where a several ranked arrangement sometimes occurs (Irmisch, 1858 and Ascherson, 1889).

2. A single plane, therefore, will intersect all points of leaf insertion on any stem (Pl. I, fig. 1, Pl. VI, fig. 25).

3. Moreover, on all branches, this plane of leaf insertion coincides with that of the main stem, a feature which combines with the methods of branching to produce the characteristic flat, fan- or sickle-shaped plants, a condition already noted in the Potamogetonaceae by Ascherson (1889) (Pl. I, fig. 2; Pl. II, fig. 6).

4. The first, or basal leaf of every branch, which is in all cases a scale leaf, originates on that portion of the branch opposite to the subtending leaf—i. e., with its back toward the main axis of the shoot (Pl. II, fig. 6, $vsl$). No internode is placed between it and the subtending leaf, so that it is therefore exactly opposite the latter.[1]

5. The second leaf of every branch, in all cases a foliage leaf, is developed on the side of the branch adjacent to the subtending leaf, and thereafter ordinary foliage leaves of the branch proceed in regular distichous order (Pl. II, fig. 6, $l_1^{(I)}$, $l_2^{(I)}$, &c.).

In an account of the leaf of Ruppia I shall describe the three sorts of leaves, beginning with the ordinary foliage leaves, taking up next the subfloral leaves, and concluding with the scale leaves.

## C. Ordinary Foliage Leaves

General Characters. The ordinary foliage leaves are "ribbon-shaped"; long (7—17 cm.), and very narrow, (about 1 mm.) (Pl. I, fig. 2; Pl. II, fig. 6). Although to the naked eye the margins appear entire, the microscope reveals at the apex a large number of one-to three-celled teeth, and these, with increasing distances between them, extend a considerable distance down the margins of the leaf (Pl. VI, fig. 26; Text-fig. 11). Upward from the base of the leaf, on each side, extending for 13—27 mm. are thin translucent appendages,

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[1] Goebel (1898, pp. 68—69) says, however, in speaking of the phyllotaxy of axillary branches in general, that, due to mechanical causes in the bud, " die bei weitem häufigste Stellung der beiden ersten Blätter des Axillar-triebs eine *laterale* ist, und erst die folgenden Blätter *median* oder mehr oder weniger schief gestellt sind."

which I have termed the stipular sheaths (Pl. I, fig. 2; Pl. II,
fig. 6). In external appearance the leaves very closely resemble
those of *Potamogeton filiformis* and *Potamogeton pectinatus*, both
also typical submerged plants (Morong, 1893). The leaves of
Zannichellia and Althenia are also macroscopically very similar:
those of all species of Naias are much shorter with strongly serrate
or toothed margins.[1]

Development of the Foliage Leaf. Different stages of leaf
development are represented in Pl. I, fig. 1; Pl. VI, fig. 25, lon-
gitudinal sections of a vegetative bud, where the youngest leaf
is the protuberance at one side of the growing point ($L^{VIII}$
in Pl. I. fig. 1). The next older leaf primordium, shown on
the right of the growing point (Pl. I, fig. 1, $L^{VII}$), has already



slightly elongated, until now it extends
beyond the growing point. Successively
older stages are represented by $L^{VI}$,
$L^V$, $L^{IV}$, &c.

Pl. VII, fig. 48 represents a transverse
section through a similar bud cut through
about the region of the line *a b* in Pl. I,
fig. 1. Therefore, we find here the different
members in exactly the same orientation
as in Pl. I, fig. 1. Thus, the outermost leaf,
$L^I$, embraces with its sheaths all the
internal complex, just as is shown by $L^I$
with its sheaths in Pl. I. fig. 1. The next
inner member is $l_1^{(I)}$, or the lowest and
largest foliage leaf of the axillary bud of
$L^I$; then a section through the apical
portion of the scale leaf which envelops
this axillary bud; next the leaf sheaths of

Figure 11.—The marginal
leaf teeth, showing variation
in cell arrangement. × 300.

the next upper main leaf, $L^{II}$, appear, which brings us to a cross
section of the central stem: without going into more detail it is
sufficient to note that the remaining parts may be correlated to their
corresponding longitudinal sections in the same way. The line *a b*

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[1] All these species belong to Hansgirg's (1903) " Vallisneria-Typus
der Strömungsblätter," a physiological group showing little differentiation
into blade and petiole and characterized by the lack of cuticle, hair
structures and stomata, by the absence of any considerable amount of
supporting tissue, and by the ribbon-like, isolateral form.

in Pl. VII. fig. 43 represents the approximate plane in which a
section similar to that in Pl. I. fig. 1 and Pl. VI. fig. 25 would
be cut.

With the help of these sections and other similar serial preparations,
I have found that the course of development and distribution of
growth in the ordinary foliage leaf is approximately as follows: the
leaf primordium first appears as a mere swelling at one side of the
growing point, Pl. I, fig. 1, $L^{VIII}$: this protuberance soon differen-
tiates into two parts, an "upper leaf," elongated in the direction of
the growth of the shoot, and a "leaf base."—to use the terms of
Eichler, 1865—consisting of lateral protuberances on each side of
the base of the "upper leaf" and extending part way around the
shoot axis (Pl. VII, fig. 39). This arrangement is also shown, though
not very clearly, by Pl. VI, fig. 28. The "upper leaf" will produce
the leaf blade and the "leaf base" the leaf sheaths. The upper
leaf now elongates rapidly, being composed entirely of embryonic
tissue, and the succeeding stages are essentially those described by
Prantl (1883): the cells at the apex are the first to commence
extension to their mature size, and this stretching gradually proceeds
toward the base of the leaf. The final developmental stage is
marked by a considerable growth of the sheaths due to the inter-
calary growth of the leaf, which Goebel (1898, p. 518) states is so
characteristic of monocotyledonous leaves.

A point of interest here is the comparative large size of the sheath
rudiments at the first segmentation of the leaf primordium (Pl. VII,
fig. 39), a circumstance which will be discussed below under the
heading of the leaf sheath.

For purposes of further description, it is best to divide the leaf
into two parts,—the blade and the sheaths, assuming the blade to
be that part of the leaf from the sheaths to the apex (Pl. I, fig. 2;
Pl. II, fig. 6).

### 1. The Leaf Blade.

A cross section of a leaf, made above the region of the sheaths,
is represented in Pl. VII, fig. 43. $l_1^{(I)}$. An epidermis of comparat-
ively narrow cells, a subepidermis of wider cells, a single axial vas-
cular bundle with one subsidiary bundle running along each leaf
margin, a lacuna or air space on each side of the axial bundle, and
a few extra interior layers of parenchyma cells toward both edges
of the leaf and surrounding the vascular bundle comprise, in brief,
the internal structure of the leaf blade.

a. Epidermis.

(1) Chloroplasts.

An interesting point brought out by a study of the epidermis, and shown especially well in the living condition, is the fact that here most of the photosynthesis is carried on, for the cells contain large numbers of chloroplasts (Text-fig. 12). The discussion of the causes of this condition, being of an ecological nature, will be postponed for the present.

(2) Marginal Teeth.    .

The teeth, mentioned above, p. 88 (Pl. VI, fig. 26; Text-fig. 11),



Figure 12.—
Portion of epi-
dermis of leaf,
showing chloro-
plasts. Drawn
from living leaf.
× 335.

have already been observed in the leaves of *Ruppia maritima* by Sauvageau (1891, II, p. 209), who says, " à son sommet [i. e. of the blade] la plupart des cellules terminales se prolongent en dents arrondies composées de 1–2–3 cellules. Des dents semblables, mais plus espacées, se retrouvent sur les bords latéraux du limbe et font légèrement saillie." In a young stage, as in Pl. VII, fig. 39, the teeth appear mostly at the end of the leaf primordium; later they may be found down the margins of the leaf anywhere from $\frac{1}{3}$ to $\frac{1}{4}$ of the whole distance from the apex. Finally, in the adult leaf they rarely extend more than $\frac{1}{4}$ of the distance from the apex.

(3) Secretion Cells.

In the epidermis I have observed also the cells referred to by Sauvageau (1891, II, p. 209) as " cellules sécrétrices," containing, in alcoholic material, a granular or a homogeneous content of a brownish color, and somewhat larger than the ordinary epidermal cells (Pl. VI, fig. 30; Pl. III, fig. 8). They appear to be identical in nature with those of the stem. Although Sauvageau asserts that they are most abundant at the edges and at the apex of the leaf, yet they appear to me often to increase in numbers towards its base, including that part which adjoins the sheaths, as well as in the sheaths themselves (Pl. VI, fig. 25). According to Sauvageau these peculiar cells occur also in Posidonia, Cymodocea and Halodule. He has ascertained the presence in them of tannin, probably in combination. In my permanent slides their contents were almost invariably granular, assuming, with the triple stain, either a crimson or a brilliantly refracting yellow color—more often the latter. With ferric chloride the contents stained a brownish black and the same result was given on treatment with potassium bichromate.

These reactions confirm Sauvageau's assertion that these cells are tanniferous. As is well known, tannin is a common byproduct in plants, and in this case is possibly deposited in special cells on account of the difficulty of its diffusion through the epidermis. That these tanniferous cells should be more abundant toward the base of the leaf is natural, for it would be more economical for the plant to devote as many as possible of the cells in the upper part of the leaf to photosynthesis.

(4) Absence of Stomata.

Another peculiarity of the epidermis is the absence of stomata, which will be discussed later (p. 118).

b. Subepidermal Layer.

Below the epidermis. at all parts of the leaf, is a distinct layer one cell thick, which I have termed the subepidermal layer (Pl. VII, fig. 43), composed of cells about twice the width, tangentially, of those of the epidermis; and longitudinally, i. e. running the length of the leaf, many times longer than the epidermal cells (Text-fig. 13). This layer contains a few chloroplasts (cf. Sauvageau, 1891, II, p. 298).

c. Vascular System.

The vascular system of the leaf is extremely simple. Three strands of conducting tissue are present—one large axial and two small marginal.

(1) Course of Vascular Bundles.

These three vascular strands enter the leaf separately at its insertion on the stem. The axial bundle maintains a median position and extends to very near the tip of the leaf (Pl. VI, figs. 26, 30; Pl. VII, fig. 43); the two lateral bundles enter one on each side of this axial bundle, and throughout their extent run parallel to it (Pl. VII, fig. 43. *1b*).



Figure 13.—Portion of subepidermal layer of leaf blade, showing a few chloroplasts. Drawn from longitudinal section cut from living leaf. × 335.

In the region of the sheaths each of these lateral bundles runs along the edges of the leaf proper, just at its line of junction with the sheaths, and above the leaf sheaths they retain this marginal position. I can confirm Sauvageau's (1891, II, p. 210) observation that they do not unite with the median bundle near the apex of the leaf, but disappear—according to my preparations, about 10 mm. from the apex.

(2) Structure of Vascular Bundles.

The axial bundle contains two or three annular tracheae in its young stage, which generally break down in the mature leaf to form an open passageway; surrounding this portion are phloem cells and parenchyma, difficult to distinguish on account of their low degree of differentiation, and about the whole axial bundle is a pretty definite endodermis, which Sauvageau has also noted. The structure of the two lateral bundles is extremely simple, consisting merely of a very few small conducting cells (Pl. VII, fig. 43, *l b*).

d. Lacunae.

On each side of the central bundle, separated from it by two or three layers of parenchyma, are the lacunae (Pl. VII, fig. 43, *l a*). In mature leaves these are of the shape of flattened cylinders, extending longitudinally through the leaf, and divided into compartments by transverse, sometimes oblique, perforated, one-layered plates of roundish cells (Pl. VI, fig. 25; Text-figs. 14–15). The perforations are round intercellular spaces, occurring regularly at the
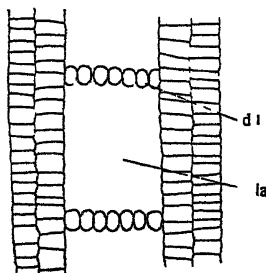


Figure 14.—Longitudinal section of lacuna, showing cross sections of diaphragms; *d t*, diaphragm; *l a*, lacuna.  × 185.
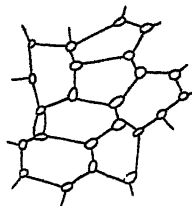


Fig. 15.—Surface view of one of the diaphragms which separate the leaf lacunae into compartments. The small circular outlines represent spaces between the cells.  × 350.

angles of the cells and affording communication for the air or gases from one compartment to the next.

e. Comparison with Other Potamogetonaceae.

The anatomy of the leaf of *Potamogeton pectinatus*, a plant so much like Ruppia in external appearance, is almost a duplicate of the leaf structure of Ruppia (Schenck, 1886, p. 16 and figure 11 *d*). The other species of Potamogeton show differences more or less great, according to the shape of the leaf and the external conditions. The leaves of *Zannichellia palustris* (Schenck, 1. c.) and *Althenia*

*filiformis* Sauvageau, 1891, II. p. 259. and Prillieux, 1864 are very similar, the main difference being the absence in both of the marginal bundles, although Althenia has in place of these, small groups of fibrous cells. Althenia is, moreover, further distinguished by a rather large number of lacunae of various sizes.

The leaves of Zostera, Phyllospadix, Posidonia. Cymodocea and Halodule, show greater differences, of all of which, together with the remaining Potamogetonaceae, Sauvageau has made an excellent comparative study in his " Feuilles des monocotyledones aquatiques " (1891, II).

## 2. The Stipular Sheaths.

### a. Structure.

At every node the leaf envelops the stem by means of basal sheaths composed of only two layers of cells. These cells lack chlorophyll and become very minute at the free edge of the sheath (Pl. I, fig. 2; Pl. VII, fig. 43, *s h s*).

### b. Development.

I have already (p. 89) mentioned the fact that the development of the sheath rudiments in the young leaf primordium is quite marked. From the record of measurements (see Table p. 94) of the length of sheaths at different stages of leaf development there is manifested a certain periodicity in the growth of the sheath. Thus its percentage of the whole leaf length in a very early stage is high, as is shown in Nos. 1 and 2 and Pl. VII, fig. 39; [1] next, during the special growth of the leaf (see p. 89) this percentage noticeably decreases, in some cases to a very marked extent; [2] finally, due to the ultimate basal intercalary growth, the proportional size of the sheath again increases until in the mature leaf the proportional length of sheath to entire leaf averages about 1 to 6 (Table, Nos. 27–38 .

Although, even by making considerable allowance for error, the data in the table are not at all uniform, as is natural, yet they do bring to light with no uncertainty the large comparative development of sheath both at the beginning and at the end of leaf growth.

### c. Function.

That this periodicity in sheath development is related to the principal function of stipules, that of protection of younger parts,

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[1] Such an early enlargement of sheath is mentioned by Van Tieghem (1898, pp. 250–251), who speaks of it as quite general.

[2] Possibly the very low percentages are results of environmental influence, e. g. position of leaf on shoot, surrounding leaf sheaths, &c.

*A. H. Graves,*

TABLE

of

COMPARATIVE LENGTH OF SHEATH

IN THE VARIOUS STAGES OF LEAF DEVELOPMENT.

| Description of specimen | No. of specimen numner | Length of entire leaf | | Length of sheath | | Approximate percent. of sheath to leaf length | |
|---|---|---|---|---|---|---|---|
| Dissected from living plant | 1 | .14 | mm. | 0525 | mm. | 37.5 | % |
| „ „ „ „ | 2 | .341 | „ | 0875 | „ | 25 6 | „ |
| From prepared slide | 3 | .586 | „ | .0875 | „ | 14.9 | „ |
| „ „ „ | 4 | .785 | „ | .11 | „ | 14.9 | „ |
| „ „ „ | 5 | .8137 | „ | .1487 | „ | 18 2 | „ |
| „ „ „ | 6 | .8137 | „ | .1225 | „ | 15. | „ |
| „ „ „ | 7 | .8575 | „ | .1575 | „ | 18.3 | „ |
| „ „ „ | 8 | 1.137 | „ | .096 | „ | 8.4 | „ |
| Living specimen | 9 | 1.382 | „ | .096 | „ | 6.9 | „ |
| From prepared slide | 10 | 1.41 | „ | .122 | „ | 8.6 | „ |
| „ „ „ | 11 | 1.47 | „ | .19 | „ | 12.9 | „ |
| „ „ „ | 12 | 1.54 | „ | .11 | „ | 7.1 | „ |
| „ „ „ | 13 | 1.80 | „ | .17 | „ | 9.4 | „ |
| Living specimen | 14 | 2.275 | „ | .262 | „ | 11.5 | „ |
| From prepared slide | 15 | 2.362 | „ | .227 | „ | 9.6 | „ |
| „ „ „ | 16 | 2.458 | „ | .166 | „ | 6.7 | „ |
| „ „ „ | 17 | 3.745 | „ | .105 | „ | 2.8 | „ |
| Living specimen | 18 | 11.14 | „ | .315 | „ | 2.8 | „ |
| „ „ | 19 | 11.81 | „ | .525 | „ | 4.4 | „ |
| „ „ | 20 | 22. | „ | 1. | „ | 4.5 | „ |
| „ „ | 21 | 31. | „ | 1 | „ | 3.2 | „ |
| „ . | 22 | 38.6 | „ | 3.6 | „ | 9.3 | „ |
| „ „ | 23 | 61.9 | „ | 9. | „ | 14.4 | „ |
| „ „ | 24 | 66.6 | „ | 4.6 | „ | 6.9 | „ |
| „ „ | 25 | 70.4 | „ | 1.4 | „ | 1.9 | „ |
| „ „ | 26 | 70.9 | „ | 2.9 | „ | 4 | „ |
| Living matured specimen | 27 | 102. | „ | 14. | „ | 13.7 | „ |
| „ „ „ | 28 | 109. | „ | 15. | „ | 13.7 | „ |
| „ „ „ | 29 | 116 | „ | 18. | „ | 15.5 | „ |
| „ „ „ | 30 | 116. | „ | 23. | „ | 19.8 | „ |
| „ „ „ | 31 | 118. | „ | 20. | „ | 16.9 | „ |
| „ „ „ | 32 | 125. | „ | 25. | „ | 20. | „ |
| „ „ „ | 33 | 130. | „ | 25. | „ | 19.2 | „ |
| „ „ „ | 34 | 130. | „ | 25. | „ | 19.2 | „ |
| „ „ „ | 35 | 133. | „ | 25. | „ | 18.7 | „ |
| „ „ „ | 36 | 134. | „ | 24. | „ | 17.9 | „ |
| „ „ „ | 37 | 144. | „ | 22. | „ | 15 2 | „ |
| „ „ „ | 38 | 163. | „ | 27. | „ | 16.5 | „ |

is well exemplified in the leaf sheaths of Ruppia. It is stated by Goebel (1898, p. 556) that no general rule can be given for the time of stipular development, but that as Massart (1894) had already emphasized before him. they are formed and called into play when their function is most needed, as is in fact true of other parts of the leaf (Goebel, 1898, p. 503,. I shall undertake presently to show why the leaf sheaths may be considered stipular as to their nature and origin.

In Ruppia the delicate parts in the region of the growing point are in great need of the protective offices of the early developed sheaths of surrounding leaves: and also the mature leaves, since they almost always contain axillary buds, require large enveloping sheaths. These considerations may explain the manner of sheath development.

d. Morphological and Morphogenetic Considerations.

The nearly related genus Potamogeton throws considerable light upon the question of the morphology and morphogeny of the leaf sheath. This genus, as is well known, embraces a great variety of forms, from those with broad floating leaves only (or with the addition merely of capillary phyllodia), passing through intermediate forms which have both narrow submerged leaves and broad floating leaves, to the typical submerged plants with narrow leaves only. In the last group we find basal sheaths on the leaves of most of its representatives, e. g. *P. filiformis, P. pectinatus, P. interruptus,* &c: but in the broad leaved forms and in the intermediates we find no sheaths, free stipules, however, being more or less prominent.

It seems clear, then, that in the submerged forms sheaths replace the stipules; and this deduction is borne out by such a species as *P. diversifolius,* which has the *submerged* leaves with stipules sometimes adnate, and even better by *P. spirillus,* where the stipules are always adnate to the submerged leaves, while those of the floating leaves in both species are free.

A comparison of any of the above mentioned typical submerged Potamogetons with Ruppia—especially *P. pectinatus* and *P. filiformis* —shows the similarity of the sheaths in the two genera. One may conclude, then that both the free stipules of the Potamogetons and the sheathing stipules of the submerged forms and of Ruppia are closely related, being connected as they are by many gradations; and it is probable that the sheathing stipule has been evolved from the free stipule, such as it is in Potamogeton.

As to the reason why this form of stipules is developed in water plants, the most obvious explanation seems to be that joined to

the leaf-base they can better perform their office of protection of
the axillary structures. In the case of the grasses, where the sheath
is also retained, Strasburger (1908, pp. 29, 30) states that be-
sides protecting the soft lower part of the internodes when inter-
calary growth takes place, the sheaths also give the stem rigidity.
Possibly this latter strengthening function applies also in some
degree to the sheaths of water plants.

3. Axillary Scales.

   a. General Characters and Anatomy.

   In the axils of all the leaves are two small, ovate, scaly forma-
tions, one on each side of the median line of the leaf (Pl. III, fig. 8;
Pl. VII, fig. 48, *a s*). These structures, common in water plants,
and first shown by Irmisch (1858, p. 12) to be of general occurrence
throughout the Potamogetonaceae, consist of generally two layers
of cells rounded in cross-section, and loosely joined together, con-
taining large nuclei and a large quantity of cytoplasm (Text-fig. 17).
A longitudinal section of a single scale (Text-fig. 16), shows that
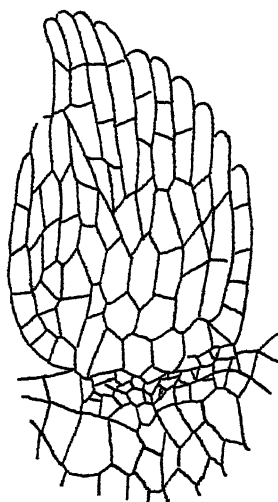


Figure 16.—Longitudinal section
through axillary scale, showing
arrangement and shape of cells.
× 170.
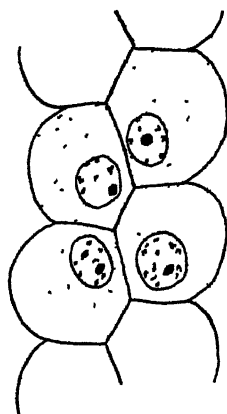
Figure 17.— Cells from cross
section through axillary scale,
showing cell structure.
× 1400.

these cells are long and arranged more or less in rows in the
upper part of the scale. Prillieux (1864) has found similar struc-

tures in *Althenia filiformis*. which he erroneously believed to be stipules.

b. Function.

The nature of these "*squamulae intravaginales*" points to their serving the purpose of secreting organs. Schenck (1886. p. 9) is of the opinion that their secretion, which covers the growing point, is a protection against parasites, but of this point he is not certain. F. Müller (1877) had already claimed that the slimy secretions of such cells protected the delicate growing points from immediate contact with the surrounding water, a position which Schenck (1886, p. 10) criticizes by asserting that it is difficult to see how the outside water could injure the growing points. But Schilling (1894), who has made a special study of the subject, returns to Müller's view. He shows that the slime is impermeable to certain salts, &c. in solution in the water, and is probably only developed until the epidermal tissue and cuticle are far enough advanced to make such protection unnecessary.

Schilling's arguments are reasonable. The growing point is an extremely delicate part and might easily be injured by the salts or other substances in solution in the surrounding water. Especially is this true of Ruppia, a plant of salt or brackish waters. This would be an omnipresent danger to the young growing parts : the attacks from parasites would be intermittent or rare.

## D.  Subfloral Leaves

Although along the stem the leaves are distributed at fairly regular intervals, just below a flower they approach each other so closely as to appear opposite (Pl. VII, fig. 35 ; Text-fig. 18), a characteristic common to many other members of the family, notably Zannichellia, Althenia, Potamogeton, &c.

These two apparently opposite leaves, which I have designated the subfloral leaves, besides differing from the ordinary foliage leaves in their mode of arrangement, possess slightly specialized sheaths and considerably shorter leaf blades. The sheath of the outer or lower leaf envelopes that of the inner, while the sheath of the inner or upper leaf surrounds the peduncle, and both together form a protective envelope for the young flower before it elongates (Pl. II, fig. 6 ; Pl. VII, fig. 35). As a result of these conditions, the sheaths are slightly wider than those of the ordinary leaves, and they as well as the leaf blades are also shorter, the latter characteristic being in harmony with the bracteal nature of these leaves.

In all other respects, however—internal structure, axillary scales, &c.—the subfloral leaves are identical with the ordinary foliage leaves.

Koch and Irmisch have called these two leaves "folia floralia," an unwieldy term, whose English equivalent is at present applied to the modified leaves forming the floral envelope in the Phanerogams. On account of their position near the peduncle and their slight modification in form, they approach the category of "bracteal leaves" or "Hochblätter" of Strasburger (1908, p. 81) and Goebel (1898, p. 578), yet the difference from the ordinary leaves is so slight that such a classification seems unwarrantable. I have therefore adopted the term "subfloral leaves," although bearing in mind their approach to typical bracts.

Figure 18.—Sketch of flowering branch, showing location of flowers and subfloral leaves. ½ natural size.

## E.  Scale Leaves

Besides the subfloral leaves and ordinary foliage leaves, two other sorts of leaf structures occur in *Ruppia maritima*, quite similar in appearance and structure. These are very small membranous formations—one kind borne at the base of the ordinary branches and the other at the base of the floral axis (Pl. I, fig. 1; Pl. II, fig. 6, *vsl*; Text-figs. 19–20, *vsl* and *fsl*). The first sort is plainly what Goebel (1898, p. 572) has in mind when he writes of "Vorblätter." He says, "Die Vorblätter sind zunächst charakterisiert durch ihre Stellung. Wir finden sie—wo sie überhaupt vorkommen—bei den Dikotylen meist in Zweizahl an der Basis der Seitensprosse, bei den Monokotylen wird gewöhnlich ein Vorblatt angenommen, welches auf der dem Mutterspross zugekehrten Seite des Tochtersprosses steht."

Strasburger (1908, p. 429) also mentions "Vorblätter," but with him they signify bracts or bracteal leaves. It is his category "Niederblätter" or "scale leaves" that includes the structures described by Goebel as "Vorblätter."

In accordance with the Bonn Textbook I have adopted the term scale leaf; the leaf at the base of the ordinary branch will be referred to as the vegetative scale leaf, while that at the base of the peduncle will be termed the floral scale leaf.

1. The Vegetative Scale Leaf.

a. General Characters.

This envelops the base of all axillary structures, except that in the axil of the upper subfloral leaf, and is developed on the side of the axillary member opposite to that of the subtending leaf. Flattened out, it is in the form of a long narrow triangle (Pl. VII, fig. 40), from 6 to 13 mm. in length; in position, its edges slightly overlap at the base, on the side nearest the subtending leaf (Text-figs. 19–20). If the axillary members are young, it completely envelops them; when they grow out, forming a branch, it remains sheathing the base of the branch.

b. Internal Structure.

Anatomically the vegetative scale leaf is composed of two layers of elongated large-vacuolate cells, rectangular in outline, with the nucleus flattened against the wall, and with a very few small chloroplasts. No vascular bundles are present. Pl. III, fig. 8 and Pl. VII, fig. 43, *vsl* show a transverse section, and Pl. VII, fig. 37 a surface view.

2. The Floral Scale Leaf.

a. General Characters and Structure.

The other form of scale leaf, which I have termed the floral scale leaf, occurs always at the base of the peduncle, on the side to ward the youngest or upper subfloral leaf (pp. 97–98 and Text-fig. 20, *fsl*) and its base is coincident with the base of the floral axis for nearly one half of its circumference. Its development can be easily traced in Pl. IX, figs. 54, 55, 56, 51 and Pl. VII, Fig. 41, *fsl*, and it will be seen that never at any time does it completely envelop the floral axis, as does the scale leaf the base of the axillary shoot, but nevertheless partly surrounds it. When the peduncle elongates in the course of floral development, the floral scale leaf remains at its base, similarly as in the case of the vegetative scale leaf. Moreover, in its growth this scale leaf follows the growth of the flower and is entirely in independence of the adjacent axillary bud.[1] (Pl. VII, fig. 41, *fsl*).

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[1] I have been unable to find a flower bud in which there was no branch in the axil of the upper subfloral leaf, which would of course, be an

In shape the floral scale leaf differs considerably from the vegetative scale leaf. It is shorter, almost symmetrically oblong and emarginate (Pl. VII, fig. 42; Text-fig. 19, *fsl*).

The internal structure is identical with that of the vegetative scale leaf.

### 3. Orientation of Scale Leaves in Floral Region.

The plan of a typical arrangement in the floral region is represented in Text-figs. 19—20. Here we find the members occurring in the following order, proceeding from right to left:—
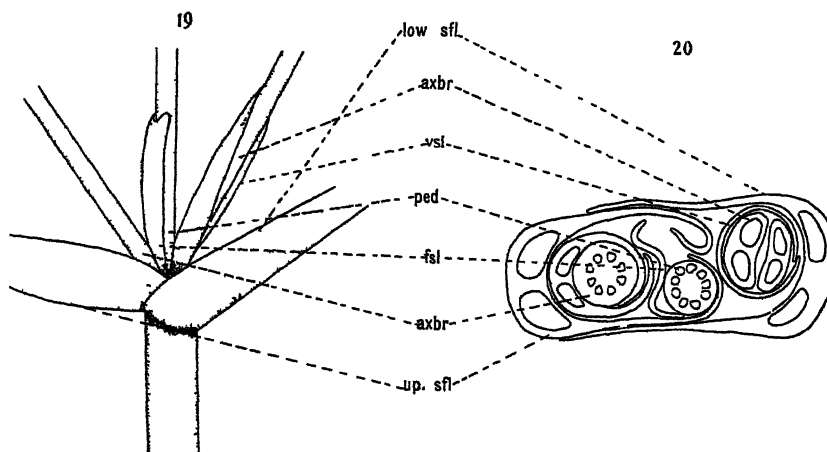


Figure 19.—Portion of the plant near the floral region, showing the two kinds of scale leaves and the subfloral leaves. The parts are somewhat separated so that they may be seen more clearly. *low. sfl.* lower subfloral leaf; *axbr*, axillary branch; *vs l*, vegetative scale leaf; *ped*, peduncle; *fsl*, floral scale leaf; *up. sfl.* upper subfloral leaf. × 6.

Figure 20.—Cross section through flower bud, below flowers, showing parts represented in figure 19. Abbreviations as in figure 19. × 35.

1. The lower subfloral leaf (*low, sfl*). sheathing the entire flower bud;
2. The branch (*axbr*), arising in the axil of this leaf;
3. The vegetative scale leaf (*vsl*), enveloping this axillary structure and borne at its base on the side furthest removed from the subtending subfloral leaf;
4. The peduncle (*ped*), bearing the flowers;

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exception to the general rule. If such cases could be found, showing a normal floral scale leaf. this would prove that the scale leaf is borne on the peduncle and not on the axillary bract.

5. The floral scale leaf *f.sl*, adherent to the base of the peduncle in a position corresponding to that of the vegetative scale leaf ;

6. The branch *axbr*, arising in the axil of the upper subflora. leaf;

7. The upper subfloral leaf *up. sfl*.[1]

## 4. Summary of Distinguishing Characters.

Irmisch (1851) has described the inflorescence in *Ruppia rostellata*. His account is in part as follows: " Die folia floralia bilden nun zwei Achseln, aus denen Zweige hervorbrechen, welche sich folgendermassen verhalten. Das erste tief an der Basis der Zweige stehende Blattgebilde ist eine dünnhäutige abgestutzte lanzettliche Schuppe, die sich um den Grund des Zweiges herumlegt. Ich will sie Vorblatt nennen. Es steht dasselbe, wie das auch sonst regelmässig der Fall ist, mit seiner Rückseite wegwärts vom Mutterblatte des Zweiges und dem Blütenstande A zugewendet";—and further, " Der Zweig in der Achsel des obern fol. flor. ist dem des untern in der äussern Bildung gleich."

In this and his description following, it is clearly evident that Irmisch considered the floral scale leaf and the vegetative scale leaf in an inflorescence to be identical, for he alludes to nothing but a vegetative scale leaf (Vorblatt) at the base of each branch. Of course it is possible that no floral scale leaf, as such, occurs in *Ruppia rostellata*, yet the two species are evidently quite similar. For example, Sauvageau (1891. II) finds no differences in the structure of their foliage leaves.

To sum up the differences between floral and vegetative scale leaves, they are in brief as follows.

1. Shape. The floral scale leaf differs markedly in shape from the vegetative scale leaf. A large number of specimens was examined and this difference was constant.

2. Position. As shown in Text-fig. 20, but more clearly were the section cut at a lower plane. the floral scale leaf does not envelop the axillary shoot as does the vegetative, but is turned toward and partially surrounds the young flower or the peduncle, according to the stage of floral development.

3. Development. The growth of the floral scale leaf is always correlated with the growth of the peduncle and flower—even when the axillary structure adjacent is as yet rudimentary. In case the

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[1] The axillary scales have been purposely omitted. The cross section represented in Text-fig. 20 was cut above them.

floral axis is already well developed, one finds a correspondingly advanced development in the floral scale leaf, showing that it is now connected with and influenced by the flower and peduncle.

5. Morphological and Physiological Nature of Scale Leaves.

That the vegetative scale leaf is a leaf and not an axillary stipule, may be accepted without question, for its development occurs much later than that of the subtending leaf and its origin is from the axillary bud of the latter. As to whether one has to deal here with a degenerate leaf, or one that approximates the primitive form is a question difficult to settle without a definite knowledge of the primitive monocotyledonous leaf form. It is probable, however, that the ordinary foliage leaves are nearer to the primitive form. Moreover, the lack of vascular tissue and chlorophyll, and the simple structure in general indicate a degenerate form.

Whatever its genetic origin, the early development and complete enfolding of the young primordia, which later grow out beyond it, point to the present function of the vegetative scale leaf as being essentially protective.

The nature of the floral scale leaf is not so clear. The striking fact, however, that the branch adjacent to this scale leaf never has a vegetative scale leaf at its base would seem to point to the conclusion that the floral scale leaf is simply this vegetative scale leaf now connected with the flower and utilized for its protection in an embryonic state.

6. Comparison with Other Potamogetonaceae.

Scale leaves (préfeuilles) are found in Althenia (Prillieux, 1864) where there are often two on a branch below the first ordinary foliage leaf. They occur also in Zannichellia (Irmisch, 1858, p. 80) and Cymodocea (Bornet, 1864). In Potamogeton (Irmisch, 1858) the scale leaves are quite abundant, occurring on the rootstock as well as at the base of the shoots. The prophyllon, or fore-leaf, described by Holm (1905) in his writings on the grasses, is an homologous structure.

ROOT

A. General Characters

The roots in Ruppia, as in many submerged plants, are simple, and strikingly reduced in structure. The primary root, formed

in the embryo, does not persist; the sole representatives or the root system are small slender adventitious roots arising at the nodes (Pl. VII, fig. 36, *r*). These occur singly or rarely in pairs and are unbranched. Each root, in the early stages of its development, is surrounded by a sort of pocket formed from the secondary growth of the epidermis of the stem. After the root has broken through the tip of this pocket, the latter remains as a persistent sheath or collar at its base. A zone of root hairs appears on every root, developed from specialized piliferous cells (Textfig. 21, *rh*).

As to the length of the root, I have measured specimens 20 cm. long, but a much shorter length is the general rule.

The life of the root is comparatively short; for as the stem advances in its growth, new roots are successively put forth from new nodes, the older roots becoming gradually discolored and ultimately dead, together with the stem or rootstock whence they originate.

Figure 21.—Sketch of portion of root-stock and root; *col*, coleorrhiza; *rh*,root hairs; *rs*, rootstock. × about 1½.

## B. Root Arrangement

Since the laws of the root arrangement in Ruppia are often obscure, as regards both the roots themselves and also their orientation with respect to the leaves, a somewhat more detailed description than that already given seems necessary.

Irmisch (1858, pp. 44–45) has described at some length the relation of root to leaf arrangement in *Ruppia rostellata*, and for a more detailed account I would refer to him. The following is to some extent a confirmation of his observations as referred to *Ruppia maritima*.

The root is developed at the node, at one side of the leafbase, and in the comparatively rare cases where a second root is developed at the same node (Pl. III, fig. 10), it assumes a corresponding position at the other side of the leaf base.

Since the roots, when occurring singly, invariably issue from the same side of the stem, and since their orientation with respect to the leaf of the node is as just stated, the following facts or laws of arrangement are established:—

1. The line connecting the points of insertion of the roots is straight, provided the stem has undergone no torsion (Text-fig. 22 .

2. On account of the alternate arrangement of the leaves, the roots appear at successive nodes on opposite sides of the median longitudinal plane of the leaf.—i. e., if at one side of the leaf at any node, then at the other side of the leaf at the next higher or lower node Text-fig. 22 .

These simple rules of arrangement are, however, not often apparent. In a great many instances the stems to a considerable extent, due probably to currents of water, light conditions, &c., become twisted.

In such cases the orientat ionof the roots with regard to the leaves remains nevertheless constant. in accordance with the rule stated in ʻ2ʻ. In other words, no matter what position the leaves may assume on the stem. the roots always emerge. in any two successive nodes, at one side of the leaf at one node, and at the other side of the leaf at the next node.

Compared with the other Potamogetonaceae. Ruppia has perhaps the simplest root system, if one considers the number of roots alone. Zannichellia ʻCampbell. 1897, p. 40 , with generally two slender, un-branched rootlets at each node. and Althenia Prillieux, 1864, p. 182) with two or sometimes more, borne on alternate nodes, most resemble it. The roots of Cymodocea Bornet, 1864 are often borne singly, but differ in being branched. The roots of many Potamogetons ʻIrmisch, 1858ʻ are more numerous. arising often in a circular line of insertion at the nodes. In Zostera ʻGrönland. 1851ʻ and Phyllo-spadix Dudley, 1893ʻ a cluster of roots occurs at each node.

## C. Anatomical Structure

### 1. The Coleorrhiza.

Extending for a distance of often 5 mm. on the root from its junction with the stem is a peculiar sheath-like structure, termed by Irmisch ʻ1858ʻ the " coleorrhiza " or root sheath (Pl. VII, fig. 38; Pl. VIII. fig. 45. Text-fig. 21 . Beyond mentioning the fact that it occurs in _Potamogeton crispus, Zannichellia palustris_ and _Ruppia rostellata._ Irmisch gives no description of it. In the related plants which Prillieux ʻ1864 , Magnus ʻ1870ʻ, Schenck, (1886), Campbell ʻ1897 , and others describe, no mention is made of such a structure. Sauvageau ʻ1889ʻ indeed merely mentions a coleorrhiza as appearing in Zostera. Bornet ʻ1864ʻ notes in Cymodocea the formation of a

sheath " more or less short   at the base of the root. but is es not give a detailed account of it. so that its nature is not entirely clear. This is. however. probably a coleorrhiza.

According to Sachs (1875, p. 148) in grasses and some other Phanerogams. the first root arises so deep in the interior of the embryonal substance that in the fully developed embryo of the ripe seed it is enclosed by a thick, sac-like layer of tissue. which is ruptured on germination and is known by the name of " root sheath (coleorrhiza)." Similar formations occur also in the first secondary roots of the germinating plants of *Allium cepa*. and occasionally elsewhere.

The coleorrhiza[1] in Ruppia, which is found on all the adventitious roots, is of entirely different nature. Its development can be observed at all the early stages of the growth of the root. The first indication of it appears during the origin of the young root within the stem. As the young root develops and appears in the stem cortex, a change in character takes place in certain of the epidermal cells of the stem, and in a few in the sub-epidermal layers. which lie in the region whence the root would naturally emerge. These cells become filled with protoplasm, their nuclei enlarge, and cell division takes place. the resulting cells being small and narrow (Pl. III, fig. 10; Pl. VI, fig. 31; Pl. VIII, fig. 44). In other words. a new growth starts up in the epidermis and a few of the cells of the subjacent layer, but principally in the epidermal cells. This stage is, of course. the most important for the proof of an entirely independent correlative origin of the coleorrhiza.

As the root grows out from the node, the epidermal or coleorrhizal cells divide by anticlinal walls, keeping pace with the root growth for some time. and forming the pocket or enveloping sheath already mentioned.

Figure 22.— Diagrammatic representation of root arrangement; *r*, root: *L*, leaf.

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[1] Since the term " root sheath " may be applied to several sorts of structures. it lacks definiteness. The subject of the present account, however. has a distinct character, as will be demonstrated more fully below, so that it seems fitting to apply the name coleorrhiza to it.

A cross section of the coleorrhiza as represented in Text-fig. 23 shows that it is composed principally of a single cell-layer. The section was cut purposely near the junction of root with stem, in order to show how toward the proximal end of the coleorrhiza a few of the cells of the subepidermal layer enter into its composition.



As is shown by longitudinal sections, the cells in the outer or more distal region of this subjacent layer are undoubtedly the results of divisions similar to those which take place in the epidermal or coleorrhizal layer,
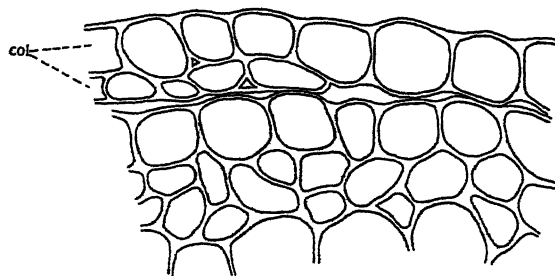
Figure 23.—Portion of cross section of root showing surrounding coleorrhizal layer. × 400.

but at the proximal or basal part of this subjacent layer the cells are larger and two or three rows deep, and those immediately adjacent to the root are considerably flattened. In this region, therefore, the subepidermal layers of the coleorrhiza are derived immediately from the cells of the stem without division, being forced outward by the development of the root and coleorrhiza.
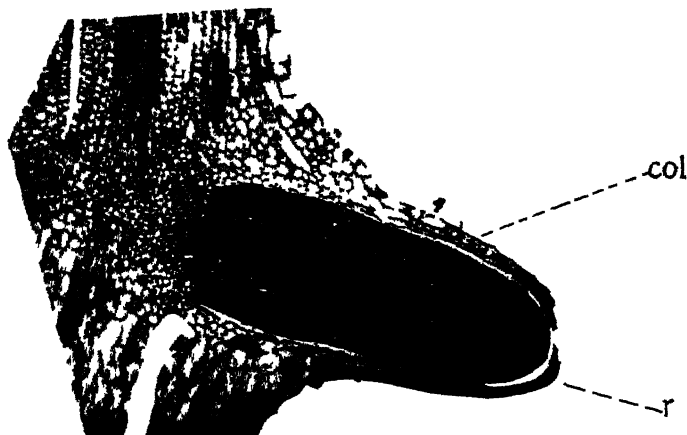


Figure 24.—Photomicrograph of longitudinal section of young root still enclosed in coleorrhiza; *col*, coleorrhiza; *r*, root. × 50.

Pl. VII, fig. 38 shows a well advanced stage, where by pressing with the cover glass the root was detached from its connection

with the stem and squeezed out of the coleorrhiza. producing the ruptured end of the latter. In this figure the coleorrhiza had attained approximately its maximum development. It then, with its enclosed root, appears to the naked eye as a small, smooth, blunt projection at the node. A photograph of a longitudinal section of this stage is shown in Text-fig. 24.

In the course of its growth the young root breaks through the coleorrhiza, which then remains at its base as a collar-like structure (Pl. VIII, fig. 45; Text-fig. 21).

An interesting incidental point in relation to the coleorrhiza, and one which indeed may be connected with its function, is the occurrence in its surface layer (when of more than one layer in thickness) of cells which are evidently secretion cells (Text-fig. 25).

It will be seen from the above description of the coleorrhiza in Ruppia that it is of entirely different nature from that to which Sachs (1875, 1. c.) alludes. It has an external origin, being the outgrowth of the epidermis (including part of the immediately subjacent region) of that part of the stem which is situated over a developing root.

Such a structure as this evidently falls into the category of "correlative" growths. Having no immediate organic connection with the root and yet taking its inception close on the development of that organ,

Figure 25.—Group of cells from surface of coleorrhiza, showing secretion cells. × 185.

the nature of its growth is analogous, for example, to the correlative growth manifested in the development of fruit and fruit-covering after fertilization of the egg cell and during the development of the embryo.

As to the function of the coleorrhiza in Ruppia, not much can be said with any certainty. Perhaps it acts as an organ of protection for the root until the latter attains some length. The facts that it occurs only in water plants, so far as known; and that also, borne on it, are abundant secretion cells, may point to a functional secretion of slime of some sort to protect the young root in the water. Goebel (1893, pp. 233–237) has noted this quite general production of slime and slime-producing organs in water plants and has explained in some detail its beneficial effects. It has already been

noted that the axillary scales probably serve a similar purpose of protection for the young growing points of the stem.

The references in literature to a similar root sheath or coleorrhiza are scanty. As stated above, Irmisch gives it a mere mention in his work, as occurring in Potamogeton, Ruppia and Zannichellia. A similar formation in Lemna is described by Caldwell (1899). Long ago Strasburger (1873) noted and described an epidermal root sheath in Azolla. According to him the sheath was formed of two layers of cells, the inner layer eventually disorganizing and leaving the root free inside the sheath, which itself surrounded the root during its entire existence. Leavitt (1902), however, finds only one layer producing the sheath in Azolla, and through this the root soon pierces, leaving it as a collar at its base. Quite recently Lachmann (1906) has described an essentially similar structure in *Ceratopteris thalictroides*.[1]

2. The Growing Point of the Root.

Four meristematic layers appear well defined at the root tip (Pl. VI, fig. 31; Pl. VIII, fig. 44). The calyptrogen is a well marked region with several rows of tabular cells. The dermatogen layer, one cell thick, encloses all of the remainder of the meristematic tissue of the root. This dermatogen layer is continous, even at the apex. Inside of the dermatogen layer the periblem rows converge until they meet at the apex in a small cell area one or often two cells deep. At this same area the plerome originates, and the sides of the plerome' cylinder are generally clearly marked off from the surrounding periblem.

Miss Daisy G. Scott (1906) in a recent paper has published the results of her investigations of the root tips of several aquatic monocotyledons. Among these are Ruppia, Naias and Zostera, in all of which the periblem and dermatogen arise from a single initial cell. These contain, then, according to Miss Scott, three apical meristematic regions, giving rise, the first to calyptrogen, the second to dermatogen and periblem together, and the last to plerome.

As will readily be seen, this arrangement disagrees with what I have described for Ruppia. Four meristematic regions are always clearly defined here. The dermatogen is invariably a continuous single layer, and the periblem arises quite often from a two-layered region at the apex, always showing at least one layer (Pl. VIII, fig. 44).

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[1] Cf. also Goebel. 1898, pp. 469—470.

A four-layered type has indeed been known to be unusual since De Bary s 1884. p. 9 work in which he describes it as appearing in only two plants, Pistia and Hydrocharis. Recently Campbell 1897. p. 40) has added Zannichellia to this list.

Beyond the papers of Miss Scott and Campbell. I find no record of other investigation on the apical meristem of the root tips of the Potamogetonaceae, if we except *Potamogeton crispus* and *natans,* studied by Treub (1876) and Flahault (1878), who assign to them three meristematic layers.

Considerable doubt and uncertainty have arisen concerning the tissue-forming capabilities of the various meristematic tissues of the growing point, a subject on which Scott (1894) has written an admirable resume. Histogenesis in the roots of Ruppia is. however, perfectly clear. The plerome can be traced in its development into the central cylinder, communicating with the central cylinder of the stem. Thus, Pl. VIII, fig. 44 shows a plerome cylinder connecting
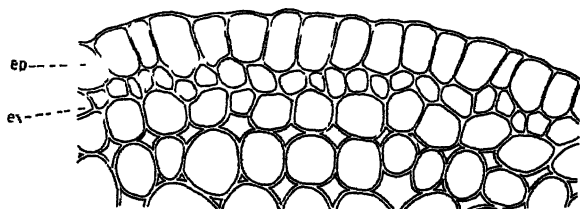


Figure 26.— Cells from outer part of cross section of root ; *ep.* epidermis, *ex,* exodermis. Showing also two or three layers of cortex with intercellular spaces. ×325.

with the central cylinder of the stem. The width of this plerome at its base is about eight cells, which corresponds to the number of cells in the diameter of the central cylinder of the mature root— i. e., 7—10 (Pl. VIII, fig. 46).

8. The Anatomy of the Mature Root.

The internal structure of the root is even simpler than that of the stem. It consists of an epidermal region and a large zone of cortical parenchyma limited on the inside by the endodermis which surrounds the axial vascular bundle (Pl. VI, fig. 27). Each of these four parts will now be described in detail.

a. Epidermal Region.

The epidermis is a layer of large, thin-walled cells (Text-fig. 26), some of which produce the root hairs, to be described in detail presently. Beneath the epidermis, but not connected with it genetically,

is a layer of small, thick-walled cells, lacking intercellular spaces —evidently a strengthening layer (Text-fig. 26, *ex*). This may be designated the epidermoidal layer, as described by Juel (1884), or exodermis, as defined by Vines (1898, p. 111) and Strasburger (1908, p. 102).

(1) The Root Hairs. In a recent paper, Leavitt (1904) has published the results of his investigations on the root hairs of the higher plants, a subject which has hitherto been much neglected and concerning which only scattering references can be found here and there in the literature.

Leavitt has ascertained that the root hairs are of two types, depending on their manner of development. The first type may arise from any ordinary epidermal cell. The second type of trichome, common to most monocotyledons, arises from a specialized cell, which, in its embryonic state, Leavitt has termed a trichoblast.

It is to this second type that, as briefly noted by Leavitt, (1904, p. 292) *Ruppia maritima* belongs. According to my investigations, the root hairs arise here in the following manner. At a short distance back of the growing point, certain of the epidermal cells begin to differentiate. They are easily distinguished by their deeply staining, granular contents, and their larger nuclei, (Pl. VI, figs. 29, 32, 33). As to their size, there is some variation, but on the whole they are of about the same size as the ordinary epidermal cells. These peculiar cells are the primordia of the trichomes, called the trichoblasts, which by the subsequent development of tube-like projections from their free walls will produce the root hairs (Pl. IX, fig. 48).

As to their mode of origin, there seems to be no regularity. Often two or three non-piliferous cells alternate with the trichoblasts, but again, as many as a dozen or more ordinary cells may occur between two trichoblasts (Pl. VI, figs. 29, 32, 33).

Not all of the trichoblasts develop root hairs. They may remain simply as large cells with granular contents and large nuclei. They are evidently, then, as Leavitt has also noted, potential trichomes, the development of root hairs depending possibly on some stimulus. This explains why we find zones, often at considerable distances apart, where the hairs are developed in abundance.

A remarkable characteristic of the root hairs is their persistency. In roots which measured over 20 cm. in length, quite a number of hairs were noted still intact and apparently functioning even at the base of the root near the coleorrhiza. This condition is very common and may be due to the fact that the hairs are not worn off easily in the soft mud. Often, however, hairs do not

persist in this way—becoming broken off. Their basal parts then appear as blackened dead cells in the epidermis.

Leavitt (1904, p. 279) lays considerable stress on the small size in general of the trichoblasts and bases of trichomes as compared with the ordinary epidermal cells. In Ruppia, while some of the trichoblasts and bases of mature trichomes may occasionally be somewhat shorter, on the whole they average about the same size as the non-piliferous cells, even from the very first appearance of the trichoblasts (Pl. VI, figs. 29, 32, 33). In her paper on root hairs, Miss Snow (1905), from the study of several species, assigns no definite length to the hair-producing cells. but announces that in the same root the average length of the trichome-cells is less than that of the atrichomic cells.

b. Cortex.

Although to the cortex proper belong genetically the exodermis described above under the head of epidermal tissue, the cortical parenchyma, and the endodermis, I am describing each separately. Varying with the thickness of the root, from six to twelve concentric rings of large, rounded, thin-walled cells with diamond-shaped, schizogenous intercellular spaces. form the cortical parenchyma (Pl. VI, fig. 27). Often. in the mature root, on account of the radial expansion of the tissues, these parenchyma cells undergo a stretching, and separate from each other laterally, producing long strings of collapsed cells.

A longitudinal section proves that the intercellular spaces of the cortical parenchyma in the region of the endodermis, are often of a somewhat peculiar nature (Pl. VIII, fig. 47, *u*). These spaces arise by a local splitting of the wall, the split parts curving outward in opposite directions, forming openings which in the longitudinal section appear from spindle-shaped to circular in outline. These spaces may occur in series or singly at intervals. Near the endodermis they are small, increasing in size in the direction of the middle cortex. Before we come to the middle cortex, however, we find them elongated into long narrow canals, of which the diamond-shaped spaces described above are cross-sections.

c. Endodermis.

A single—occasionally, in places, double—ring of cells surrounding the vascular bundle, comprises the endodermis (Pl. VIII, fig. 46). Its radial walls show to a very slight degree the typical endodermal thickenings. The tangential walls are minutely wavy at irregular intervals. After treatment of cross-sections of the root with concentrated sulphuric acid, the endodermis as well as the epidermis and exodermis remain clearly defined.

d. Vascular System.

This is represented by a single vascular bundle in the center of the root, with a structure similar to that of the stem (Pl. VI, fig. 27 : Pl. VIII, figs. 46, 47). No longer a typical radial root bundle, its present concentric structure admits of the same interpretation as regards adaptation as does the simplified bundle of the stem (see pp. 81—82). It differs from the stem bundle in only a few minor details. Barely as large as that of the stem, in all cases observed the root bundle retains intact the axial vascular area composed of a few tracheae. These tracheae are peculiar in that they possess no thickenings of any sort. Their walls, although extremely thin, can be distinctly seen in cross and longitudinal sections (Pl. VIII, figs. 46, 47, *tra*). Schenck has observed a similar peculiarity in the root of *Potamogeton densus*. He says (1886, p. 61), "Die Gefäße aber erfahren keine deutliche Differenzierung der Wandung; auf Längsschnitten sieht man keine Verdickungen oder nur schwache Spuren, so daß die Gefäße als Gänge erscheinen. Überhaupt erfahren in den Wurzeln der monocotylen submersen Gewächse die Gefäße, obwohl sie überall sofort zu erkennen sind, keine weitgehende Ausbildung. Die Resorption der Querwände tritt meist ein, bevor Verdickungen angelegt werden."

Five or six sieve tubes, with their companion cells, are situated at more or less regular intervals just inside the endodermis, forming an irregular ring. These can be distinguished in cross-section by their generally larger, nearly empty cell-cavities, their position just inside the endodermis, and their accompanying companion cells. In longitudinal section the sieve plates may be seen, as also the accompanying companion cells with their protoplasmic contents (Pl. VIII, figs. 46, 47 *s*).

The remaining tissue of the bundle is parenchymatous, and on account of its disposition in all parts of the bundle, it is impossible to distinguish phloem and xylem parenchyma. It may therefore be termed "connecting tissue" in accordance with Van Tieghem's (1870—71) "tissu conjonctif" or Schenck's "Verbindungsgewebe" (1886, p. 59).

e. Brief Comparative Study of Roots of Other Potamogetonaceae.

In general, *Potamogeton pectinatus* and *Zannichellia palustris* agree with Ruppia in the vascular structure of their roots. The main difference is the presence of only one central trachea, which is considerably enlarged (Schenck. 1886, p. 61, figs. 80, 81). I find no reference in the literature to the cortex or epidermis of these plants, but see no reason why it should differ to any extent from that of Ruppia. Very probably also other slender sub-

merged Potamogetons. such as *P. filiformis*, have a similar root structure.

Another plant, whose roots resemble closely in internal structure those of Ruppia. is *Althenia filiformis*. Epidermis. exodermis and cortex are practically identical with the same tissues in Ruppia, and although Prillieux's (1864) description and figures of the bundle leave much to be desired, there is evidently a strong resemblance to Ruppia.

The structure of the mature root of Cymodocea apparently differs entirely from our plant, most certainly in the composition of the central cylinder, according to Bornet's (1864) description and figure.

## D. Functions of the Root

As is well known, the two main functions of the root are—1. to absorb water and watery solutions from the soil, and 2. to act as organs of attachment. That these functions are both of much importance in the roots of land plants is an established fact; that their relative importance assumes the same proportions in submerged plants is not so well established.

Some authorities, such as Sachs (1887) and Vines (1898), have expressed the view that the roots of submerged plants are used mainly as organs of attachment. Schenck (1886, pp. 57, 58) modifies this somewhat, concluding that the absorption is of not much importance, but may possibly supply mineral solutions from the soil.

Strasburger's (1891) view, as expressed in his work "Über den Bau und die Verrichtungen der Leitungsbahnen in den Pflanzen," is similar to Schenck's. In the Bonn Textbook (1908, p. 165), however, Noll says, "Wasserpflanzen .... vermögen Wasser und gelöste Stoffe überall an ihrer Oberfläche aufzunehmen. .... Die in den Boden eindringenden Wurzeln vieler submersen Pflanzen tragen aber zur Ernährung und zum Gedeihen dieser Wasserpflanzen wesentlich bei"—thus laying considerable emphasis on the absorptive power of the root.

Pond (1905) has indeed recently proved quite conclusively that this absorption is of more importance than was formerly supposed— to such a degree that, for example, in *Ranunculus aquatilis tricho-phyllus,* one of the various submerged plants he has experimented upon, specimens rooted in soil exceeded in growth those rooted in clean washed sand 62.96%. Similar results were obtained by him with other submerged plants.

Not only the evident carefulness with which he has conducted his experiments. but also the variety of species and number of specimens used, joined with his very positive results, should put at rest all doubt as to the importance of the absorptive rôle of the roots of submerged plants.

On the other hand, Pond's experiments fail to show conclusively whether or not water and dissolved salts are absorbed by the part of the plant above the soil—a condition which is indicated by the reduction of the xylem area of the root.

Too much emphasis, therefore, must not be laid upon the absorbing capacity of the root, as is shown by the following considerations:—1. the absence of transpiration as we are acquainted with it in land plants; 2. the probable absorption by the parts of the plant above the soil of nutrient solutions from the surrounding water; 3. connected with these two conditions a reduction of the xylem area to a few tracheae of rudimentary nature; and 4. the total lack of branches and the slenderness of the roots.

In brief, the absorption carried on by the roots of submerged plants and the importance of this function in the economy of the plant is much greater than is implied by Schenck, as the experiments of Pond demonstrate; but, on account of the peculiar environmental conditions of submerged plants, it can never equal in importance the absorption of the roots of land plants.

# THE ECOLOGY OF THE VEGETATIVE ORGANS

In view of the fact that Ruppia is a submerged plant, living under water at all seasons of the year, and unable to live out of it, a study of the methods by which it accommodates itself to this life is of interest, especially when one compares it with a typical land plant.

Of course, in a comparison of these two types, the one point of vital importance to be noted first of all is the radical difference in the nature of their environment: the land plant pushes its stem and leaves into the air, a gaseous medium; while the submerged plant extends its shoots entirely surrounded by water, a liquid medium.

The striking formal, structural, and physiological dissimilarities which obtain between land and submerged plants have their fundamental origin in the physical differences of these two media and the concomitant variations in quality and intensity of light, temperature, &c. These physical factors have already been clearly stated by Warming (1902, pp. 127 ff.), Schimper (1898), and others, so that it is unnecessary to recount them here.

On account of the various methods of ecological classification by different authors, the assignment of Ruppia to a definite ecological group is not as easy as might seem at first sight. Among his four ecological groups Warming (1902, p. 121) defines the hydrophytes as being those plants that are surrounded wholly or for the most part by water, and those that grow in very moist earth. Of the subdivisions of this group the "Enalid society or sea grass vegetation" includes such plants as Zostera, Cymodocea, Phyllospadix, Potamogeton, Althenia, Ruppia, &c. (l. c. p. 156).

More recently, however, a tendency has been manifested to restrict the term hydrophytes to plants of fresh water only (e. g. Atkinson, 1905, p. 484). According to this view Ruppia is excluded and must be classified as a halophyte, an arrangement which seems reasonable, if one accepts the literal meaning of the term "halophyte."

As a matter of fact, a study of the plant shows that the great majority of its adaptive characters fit it for membership in the hydrophytes, and only a few—possibly not more than one or two —features added to its hydrophytic characters would qualify it for

a halophyte. And when one reviews the strikingly diverse char-
acters exhibited by plants living in salty soil—which is the vegetation
one naturally thinks of as halophytic—and by those living in salt
water, it seems best to avail ourselves of Schimper's (1898, p. 817)
terms "land halophyte" and "water halophyte" as a solution of the
difficulty.

Considered as a water halophyte, therefore, Ruppia has both
hydrophytic and halophytic adaptations. I shall outline first the
former, taking up in order those modifications that occur in the
shoot and in the root, and secondly describe the halophytic adaptations.

## HYDROPHYTIC ADAPTATIONS

## A. The Shoot

### 1. Gross Morphological Adaptations.

In its external form and style of branching the stem of Ruppia
presents a marked contrast to the stem of a typical land plant.
Since water has such a greater density than air, it becomes no
longer essential for the plant to have a stout, firm axis capable of
holding itself erect and supporting numerous branches. Instead we
find the stems of Ruppia long and slim, and except at the very
apex, of equal diameter throughout. For the same reason the
branches are similar, and as long or longer than the principal axis
from which they arise. Hence the peculiar wide-spreading branch
system which I have already described—a type impossible in a
land plant without a copious development of strengthening tissue.

The leaves of Ruppia, arising from such a weak, slender stem,
are of enormous length compared with the leaves of an ordinary
land plant having a main axis of similar diameter. This extreme
length is again made possible by the greater density of the sur-
rounding medium. A leaf of similar proportions in a land plant
would have great difficulty in holding itself out in a plane suitable
for receiving the rays of light.

The advantages of this extreme length seem quite apparent.
Coupled with the narrowness of the leaf, the effect is just as in
aquatics with finely divided leaves, such as Batrachium, Cerato-
phyllum, &c., i. e., to present a large amount of surface to the
water in proportion to the volume of the leaf. This increase in
leaf surface is beneficial for several reasons. In all probability much
of the nutrient mineral solution requisite for metabolism is absorbed
by water plants directly from the surrounding water. Moreover,

both oxygen and carbon dioxide are necessary for the life of the water plant just as in the case of the land plant. Of the gases in solution in water, both oxygen and carbon dioxide are present. indeed, in a greater proportion than in air. The diffusion of these gases, however, takes place much more slowly than in the air, so that the larger the area of leaf surface. the more readily will the requisite amount of gases be absorbed (Warming, 1902, p. 127 and Goebel, 1898, pp. 248 ff.).

It is obvious, therefore, that for the purpose of carrying on its absorption both of gases and nutrient mineral solutions in the best manner possible, the long, slender leaf of Ruppia is an ideal form.

In the salt-marsh creeks and ditches where it makes its home, Ruppia is constantly subjected to the action of fairly strong tidal currents. On this account also it is advantageous to the plant to have slender leaves; for such easily accommodate themselves to these alternately inflowing and outflowing tidal currents. This accommodation becomes the easier from the fact that the leaves are not dorsiventral. but alike on both flat surfaces and, therefore, bisymmetrical; a condition which may be referred to the diffuseness of the light in the water, as well as the constant moving about of the leaves, resulting in a tendency to equality of conditions on both sides of the leaf.

A condition of general ecological significance, which affects all ·of the vegetative organs, but particularly the shoot system, has to do with the tidal currents mentioned above. At times these currents are even so strong as to detach portions of plants, which I have often observed floating up or down stream, according to the tide. These plant segments become entangled in grass, &c. along the stream edge, or collect in some miniature cove and then begin an independent existence, at perhaps a considerable distance from the parent plant. This is probably a common mode of vegetative reproduction among such water plants.

2. Internal Structural Adaptations.

a. Epidermal modifications.

In many respects the epidermis of stem and leaf clearly reflects environmental influence.

In contrast to the thick outer or free walls of the epidermal cells of land plants, we meet here with a thin wall. Surrounded by water, there is no danger of the drying up of the plant by evaporation from the epidermal cells—a process which, on the other hand, commences immediately on exposure of the plants to the air—and consequently a thick wall would here be superfluous. The fact that even in Ruppia the outer wall of the epidermal cell is slightly

thicker than the others, may possibly be attributed to the necessity for some slight degree of firmness in the covering of the shoot.

Besides the thinness of the walls, the epidermis of the leaf exhibits the following two remarkable peculiarities, which have already been observed in similar aquatics by Warming (1902), Schenck (1886), Goebel (1893) and others, and need not, therefore, be entered into in detail here.

The light is weakened to such an extent by reflection on the surface of the water, absorption in the water, &c., that most of the chloroplasts, for the purpose of the best illumination possible, are located in the epidermal cells, which therefore assume the rôle of photosynthesis, but yet have not at all the shape of the palisade cells of land plants.

As in the majority of other submerged plants, no stomata occur in Ruppia, nor, as already ascertained by Sauvageau (1891, II, p. 209) any of the apical leaf pores found by him in other water plants, so that openings of any kind are lacking in the epidermal covering. The reasons for this, dependent on the characteristic mode of food absorption, the lack of a transpiration current as it occurs in land plants, the extreme permeability of the leaves of aquatic plants to gases, &c., have been fully elaborated by the authorities quoted above (Schenck, 1886 and Goebel, 1893) and need not be dwelt upon here.

It seems to be generally admitted that where stomata do occur in submerged species, they are to be looked upon as hereditary structures. rather than as possessing any ecological significance.

Schenck (1886, p. 6) claims that stomata in submerged leaves are positively harmful, admitting the water into the air reservoirs located in the lacunae. Sauvageau (1891, II), although admitting their uselessness, maintains that they are not harmful to the plant. They have gradually disappeared from the leaves of water plants, not because they are harmful, but because they are useless.

Development of slime. The slime developed by the axillary scales in the shoot has been already treated in detail. It is of ecological significance in that a protection is thus effected for the delicate growing points against their aquatic environment, the protective function of slime being well known (cf. Goebel, 1893, pp. 232–237).

b. Development of Air Spaces.

The formation of large and small intercellular air spaces, most pronounced in stem and leaves, is one of the most striking histological characters of the shoot system. In general, the larger of these air spaces, such as the zone occurring in the stem, and the two

longitudinal rows present in the leaves, receive the special name "lacunae."

The function of these lacunae has not yet been sufficiently investigated. So far as is known, they occur in all water plants. Schenck (1886, p. 49) states that water plants grown on land diminish the size of their air spaces and, conversely, land plants grown under water reveal a tendency toward a loosening up of the cortical parenchyma. This necessarily indicates that the formation of air spaces is in some way connected with and necessitated by an aquatic environment. Haberlandt (1896) has probably hit the truth of the matter when he explains that this is a method of obviating the difficulty of osmotic interchange of gases in submerged plants : " Es wird eine " innere Atmosphäre " geschaffen, mit welcher die geschiedenen Gewebe einen lebhaften Assimilations- und Atmungsgaswechsel unterhalten." The larger these inner air reservoirs are, so much less will the plants have to suffer the difficulties of direct interchange of gases with the surrounding water. It is probable also that the considerable amount of air and gases enclosed in these intercellular spaces is of advantage for submerged as well as for floating plants, by its lessening of the specific gravity of the plant.

c. Absence of Mechanical Tissue.

As has been, in part, pointed out above (see p. 116), a land plant with dimensions of stem, branches and leaves similar to those occurring in Ruppia, must of necessity develop considerable mechanical tissue. But in no part of the vegetative organs of Ruppia is a characteristic thick-walled tissue developed. That such mechanical tissue is entirely absent is to be explained in the greater supporting capacity of water as compared with air, resulting from the greater density of the aqueous medium. In this way this lack of supporting tissue is to be considered as an adaptation to environment.

d. Reduction of the Vascular System.

The vascular system is greatly reduced. If we except the minute cortical bundles of the stem and the small lateral leaf nerves, the entire vascular system of the shoot is represented by a single axial vascular strand. The xylem portion mostly disappears in the mature stages except at the nodal regions, leaving a central cavity in the vascular area. The phloem portion is, however, in all cases intact, consisting of sieve tubes, companion cells and phloem parenchyma.

One may lay this vascular reduction entirely to the different mode of nutrition employed by submerged plants. On the one hand, the food solutions formerly conveyed from the roots by way of the stem are probably now absorbed in large part from the surrounding water,

and on the other hand, the transpiration current, unaided in its work by stomata or even by apical leaf pores, and, moreover, rendered unnecessary by the presence of water on all sides. must inevitably be diminished.

That the sieve tubes and their companion cells should remain intact simultaneously with a complete degeneration of the xylem, seems reasonable when we consider that the function of the phloem is the transportation of *elaborated* food.

In another respect the reduction of the vascular system is of ecological importance ; for not only are the vascular elements reduced quantitatively, but also the fact that practically all of the vascular system is concentrated into a single bundle, which is axial, deserves especial note.

That this axial position of the bundle in the stem is considered to have been attained phylogenetically, through a gradual displacement of the more peripheral bundles toward the center of the stem and fusion there into a single concentrically arranged bundle, has already been touched upon in the account of the morphology of the stem (p. 81–82).

Ecologically considered, this axial arrangement enables the plants to bend about easily and accommodate themselves to the movements and currents of water. For the axial bundle of the stem, with its slightly thickened endodermis enclosing the long cells of the vascular tissue, is naturally the region most resistant to bending movements. Now, it may easily be seen that a plant with its vascular area in such a position is capable of bending much more readily than one in which this area is more or less peripheral, as in the typical land plant. Such an arrangement is analogous to the axial strand of roots, and there subserves a similar purpose.

## B.  The Root

### 1. Gross Morphological Adaptations.

Another result of the absorption of nutrient solutions by the shoot system directly from the surrounding medium is the great reduction of the root system. This consists entirely of slender, unbranched, adventitious roots arising singly at each node of the creeping stem —a simplicity of form and development which is correlated with the function of the root, and which has been already discussed in detail under the morphology of the root (pp. 108–114).

As has already been pointed out, the development of a coleorrhiza or root sheath may have some bearing on the environment, protecting

the root in its very young stages. It may be here again noted
(cf. p. 107) that possibly the soil or the water, containing common
salt in solution, would injure the very young root if not thus protected.

2. Internal Structural Adaptations.

a. Air Spaces.

The root cortex, as is the case in the shoot, shows many inter-
cellular spaces, but much smaller. These, however, are often made
larger by the collapsing of the cells. The function of these air
spaces may probably be explained in the same way as in the shoot
(p. 119).

b. Reduction of the Vascular System.

The vascular bundle of the root, of essentially the same structure
as that of the stem, admits of the same interpretation, as regards
the reduction of its elements, as in the stem. Especially significant
is the reduction in this case, however, since the root is properly
the absorbing organ of an ordinary plant, and as such should possess
at all events a well developed vascular system. The absence of such
a one here is the most conclusive proof of the comparative small
amount of absorption carried on by the root system.

In this connection the presence of thin-walled tracheal cells without
thickenings, still intact in the xylem area of the bundle, is inter-
esting in comparison with the axial canal caused by their dissolution
in the stem. Their presence may indicate a need for them in the
root. and therefore some degree of absorption by this organ.

### HALOPHYTIC ADAPTATIONS

## A. General

It was pointed out by Schimper (1890, p. 1047, and 1891, pp. 25 ff.)
that any considerable amount of salt in the cell sap is detrimental
to the plant. and that here we have the probable cause of the
characteristic halophytic modifications, which aim, therefore, at a
lessening of the transpiration current. To this Warming (1902, p. 809,
810) replied that even if transpiration were diminished to a very low
degree, slowly but surely an amount of salt would be collected in
the plant which would eventually prove fatal. On the other hand,
Warming saw better logic in another idea of Schimper (1890), which
has become the widely accepted view at the present time—namely,
that the protective contrivances against strong transpiration are
necessary in halophytes, because absorption of water from a salt
solution is slow and difficult.

On account of this difficulty in water absorption, then, one finds exhibited in land halophytes many water-storing devices and typical xerophytic methods of diminishing transpiration.

The water halophytes, on the contrary, especially the submerged aquatics, such as Ruppia, Zostera, Phyllospadix, &c.—members of the Enalid hydrophytic society of Warming (1902, p. 156) as above noted—evince none of these modifications. The hydrophytic adaptations already described for Ruppia show how closely this plant resembles a typical submerged fresh water hydrophyte. Although surrounded by salt water to a much greater extent than land halophytes, yet it shows none of the characteristic xerophytic modifications which are associated with land halophytes in general.

The explanation of this may be based on the fact that transpiration, as such, does not appear in Ruppia. The comparatively small amount of water absorbed by the roots, the absence of any openings in the leaf through which water could pass, such as Sauvageau (1891, II, pp. 127 ff.) has described for Zostera, Phyllospadix, Halodule and Potamogeton; moreover, the fact that the epidermal cells of the leaf, with the exception of the secretion cells, are all photosynthetic and absorb solutions from the outside into the interior, indicate the giving off of a very small amount of water, if any from the leaves. Waste gases, however, can easily pass out through the cell walls in solution. On account of these conditions, structures adapted to the retention of a supply of water and reduction of transpiration, are unnecessary, and we accordingly find an absolute lack of such halophytic adaptations in Ruppia.

## B. The Adaptation to a Salt Water Environment

In one particular, however, Ruppia shows a distinct halophytic adaptation. This is exhibited in its power to withstand the plasmolytic action of salt water. Ganong (1908) has found that the root hairs of certain land halophytes possess specific abilities to resist plasmolysis in various solutions of sea water, showing in this way a greater or less halophytic adaptation. He says (1908, pp. 353, 354), "I found a close correspondence between the halophilism of a plant and the power of its root hairs to resist plasmolysis. This power has of course been gradually acquired. but what its physical basis is I do not know; though we shall probably find that substances osmotically equivalent to the salt of the sea water have been formed in the sap of the hairs."

But Ruppia, and all of the allied salt water genera, such as Zostera, Phyllospadix, &c., show an even greater adaptation than

these land halophytes, since they live and flourish entirely in a salt-water medium.

Wishing to ascertain how great a concentration of salt water *Ruppia* would stand without plasmolyzing, I tried first a 105 per cent solution, obtained by evaporating salt water (taken at high tide at Savin Rock, near the entrance to New Haven Harbor), to the desired concentration. As is indicated in the following table, (p. 125), both leaves and root hairs showed occasionally a very slight plasmolysis. In the root hairs, indeed, the protoplasmic movement continued, although in most cases the ends of the hairs, especially, showed a slight plasmolysis. A solution of 110 per cent strength, however, prepared in the same way, produced a marked plasmolysis in both leaves and root hairs.

In order to get a comparison with submerged fresh water plants, I experimented with the leaves and root hairs of Elodea and Callitriche, with the results as shown in the table (p. 125).

Compared with Ganong's results, a few of which also are given in the table, my experiments show that the plasmolysis index in Elodea and Callitriche is about equal to that of Atriplex and Hordeum, so that there is very little halophytic adaptation in this respect in the latter plants. This is to be expected, since Ganong (1903, pp. 359. 360, 364) expressly states that these grow in the higher and drier places of the salt marsh.

Another point illustrated by the experiments with Elodea and Callitriche is the slightly greater resistance in both cases of leaves as compared with root hairs. This is hardly to be looked for, since it would naturally be supposed that the roots would be accustomed to somewhat stronger solutions in the soil and would therefore evince a greater resistance to the plasmolyzing action.

Elodea, as is natural, being a very delicate plant, is more sensitive in both cases than Callitriche.

Being curious to know how much pure salt (sodium chloride) Ruppia would stand, I used solutions of 2.5, 3, and 5 per cent with the following results.

1. Leaves of *R. maritima*. placed in 2.5 $^o/_o$ NaCl solution. did not plasmolyze.
2.   „   „  „    „      „   „ 3  $^o/_o$  „    „ plasmolyzed in 4–5 min.
3.   „   „  „    „      „   „ 5  $^o/_o$  „    „    „ in about 1  „

Since ocean water in general is known to contain about 3.5 per cent of salts (Atkinson, 1905, p. 622), it would seem from the above that Ruppia could not live in it. And yet, as the table (p. 125) shows, Ruppia does not plasmolyze in the salt water of New

Haven Harbor. and not often even in a 105 per cent strength solution of this.

This would lead one to conclude *a priori* that the water of New Haven Harbor does not contain as much salts as the 3 per cent quantity of NaCl of the above experiment, and certainly not as much as the 3.5 per cent of ordinary ocean water.

On next ascertaining the percentage of salts contained in the water of New Haven Harbor (procured, as in the first experiment, from Savin Rock, near the entrance to the Harbor, at high tide), I found by evaporation that the proportion was about 2.8 per cent. as would be expected from the *a priori* conclusion. This amount, therefore, explains why Ruppia plasmolyzed in a 3 per cent salt solution, but did not plasmolyze in the water of New Haven Harbor.

The sea water used, although purposely obtained at the entrance to the harbor and at high tide, is, therefore, quite brackish. The fact that Long Island Sound is considerably shut off from the ocean and also has several large rivers emptying into it, probably accounts for this. Very probably a similar percentage of salts prevails all along the Connecticut coast.

It is possible that the sea water used in Ganong's experiments had a greater content of salt. If so, his results with Atriplex and Hordeum mean somewhat more than the comparison in the table indicates.

An interesting point which should not be overlooked, since it shows how delicately adjusted these plants are, comes to light in the sometimes slight plasmolysis of Ruppia in the 105 per cent solution. A little calculation shows that this is about identical in strength with the 3 per cent salt solution, at which plasmolysis occurs very slowly. It is perfectly clear, then, that Ruppia is adapted to life in water containing a solution of sodium chloride and other salts ; that this solution does not equal in strength that of the ocean in general and hence may be termed brackish; and furthermore, that Ruppia could not live in such ocean water; that this adaptation of the plant is brought about through an ability to resist plasmolysis by maintaining in some way a higher osmotic tension than prevails in submerged fresh water plants, probably by the presence of an equalizing salt solution in the cells themselves.

## RELATIVE RESISTANCE TO PLASMOLYSIS.

| Percentage of Sea-water | LEAF (epidermal cells) | | | ROOT (root hairs) | | | | | Comparison with Results of Prof. Ganong on Root Hairs of Land Halophytes [1] | |
|---|---|---|---|---|---|---|---|---|---|---|
| | Elodea cana-densis | Calli-triche palustris | Ruppia maritima | Elodea canadensis | Calli-triche palustris | Ruppia maritima | Salicornia herbacea | Suaeda maritima | Plantago maritima | Atriplex patulum |
| 10 | — | — | — | — | — | — | — | — | — | — |
| 20 | — | — | — | — | — | — | — | — | — | — |
| 30 | — | — | — | — | — | — | — | — | — | — |
| 10 | — | — | — | — | — | — | — | — | — | — |
| 50 | very gradual appearing in about 7 min. | — | — | very slight | — | — | — | — | — | — |
| 60 | bet. 40 s. & 2 m. | very gradual 7—9 m. | — | gradual, but finally marked | rather gradual | — | — | — | — | — |
| 70 | 35—75 s. | 45 s.- 2½ m. | — | rather grad-ual but final-ly marked | distinct | — | — | plasmo-lysis | — | — |
| 80 | 12—15 s. | 45 s.—3 m. | — | 1 min. | — | — | — | plasmo-lysis | — | — |
| 90 | 10 s. & under | 25 s.— 1¼ m. | — | 1 m. | — | — | — | — | — | — |
| 100 | immediate | 15  45 s. | — | 30 s. | — | — | — | — | — | — |
| 105 | immediate | Im-mediate | — | 20 s. | immediate | occasionally very slight | plas-molysis | plas-molysis | plasmo-lysis | — |
| 110 | immediate | im-mediate and fairly rapid | — | immediate | immediate | rather slow but finally distinct | — | — | plasmolysis | — |

[1] Ganong, in all cases, gives solution which the plant "would endure without plasmolysis." I assume, therefore, that plasmol would occur at the next hi[gher] strength of solution.

## ADAPTATION AND HEREDITY

When one reviews the manifold ways in which Ruppia is adapted to its environment. it becomes clear that this plant represents an advanced stage of special evolution, resulting in a particular type of plant. growing entirely below the surface of the water and almost ideally adapted to the novel conditions which present themselves.

That the characters of aquatic plants cannot, however, always be explained on the hypothesis of adaptation, Sauvageau has well illustrated. Another factor should be reckoned with, namely, heredity. For instance, among other examples, Sauvageau (1891, II, p. 121) speaks of *Althenia filiformis* and *A. Barrandonii*, which grow side by side in certain ponds near Montpellier: "Les feuilles de la première ne possèdent jamais d'éléments épaissis; celles de la seconde, au contraire, ont non seulement leur unique nervure entourée d'un endoderme puissant, mais l'épiderme a ses parois plus épaisses. le limbe est parcouru par deux gros cordons fibreux plus ou moins lignifiés, et la gaine en montre plusieurs semblables. Ces faits sont complètement inexplicables si l'on admet l'action prédominante et quasi exclusive du milieu."

It is evident that as Schenck (1886. p. 7) has declared, adaptation and heredity are two opposing factors in the transformation of an organism, and that "l'état anatomique . . . . maintenant dépend assurément non seulement du temps depuis lequel l'adaptation a commencé, mais aussi de leur structure originelle et de leur résistance spécifique à l'adaptation. autrement dit, des caractères qui leur ont été legués par hérédité." (Sauvageau, 1891, II. p. 120.)

One meets in Ruppia very few of these useless structures handed down presumably from former generations. We have seen that the cortical bundles of the stem are evidently rudimentary structures, but one cannot affirm with certainty that they are now useless. Again, the tracheae of the axial bundle in the shoot become so disorganized in the mature condition that it would seem as if here were an ancestral structure which is no longer needed. And, as if to carry out this idea, we find the tracheae absolutely lacking in such highly adapted plants as Ceratophyllum and Naias (Schenck, 1886, p. 30).

## SUMMARY

1. It is best to classify Ruppia ecologically as a water halophyte. As such it exhibits both hydrophytic and halophytic adaptations.

2. The hydrophytic adaptations of the shoot are as follows: first, in its external form it shows a weak, wide spreading branch system, with slender stems and long grass-like leaves; second, in its internal structure, it reveals a thin-walled epidermis, photosynthetic and without stomata, in the leaves, a production of slime by the axillary scales for protective purposes, a copious internal development of air spaces, a complete lack of mechanical tissue and a reduction of the vascular system to a single main axial bundle and two small lateral bundles in both stem and leaf.

3. The hydrophytic adaptations of the root are as follows: first, in its external form, a reduction of the root system to small, un-branched, adventitious roots, borne singly at the nodes, and the formation of a protective coleorrhiza; second, in its internal structure, the presence of numerous air spaces and the reduction of the vascular system to a single, axial, concentrically arranged bundle similar to that in the shoot.

4. The characteristic adaptations of land halophytes are wanting here, for the reason that their cause, the need for reduction of transpiration, is lacking.

5. Ruppia shows, however, a remarkably interesting halophytic adaptation in its power to live in salt water, which, when applied to submerged fresh water plants, causes instant plasmolysis.

6. This salt water is not equal in strength to that of the open ocean, containing in New Haven Harbor, about 2.8 per cent of salt, and may therefore be termed "brackish".

7. My experiments show also that this power is confined to a very slight margin, i. e., that slightly concentrated harbor water causes plasmolysis in both root hairs and leaf cells of Ruppia proving that the plant, as now constituted, could not exist in ocean water.

8. The tracheae and cortical bundles are evidently more or less rudimentary and possibly represent useless structures handed down by heredity.

# THE REPRODUCTIVE ORGANS

As a preliminary to the account of the development of the flower and of the reproductive organs, it will perhaps be in the interests of clearness to preface a short description of the mature flower. In this connection I shall also take occasion to discuss briefly some of the more important morphological aspects of the flower.

The flowers of *Ruppia maritima* are small, (3–5 mm. in diameter), consisting of two stamens and generally four pistils, with no perianth, and are borne in a pair at the apex of the peduncle, occurring one above the other, on opposite sides of the rhachis (Pl. IX, fig. 49). The inflorescence is thus spadix-like, a type which is more pronounced in Potamogeton and Zostera.

The mature stamen resembles two thick, rounded, semicircular bands closely appressed to the rhachis (Pl. IX, fig. 49), and meeting each other on opposite sides of it, each band being the half of an anther. In the Bonn Textbook (1908, p. 422), these anther-halves are called " thecae," and because this term is shorter and more specific, it will be used in this paper. A comparison with figure 474 of that textbook (1908, p. 422) demonstrates how a stamen of this sort could easily be evolved by a gradual separation of the thecae. In Ruppia their complete separation at maturity has led them to be interpreted as single stamens with bilocular anthers, for Roze (1894, p. 476) says: "Je n'ai pu y parvenir, car je n'ai jamais trouvé, dans les anthères même jeunes, une adhérence, une soudure quelconque qui le (i. e., cette diminution du nombre des étamines) fît supposer. Et il est bien certain qu'à la maturité des organes, il est impossible de ne pas reconnaître que chaque fleur présente quatre étamines parfaitement libres, ce qui est le point essentiel." Pl. IX, fig. 50, however, gives a correct idea of the morphology of these thecae; for at this young stage the connective shows clearly that the structures on each side of it are merely the two thecae of the same stamen.

At this point, it is interesting to note in Pl. IX, fig. 50 the extension of the connective out beyond the plane of its attachment to the thecae. forming what is described by Irmisch (1851, p. 84) as " ein

kurzer, abgestumpfter. zuweilen etwas ausgehohltei freier Fcitsatz. This develops still further during the growth of the young flower. and sections of it are shown in Pl. IX, fig. 51. Irmisch suggests that this structure corresponds to the pseudoperianth-segment of Potamogeton, which Ascherson (1889) describes as a perianth-like dorsal appendage of the connective. A similar development appears in Posidonia (Ascherson, 1889). That Ascherson agrees with Irmisch's interpretation, is shown by his generic characterization of Ruppia: "Stb. 2, mit sehr kurzen, von den Antherenhälften überragten Anhängseln des Mittelbandes " (1889, p. 207). Eichler (1875, Pt. 1, pp. 89–91) had also supported Irmisch's view.

Čelakovský (1896, pp. 48, 49), on the other hand, believes that these scaly outgrowths from the connective represent reduced floral leaves (Perigonblätter), and deprecates the supposition that they are morphologically portions of the anther connective. Eichler and Ascherson get their strongest argument, of course, from Potamogeton, which shows such a stronger development of this anther-connective structure. Even here, however, Čelakovský (1896, 1. c.) sees only floral leaves which have become attached to the connective at its base, mentioning Hegelmaier's (1870) work as one foundation for his opinions.

The development from the connective in Potamogeton resembles strongly a floral leaf, although its connection with the anther-connective is quite pronounced. In the mature flower of Ruppia, after the stamens have fallen away, the same sort of structures may be seen, two in number, at the base of the group of four pistils, and opposite each other. These small structures show a very minute projection, the outgrowth of the connective, as was noted in the young flower (Pl. IX, fig. 50) and below, on opposite sides of it, the scars showing the places where the thecae were formerly attached.

Without going into detail, my own opinion is that the connective outgrowths in Potamogeton represent morphologically perianth segments; that is, I agree with Čelakovský, and if this interpretation is true for Potamogeton, it must be true also for the evidently closely related Ruppia, which, as Čelakovský (1896, p. 49 and 1900, p. 49) emphasizes, is a reduced flower. The reduction is shown not merely in the smaller number of floral whorls, but in this rudimentary condition of the perianth segment.

It will be seen that the appearance of the thecae in the young stage represented by Pl. IX, fig. 50 is much different from that in Pl. IX, fig. 49. As the rhachis elongates, the thecae grow in a

horizontal direction, around the rhachis. closely following its sur-
face, until finally they nearly meet, as in Pl. IX, fig. 49.

The pistils, occurring between the thecae in a group of four, form
a diamond-shaped pattern arranged in a transverse position on the
rhachis. Each pistil develops, in the mature flower, into a cyl-
indrical structure tipped with a sessile peltate stigma and con-
taining a single ovule.

As regards the number of the pistils, I have never found it to
vary ; but Ascherson (1889), who divides *Ruppia maritima,* his only
species of the genus, into three subspecies, says in his generic
characterization, "4 (selten bis 10)". Roze (1894, p. 479), indeed,
makes the number of pistils the main specific difference between
*R. maritima* and *R. rostellata,* alleging that the former has eight
pistils and the latter four. In this vicinity, however, *Ruppia maritima*
seems to have always four.

Eichler (1875, p. 89), Irmisch (1851), and other older authorities
do not hesitate to allude to the subfloral leaves of Ruppia (the pair
at the base of the inflorescence) as the spathe leaves. This homol-
ogy is not used, however, by more recent authorities. For reasons
already adduced, I have applied to them the name of subfloral
leaves. As to the spathe, however, it seems quite probable that
a rudiment of it is represented in the floral scale leaf. As has been
shown (pp. 99–102), its manner of origin, development orientation,
etc. all connect it ultimately with the flower, and although its mor-
phological origin is clearly identical with that of a vegetative scale
leaf, its relations to the spadix indicate a spathe-like nature. Since
the flower of Ruppia represents a much reduced type. we should
expect to find such reduced structures here.[1]

### FLORAL DEVELOPMENT

With these remarks on the general morphology of the mature
flowers, I shall describe briefly the developmental stages of the
young flower, from the time when it is first recognizable until about
the period when the archesporial cells first appear. From that

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[1] I have already indicated (p. 101) that Irmisch overlooked this spathe-
like floral scale leaf, and I had been unable to find any reference to it
in the literature. At the last moment, however, I find that Griffith (1851
I, pls. 257, 258 and 259: II, pp. 196 and 198) figured and described this
structure, considering it a true spathe, and explaining its origin and
development essentially as I have done.

point, however, the gross morphological development will be followed during the history of micro- and megasporangia and male and female gametophytes.

Very early the floral rudiment can be distinguished from the vegetative cone by its greater lateral development, giving it a comparative thickness, and because even here the two protuberances, each of which is to develop into a flower, are already apparent (Pl. IX, fig. 52). In Pl. IX, fig. 53 these two floral primordia have become more distinct. Pl. IX, fig. 54 shows a more advanced stage with the floral primordia acquiring the flattened disk form which characterizes them at this period, and the floral scale leaf which, as has been shown, corresponds to the spathe just appearing. In Pl. IX, fig. 55 all of these parts are still more developed, and in Pl. IX, fig. 56 one first sees evidence of a segmentation of the several parts of the flower. At this point each flower is still disk-shaped with the thecae of the young stamens—or their primordia —as four swellings situated diametrically opposite one another—the central part of the disk being elevated and representing the region of the future pistils. The rhachis also first appears well developed here, at least at its base.

Three points of interest will be noted in this early floral development: —

1. The origin of the flowers is lateral, forming a type of inflorescence which is not uncommon, and explained on the hypothesis that the nourishment of the vegetative cone is appropriated by the lateral members (Goebel, 1898, pp. 178, 179).

2. Both flowers in the spadix are of the same age. Their origin in the periblem takes place almost simultaneously, and they remain contemporaneous throughout their entire development. This occurs also in Potamogeton, according to Hegelmaier (1870).

3. The development of the anthers precedes that of the pistils.

Subsequently, the pistils appear as four rounded prominences. These develop by degrees, (but always much behind the staminal development) into the mature, rather cylindrical ovaries surmounted by sessile, peltate stigmas (Pl. X, figs. 58–66).

## MICROSPORANGIUM

The initial cells of the archesporium are first clearly recognizable at such a stage in the floral development as is represented in Pl. XI, fig. 68, where the length of a single young flower in section is

about 0.2 mm. and is about midway between the stages re-
presented in Pl. IX, figs. 55, 56. This stage is signalized extern-
ally by the definite appearance of the primordia of pistils and
stamens.

Pl. XI. fig. 67 shows the region marked x in fig. 68, being the
same section at a higher magnification, and reveals the cells in
one of the thecae of the upper stamen. The initial cells, indicated
by shading, are distinct from the surrounding tissue by reason of
their large size, their large nuclei, their dense cytoplasm, and
especially their strong reaction to stain. In these respects, all are
essentially alike.

As is apparent, not only the hypodermal layer, but also several
of the deeper-lying plerome cells contribute to this group, and
since they grade off imperceptibly into the sterile tissue below the
theca, it is well nigh impossible to draw a hard and fast line of
separation. Thus, it is quite probable that more of the interior cells
than I have designated are archesporial.

A point of interest in this connection, and, indeed, an additional
proof of the identity of these cells, is their previous history. Up
to about the stage represented in Pl. IX, fig. 55, the divisions of the
meristematic tissue comprising the flower rudiment follow one
another in rapid succession. From that period on, however, there
is a slight pause in karyokinesis, with the exception of the divisions
in the epidermal region, so that a count of the cells reveals
practically the same number in Pl. XI, fig. 67, as at the end of the
meristematic condition. But, in the meantime, a considerable
enlargement, cell for cell, has occurred. There is, then, previous
to the first unmistakable appearance of the archesporium initials
as shown in fig. 67, a brief cessation of cell division, more or less
complete, during which occurs a marked increase in their size.

In one of the Potamogetonaceae at least, namely, Zannichellia,
more or less uncertainty has always invested the origin of the
archesporium. Warming (1878, p. 28), long ago, in his study of
this plant, was of the opinion that the sporogenous cells did not
arise from a single archesporial layer, but was unable to state just
how they did originate. Recently, Campbell (1897, p. 41), in his
study of the same plant, says,—"The origin of the sporogenous
tissue of the anther is not easy to trace, as the archesporial cells
are at first hardly distinguishable, either in form or contents from
the adjacent cells. As soon as they are recognizable, there is
already a group of them whose relation to each other is not
entirely clear."

In his work on *Ruppia rostellata* (1902. pp. 4 and 5) Murbeck, indeed, locates a hypodermal archesporial initial layer, which is quite in line with the vast majority of results so far obtained among the Angiosperms. These hypodermal cells divide into primary parietal and into primary sporogenous cells, which develop in the usual way.

As is evident, Murbeck's report does not correspond with what I have seen in *Ruppia maritima.* But the statements of Warming and Campbell are strong evidence that the state of affairs I have described is quite possible. It is well known that Campbell has found a plerome origin for the archesporial cells in *Naias* (1897, p. 13) and in *Lilaea subulata* (1898, p. 8). A careful study of the history of the archesporial initials in *Ruppia maritima* from the meristematic stage, as outlined above, leaves no room for doubt that we are here dealing with a comparatively large group of cells which originates simultaneously both in plerome and periblem. Very probably, as Warming's and Campbell's studies indicate, Zannichellia develops in a similar way. The archesporial initials of *Lemna minor*, as figured , by Caldwell (1899, figure 13), to whose paper I shall have occasion to refer more at length later, closely resemble those of *Ruppia maritima.* On the other hand, Wiegand (1899, p. 344) finds the archesporium in Potamogeton traceable to a single hypodermal cell.

Pl. XI, fig. 69 shows a more advanced stage, the length of the young flower (fig. 70), measured as in the preceding case, being about 0.25 mm. One or two divisions have occurred evidently in all the cells. Although the cells representing the parts of the mature microsporangium are as yet entirely undifferentiated as to their contents, yet the manner of cell-division and the orientation of the walls give evidence of a commencement of a differentiation. First, the majority of the hypodermal cells have divided by a periclinal wall, thus separating off the primary parietal layer. Second, at the left of the top of the figure, there is the first indication, by characteristic periclinal and anticlinal divisions, of the future septum dividing the two sacs of the theca.

In Pl. XI, fig. 71, with a length of flower about 0.3 mm, this wall of separation between the two sacs becomes quite distinct. Its cells have a much less dense content than the archesporial cells, and are thus clearly marked off from them, as well as by the orientation of their walls.

A parallel case of a wall formed in an exactly similar way has recently been found by Caldwell (1899, pp. 47, 48) in *Lemna minor*. Coulter and Chamberlain (1903, pp. 39, 40) in commenting

on this condition in Lemna. say, "To divide a large sporogenous mass by sterile plates for better nutrition is too common to call for special remark." As mentioned by Caldwell and Goebel (1898, p. 770 , Isoetes presents a similar condition of formation of sterile plates of tissue from a fairly large archesporial mass.[1]

The archesporial cells. therefore, now appear as two definite, rounded, densely staining masses. composed of sporogenous cells, surrounded by a primary parietal layer, which has undergone a periclinal division in two or three places.

The first periclinal divisions in the primary parietal layer have become more general in the next stage, Pl. XI, fig. 73, which is from a flower about 0.33 mm. in length. The septum between the two sacs is also more conspicuous, and divisions continue in the sporogenous tissue.

At a considerably later period, with the length of the flower about 0.5 mm. Pl. XI. fig. 74), the parietal layers are still two or occasionally three in number. Indications appear here that the tapetum is forming from the marginal sporogenous tissue. Nuclear divisions continue among the sporogenous cells.

Soon after this stage, however. the sporogenous cells attain their final number, and all division ceases, followed by an enlargement to the mature pollen mother-cells, just as Murbeck (1902) has recorded for *Ruppia rostellata.*

Pl. XI, fig. 75 shows how the tapetal cells, now unmistakable in form and structure, bound the sporogenous cells—which may now be termed the pollen mother-cells—and are undoubtedly derived from them. According to Rosenberg (1901, II), Zostera also forms tapetum from the ends of its long sporogenous cells. and Coulter and Chamberlain (1903. p. 37) have shown that this is not unusual nor unnatural. In this respect, together with the number of chromosomes in the dividing sporogenous cells, which I have found to be 16, and also the three or four parietal layers between the epidermis and tapetum, *R. maritima* corresponds exactly with *R. rostellata,* as described by Murbeck (1902, p. 5). It is interesting to note here that Wiegand (1899, pp. 344 and 345), finds in *Potamogeton foliosus* that the tapetum is "differentiated from the wall rather than from the archesporium."

There remain to be mentioned the dissolution of the tapetal cells (Pl. XI. fig. 76), the development of thickenings in the subepidermal layer. and the final dehiscing of the anther by a longitudinal split.

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[1] A like situation has been carefully described by Bower (1897. pp. 41 ff.) for Danaea and other Marattiaceae.

## MEGASPORANGIUM

The usual method of development of the megasporangium corresponds in every particular to that of *Ruppia rostellata*, as described by Murbeck (1902. pp. 10. 11), so that it would be useless to duplicate his careful description here. Stated briefly, the process is as follows: At a stage in the growth of the pistil represented in Pl. X, fig. 62, or when it is about 0.25 mm. in length, a hypodermal cell in the young nucellus, in the region marked x, becomes considerably larger than its neighbors, with a larger nucleus, and more densely staining contents. This then divides by a periclinal wall, forming an outer, primary parietal cell, and an inner, megaspore mother-cell. The primary parietal cell now divides twice successively by anticlinal walls, at right angles to each other, forming a plate of four cells, or, through a third anticlinal division, sometimes six cells.[1] During this time the two integuments successively make their appearance. Meanwhile, the megaspore mother-cell enlarges, and with preparations for the first reduction division the history of the female gametophyte begins.

In Zannichellia, Campbell (1897, pp. 45, 46) finds two parietal layers formed at first, but these later divide into several layers. A much greater development of parietal tissue has been observed by Wiegand (1900, pp. 31, 32) in *Potamogeton foliosus* and by Holferty (1901, p. 341) in *Potamogeton natans*. In the latter case it is definitely stated that sometimes eight layers lie between the embryo sac and the epidermis. In other respects the development of the megasporangium in both of these genera is essentially the same as in Ruppia. In Lemna, according to Caldwell (1899, pp. 56, 57), there are not more than two parietal layers formed, and the other details of the megasporangial development are practically the same as in Ruppia.

Coulter and Chamberlain (1903, p. 65) state that the suppression of the parietal tissue among the monocotyledons " is usually associated also with the greater or less development of this tissue," a point which is illustrated here in the Potamogetonaceae by the condition in Potamogeton. " The strongest argument," to quote these writers further, " that suppression of the parietal tissue of the megasporangium is a strong tendency among Angiosperms, is that this condition is universal among the Sympetalae so far as investigated."

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[1] Rarely two layers of parietal cells are formed (Fig. 78).

Before I leave the account of the megasporangium, two cases in which the archesporium was undoubtedly two-celled should be recorded. Murbeck (1902, p. 11 and figure 35) has figured a double megaspore mother-cell, which, however, according to his explanation, is caused by the very oblique orientation of the wall between primary parietal and primary sporogenous cells, making this wall almost perpendicular to the epidermis and hence resulting in two large cells, apparently both potential megaspore mother-cells, and bounded exteriorly by the epidermis.

But my first illustration (Pl. XI, fig. 77) shows a clearly differentiated, single, parietal layer and two large megaspore mother-cells with their common wall very distinct and quite perpendicular to the epidermis. The second example has developed somewhat further (Pl. XI, fig. 78), the two megaspore mother-cells having passed through the first division, a cross wall being formed, which divides each into two essentially equal daughter-cells. Here may also be noted the rather uncommon occurrence of two parietal layers.

On the analogy of the microsporangium of Angiosperms, it would seem most natural that multicellular archesporia should occur also in the megasporangium. Through the investigations of Strasburger (1879), Fischer (1880), and among others, especially Péchoutre (1902), we have come to know that such a multicellular archesporium is quite general in the megasporangia of the Rosaceae; and that it also occurs in many other dicotyledonous groups has been sufficiently proven.

On the other hand, the reports of an archesporium of more than one cell in the megasporangium of monocotyledons are meagre, and, as reviewed by Coulter and Chamberlain, may be embraced in two cases, *Ornithogalum pyrenaicum* (Guignard, 1882), and *Lilium candidum* (Bernard, 1900). In these instances the archesporium is presumably always more than one-celled. There are, however, many cases, such as some of the Ranunculaceae, when the archesporial cells vary from one to many (Mottier, 1895, and Coulter, 1898). To such as these last the condition in *Ruppia maritima* is similar.

## FEMALE GAMETOPHYTE

As it is now regarded, the history of the female gametophyte commences with the preparations for the first division in the megaspore mother-cell. As regards this preparatory stage, I find that *Ruppia maritima* does not deviate essentially from *R. rostellata,*

and I have, therefore. no occasion to alter or add to Murbeck's excellent description.

In brief, the changes are very similar to those which lead up to the first division in the pollen mother-cells. The megaspore mother-cell and its nucleus enlarge, while the latter goes through the synapsis and succeeding stages, the staining reactions being essentially the same and even the fine kinoplasmic fibers appearing in the cytoplasm the same as in the corresponding stages in the pollen mother-cell.

In the spindle formed for the first reduction division, eight chromosomes appear (Pl. XII, fig. 80), as Murbeck also announces in *Ruppia rostellata*. Although the chromosomes are here much thicker than in the sporophytic karyokinesis, they are nevertheless still so small that any definite declaration concerning their shapes and method of splitting is well nigh impossible. Still, as Murbeck has noted, the ring and Y-shaped forms characteristic of the heterotypic division are occasionally apparent.

After the nucleus of the megaspore mother-cell has divided, we find a wall laid down separating the two daughter-cells (cf. Murbeck. 1902. fig. 45). In this connection, reference might be made to the case already noted under the megasporangium (p. 136), where a double megaspore mother-cell was found. producing in each case two such daughter-cells (Pl. XI. fig. 78).

The second division follows closely on the first, with a very slight pause, similarly as in the pollen mother-cells. The two walls resulting from these divisions are, however, laid down at quite different planes with respect to each other. that is, the wall dividing the two inner cells is periclinal, while that separating the two outer is anticlinal. Thus the two outer cells are both in contact with the third cell, and are separated by it from the innermost cell. The plane of the anticlinal wall is, however, obliquely situated with regard to the plane of a horizontal or vertical median section of the megasporangium; in other words, it is oblique to the plane of the paper on which such a section is represented, so that an oblique position of the two upper cells with respect to this plane results.

This arrangement is shown by Murbeck (1902, pp. 13, 14, fig. 51). But often, due partly to the manner of cutting the section and partly to the orientation of the outer anticlinal wall, the position and even the number of cells is not so apparent, since one of the two upper cells then lies more or less completely over the other. Such a case is represented in Pl. XI, fig. 79, which a hasty

glance might have interpreted as three megaspores, the upper two becoming resorbed. But careful focussing discloses another outer cell at a somewhat lower plane. Murbeck (1902) figures a similar case in *Ruppia rostellata*.

This departure from the usual method of division of the megaspore mother-cell, in which, in general, the resulting cells are formed in a straight row, is fully commented upon by Murbeck (1902, p. 13), who states that it has been found also in *Allionia nyctaginea, Helleborus foetidus* and *Ceratophyllum demersum* ; and to his work I refer for a fuller account of the whole matter and for literature bearing upon the subject. A concise morphological consideration of such a location of the potential megaspores is also set forth by Coulter and Chamberlain (1903), who although they do not mention the case of *Ruppia rostellata*, note a similar arrangement of the outer two cells as occurring in Butomus (Ward, 1880), Jeffersonia (Andrews. 1895), and Potamogeton (Holferty, 1901).

Moreover, that this position of megaspores is not an invariable rule in *Ruppia maritima*, is shown by such a case as is illustrated by Pl. XII, fig. 81. where the four cells appear in a row, the two outer ones already much disorganized.

Before proceeding further. the condition shown in Pl. XII, fig. 82 should be noted, where the nucleus of the upper of the two daughter-cells has divided, but no wall has been formed, and the whole cell, along with its neighbor below, is undergoing resorption. Such a happening seems natural when one reflects that the division in the lower daughter-cell in general precedes that in the upper cell, producing a tendency by the earlier development of the former, to reduce activity in the upper daughter-cell. It will be seen later that this omission of the wall in the upper daughter-cell is the ordinary occurrence in *Potamogeton foliosus*.

In all cases the lowest of these four cells becomes the functional megaspore, the upper three cells becoming resorbed, as in *Ruppia rostellata* (Murbeck, 1902, pp. 14, 15).

In *Potamogeton natans*, Holferty (1901) describes cases of four megaspores with the same arrangement as in Ruppia, the innermost functioning.

In *Potamogeton foliosus*, described by Wiegand (1900), the condition is quite different and yet exhibits points of similarity. The first reduction division produces two daughter-cells, separated by a wall. The second division then takes place in each of these cells, and the resulting nuclei occupy practically the same positions as they do in Ruppia. No walls are formed after this division,

however. If they were, it is evident that a tetrad similar to that in Ruppia would have resulted. The upper daughter-cell with its two nuclei now becomes disorganized, and the lower cell becomes the embryo-sac, its nuclei producing directly the embryo-sac nuclei.

*Zostera marina* forms three megaspores, according to Rosenberg (1901, I, p. 9), the lowest functioning. The uppermost. however, in his Figure II looks much like Figure 51 of Murbeck (1902), and like many of my own preparations. gives indications of two cells obliquely arranged.

In Zannichellia the state of affairs is surprisingly different. According to Campbell (1897, pp. 45, 46). a row of three cells is formed, of which the uppermost becomes the functioning megaspore. Since this is so strikingly diverse from what takes place in Ruppia, Zostera and Potamogeton, it would seem as if it needed confirmation.

The functional megaspore now proceeds to the formation of the embryo-sac by a series of stages which are quite in line with those which have been found to be so remarkably constant among the Angiosperms. Pl. XII, fig. 82 shows the megaspore nucleus in process of division, while Fig. 83 discloses the resulting two nuclei, one at each end. Fig. 84 shows the four nuclei coming from these two. Fig. 85 represents a mature embryo-sac with synergidae and egg of characteristic form and structure. The antipodal cells are always three in number, their nuclei being surrounded by a definite layer of cytoplasm and apparently by a thin, membranous wall. Often they appear rounded in form (Pl. XII, fig. 85) and again angular (Fig. 86), in the latter case showing clearly their relations to each other. They resemble strongly those figured by Murbeck (1902, fig. 53) in *Ruppia rostellata* and by Campbell (1897, fig. 109) in *Zannichellia palustris*, and like them are situated in a small pouch at the base of the embryo-sac.

A peculiarity of these antipodals is the conspicuous blue color of their nuclei with the triple stain, showing an unusual tint of the blue, and possibly indicating degeneration. Thus they are strongly contrasted with adjacent nucellar nuclei and may be readily distinguished. They were not observed to divide, however, as in the case of certain recently investigated monocotyledons. Apparently their life is short, for they disappear in stages slightly older than Pl. XII, fig. 85.

## MALE GAMETOPHYTE

Since my studies of the male gametophyte of *Ruppia maritima* agree closely with those of Murbeck (1902), it would be useless here

to go into a lengthy description. For the sake of completeness, however, I will review the more important points, noting a few minor differences from Murbeck's results, and adding a few observations of my own.

We may regard the history of the male gametophyte as beginning with the appearance of the synapsis stage in the pollen mother-cells, which precedes the first reduction division and signalizes the special preparations for that process. Seven different flowers chanced to be fixed during this stage, and the appearance of the pollen mother-cells in all was typical (Pl. XII, fig. 87). The nuclei are very large, but no knoblike processes are appended to them, such as Wiegand (1899) figures for *Potamogeton foliosus.* Although some such appearance was occasionally found, it was not sufficiently general to be called typical of this stage.

In the first reduction division I succeeded in finding several cases of multipolar spindles, as Murbeck (1902, p. 7) has also reported. The two reduction divisions follow one another in rapid succession, forming a tetrad, whose members are oriented to each other after the manner of the four quadrants of a somewhat elongated sphere (see Murbeck, fig. 16). A similar arrangement is figured by Bornet (1864) in Cymodocea, and is not uncommon in the monocotyledons in general. During the two reduction divisions I was able in several cases to count eight chromosomes, which is, therefore, the reduction number, as Murbeck (1902) also found in *Ruppia rostellata.*

In the study of the development of the pollen-grain, as in other structures, I found it of advantage to use a certain definite method of external measurement as an index to the stage of internal development. Thus, in the case of the pollen-grain, I chose the length of the grain, which, from its first formation in the tetrad to the mature condition, increases from about 175 $\mu$ to about 550 $\mu$. Although these dimensions are subject to some variation, even in the same pollen-sac, yet they are fairly constant for the same period of growth.

At the time of the formation of the tetrads the nucleus is in an approximately central position. Very soon after, or while the grain is still not much more than 175 $\mu$ in length, the nucleus shows a position nearer to one end of the grain (Pl. XII, fig. 88), and a central zone of small starch grains has appeared. Almost immediately thereafter one finds the nucleus at the end of the grain, the starch grains having become considerably larger and uniformly distributed (Pl. XII, fig. 89). A nuclear division now ensues (Fig. 90), and as a result, the small lenticular cell at the end of the grain, the generative cell, is formed (Fig. 92).

As to the number of chromosomes distributed to the daughter-nuclei in this division, it is impossible to state it exactly, after a careful study. Murbeck, however, succeeded in finding a very clear case where eight appeared in the nuclear plate, and judging from the count in the two preceding mitoses, and also from general appearances in the division, it appeared to me as if eight was the most probable number in *Ruppia maritima*. The chromosomes in this division, as in the much later one when two male cells are formed (p. 142), are extremely small—much smaller than in the first and second divisions in the pollen mother-cell—so that even with the highest powers of the microscope they are very difficult to identify.

A few words should be added regarding the generative cell at this period. As Murbeck has noted, its nucleus is smaller than the tube-nucleus, but, judging from the staining reactions of the two, it contains a comparatively larger amount of chromatin. Moreover, a definite aggregation of cytoplasm surrounds the generative nucleus. As Murbeck has announced in *Ruppia rostellata*, so here no membrane separates this cell from the rest of the microspore, and it is evident that one sees here a naked cell, such as occurs in the case of the egg or synergidae of the embryo-sac. On the other hand, Wiegand (1899, pp. 352, 353, figs. 41, 42) finds the generative cell in Potamogeton—which, by the way, is not formed until the "spores reach their full size"—enclosed in a definite membrane. A distinct wall is also found in Typha by Schaffner (1897. I.) and in Naias and Sparganium by Campbell (1897, 1899). Rosenberg (1901. I.) in Zostera mentions only a very indistinct wall at a later stage, when the two male cells are formed, separating them from the surrounding pollen-plasm.

In most cases, though by no means always, a narrow space separates this cell from the rest of the microspore, which a study of a large number of sections shows is undoubtedly due to a contraction. (Pl. XII, fig. 92).

The microspore now enters upon a long period of growth, during which it increases in length from the 175–225 $\mu$ of the above described stages to the 500–550 $\mu$ of the mature condition. During this period it gradually assumes its curious bow-shaped form, with the characteristic dumbbell-like expansions at the ends, and also a considerable dilatation centrally at the convex side of the bow (Pl. XII, fig. 95).

A pollen-grain so remarkable in shape is not unique among the Potamogetonaceae; for the pollen-grain of Cymodocea is also unusual, being a long filamentous grain measuring, according to Bornet

(1864), ten by about two thousand $\mu$, while the microspores of Zostera (Rosenberg 1901, II) are three by about two thousand $\mu$ when mature. Moreover, in Phyllospadix. grouped by Ascherson (1889) with Zostera in the subfamily Zostereae of the Potamogetonaceae, Dudley (1893) has found the pollen grains to measure about four or five by one thousand $\mu$. Dudley (1893, p. 412) states that "They are slightly flattened at the extremity and some are enlarged toward the middle."

During this long period of development to the mature form, several noteworthy internal changes occur. The tube-nucleus (Pl. XII, figs. 92, 93 *tn*) gradually undergoes degeneration. until in the mature grain it often appears fragmentary or angular. The starch grains, so prominent in the early stages. become for the most part smaller and fewer, and one seems warranted in concluding that a part of their substance has been utilized in the formation of the grain.

It is not until the mature condition, or when the pollen is about ready to be discharged. that the generative cell divides. When this is to take place, the latter assumes a position in which its long axis is more or less parallel with the long axis of the microspore. The two resulting male cells remain united as in *Potamogeton foliosus* (Wiegand. 1899). each surrounded by a considerable layer of cytoplasm (Pl. XII, fig. 94).

Moreover, even at this stage, no wall separates the male cells from the cytoplasm of the pollen-grain, but there is a fine cell-plate formed between the two cells. As a slight variation from Murbeck's figure, I find that for the most part in *Ruppia maritima* a moderate constriction occurs between the two male cells, in the region of the cell-plate.

One feature which has not been thoroughly worked out in the pollen-grain of Ruppia and indeed has been much neglected in the study of the male gametophyte in general, is the origin and growth of the microspore wall.

Murbeck (1902) has described the peculiar thickenings of the mature wall, the latter consisting of a single thin layer.

Since I was fortunate enough to have a large number of sections of all stages of the growth of the pollen-grain, the development of the wall from the pollen mother-cell stage to the mature microspore was comparatively easy to trace.

The wall of the pollen mother-cell, after it has separated from its neighbors, is very thin. After formation of the tetrads, this wall thickens and becomes the free or outer wall of the tetrads, while the interior walls of the tetrad are laid down immediately after the two reduction divisions. Pl. XII, fig. 88. These tetrad cells do

not separate: the protoplasts forming the future pollen grains may be said to simply occupy the four quadrant-like compartments in the spherical structure comprising the tetrad. The walls at this stage are of noteworthy appearance by reason of their considerable thickness—being thicker than the wall of the pollen-grain at any future time—and also from their transparency. refracting the light in such a way that they appear gelatinous.

The tetrad walls appear now to expand. leaving the pollen protoplasts naked within them. Such a condition is shown in Pl. XII, fig. 97. Finally the walls entirely dissolve.

Judging from Murbeck's account, these conditions escaped his notice, for he speaks of the *membrane of the tetrads* as showing " noch keine Spur von Oberflächenstruktur " (1902, p. 8).

The process is, however, essentially the same as that described by Strasburger (1882, pp. 87 ff.; 1889, pp. 36 ff.) for the pollen-grain of *Malva crispa* and other plants. Only one marked difference occurs, namely, that in *Malva crispa*, &c. the old pollen mother-cell wall is apparently cast off and does not take part in the wall formation of the tetrads.

From now on a wall develops around the young pollen-grain, but. as in the cases of Naias and Zannichellia, remains a single thin membrane (Pl. XII, fig. 94).

Very early (Pl. XII, fig. 97) the local thickenings begin to be formed on the outer surface of the wall. These in this early stage are difficult to see clearly on account of their transparence. They seemed, however, to be the result of depositions on the wall and not do develop from radial bands in its interior. Ultimately these thickenings appear in the shape of low ridges arranged to form irregular polygons, at whose intersection are short spines, with a slight knob at the end (Pl. XII, fig. 96, *a* and *b*). At the ends of the grain and at its expanded center, these spines become shorter, or are absent altogether (Pl. XII, fig. 94). It is probable that at these localities the pollen tube commences its formation.

## POLLINATION

Ruppia is one of the few of the higher plants to which, in respect of the process of pollination, the term hydrophilous may be applied. In other words, water is the transporting agent for the pollen, instead of the commoner methods by insects or the wind.

According to my own observations, which agree in the main
with those of Roze (1894), the process of pollination takes place
about as follows. Soon after the extension of the peduncle above
the surface of the water, the anther sacs split open by a longitudinal
cleft, and the pollen, shed in large yellowish masses, may be seen
floating on the surface of the water.

Immediately subsequent to the shedding of the pollen, the rhachis,
which up to this time has been erect, commences to incline toward
the surface of the water. It becomes more and more horizontal
until eventually, after two or three hours. it comes to lie on the
surface of the water.

In this position, the stigmas are of course so situated that the
floating pollen grains, with which, in the height of the flowering
season, the water is fairly well covered, have easy access to them.
The final step occurs when the currents of water, always moving
in one direction or another in a tidal ditch—or sometimes set in
motion by gusts of wind—bring the pollen grains into contact with
the stigmas.

Wylie (1904), has described a similar mode of pollination in
Elodea. He attributes the floating of the pollen-grains to the air
imprisoned between the spines of the pollen-grain and the surface
of the water, this being sufficient to keep the grain afloat. Pol-
lination on the surface of the water also takes place in Vallisneria,
as is well known. In Zostera, however (Strasburger, 1908, p. 258),
pollination is performed below the surface of the water.

<center>FERTILIZATION</center>

No investigator has yet been able to demonstrate the process of
fertilization in Ruppia. Murbeck (1902, p. 15) has, indeed, found
the pollen tubes in the ovary; but has been unable to distinguish
either of the sperm-nuclei in the embryo-sac. For various reasons.
he concludes that the act of fertilization takes place very rapidly.
If this is true, it may account for the fact that in my own prep-
arations I also have been unable to find any unmistakable evidence
of the sperm-nuclei.

As is shown in Pl. X, figs. 62—66, a definite stylar canal exists
from the stigma to the cavity of the ovary, but Murbeck (1902, l. c.)
claims that the pollen tubes, however. penetrate through the cellular
structure of the style to the cavity of the ovary.

ENDOSPERM

The endosperm nucleus is large, and located near the antipodal end of the embryo-sac, or sometimes near the center. Just what nuclei enter into its formation was not ascertained, nor could I determine whether or not in the matter of commencing division it has a slight advantage over the fertilized egg, as Murbeck (1902) declares is the case in *Ruppia rostellata.*

The endosperm in its most fully developed stage (Text-fig. 27), is never more than a thin layer lining the embryo-sac and containing free nuclei. In the mature seed, traces of it may still be seen as a thin protoplasmic lining, with the nuclei now much reduced in size. Thus it is clear that the endosperm is here only temporary. The function of more lasting nourishing tissue is assumed by the enlarged hypocotyl.



Figure 27.—Portion of endosperm which lines embryo-sac, this particular example being taken from the embryo-sac which contains the embryo shown in Pl. XIII, fig. 106. × 575.

A similar development of endosperm appears in other Potamogetonaceae (Coulter and Chamberlain, 1903, p. 171).

# EMBRYO

The fertilized egg divides transversely, producing al arge lower cell and small upper cell. As in Naias (Campbell, 1897, p. 26), Zannichellia (Campbell, l. c., pp. 27, 28), Zostera (Rosenberg, 1901, I; Hofmeister, 1852,, and Potamogeton (Wiegand, 1900), the former divides no further, but subsequently increases vastly in size, developing large vacuoles, and becomes the suspensor, which is thus in this case restricted to a single large, basal cell.

I was unable to discover any stages between the two-celled proembryo and the twelve-celled stage—the latter represented in Pl. XIII, fig. 100. Murbeck, however, figures a three-celled stage in *Ruppia rostellata*, which I reproduce (Pl. XIII, fig. 98), showing two small cells, which may be called the " embryo-cells," arisen from the transverse division of the small upper cell; and also the basal large cell, namely, the suspensor-cell. Wille (1883) figures and also describes a similar stage in his work on the embryo of *Ruppia rostellata*.

Wille has carefully followed the development of the proembryo in these early stages, and his observations and figures accord well with my slightly older embryos. His report of the succeeding divisions is briefly as follows: The lower of the two embryo-cells divides longitudinally, followed by a similar division in the same plane in the upper cell, making four cells in all : next. by longitudinal divisions in both segments, in a plane at right angles to the last, an eight-celled structure is formed. The embryo now consists, therefore, of two four-celled segments lying one over the other, and borne on a single large suspensor-cell (Pl. XIII, fig. 99.)

It should be borne in mind that this suspensor cell is the basal segment resulting from the first division of the fertilized egg, and never again divides. In this connection it may be well to follow out the subsequent history of this peculiar suspensor before going into an account of the embryo proper.

After the first unequal division in the fertilized egg-cell, the suspensor-cell becomes rapidly larger, and at the time when the two four-celled segments appear, as described above, it is much larger than these combined (Pl. XIII, fig. 99). At the period shown in Fig. 100, when the embryo contains twelve cells, the suspensor-

cell nucleus, with the majority of the cytoplasm, occupies a position at the base of the cell, the upper part containing one or more large vacuoles. This appearance, although not invariable (cf. fig. 101), seems a quite general one in the disposition of the cell-contents at all stages except the very earliest. The same has been noted in Zostera (Rosenberg. 1901, I, pp. 11, 12) and Potamogeton (Wiegand, 1900, p. 37).

Although the suspensor-cell is now (Pl. XIII, fig. 100) enormous in size, this is in fact only a stage in its enlargement. Fig. 101 testifies to what proportions it eventually attains. At the same time its nucleus attains a large size. In somewhat later stages the suspensor-cell commences a gradual disintegration, until, at a period of embryo development, such as in fig. 106, the outlines of the cell can be no longer distinguished, although its nucleus is still apparent in a condition of disorganization.

During its development, the suspensor-cell is in contact with the micropylar end of the embryo-sac for some time, but may eventually become free, as Wiegand (1900, pp. 37, 38) has also noted in Potamogeton.

In connection with this remarkable growth, the statements of Rosenberg (1901, I, p. 12), who describes a similar suspensor in Zostera, may be quoted. In writing of the nucleus of this cell he says: "Der Kern des Embryoträgers .... macht eher den Eindruck, als ob in demselben eine rege Stoffbildung vor sich ginge; etwa eine Aufnahme von Stoffen, die in dem Kern weiter umgebildet werden, um später in das Embryo zu gelangen." It might be added that not only the nucleus, but also the whole cell, judging from its appearance and time and manner of development, has obviously the function of reception and preparation of nourishment for the embryo. It is significant also to note that at the time of the degeneration of this large suspensor-cell, the endosperm nuclei are relatively abundant.

Although a similar large basal cell occurs in Naias (Campbell, 1897, p. 26), Zannichellia (Campbell, 1897, pp. 27, 48), Zostera (Rosenberg, 1901, I, pp. 11, 12) and Potamogeton (Wiegand, 1900, p. 37), only in Zostera does it represent the entire suspensor as in Ruppia. In the remaining genera other small secondary suspensor-cells are formed above the large basal cell.

It will be seen that the three-celled stage of the proembryo (Pl. XIII, fig. 98) is essentially similar to that of Sagittaria (Schaffner, 1897, II, p. 262 and Pl. XXIV figures 46, 47) and Alisma (Schaffner, 1896, pp. 129, 130), which has been regarded as typical of the

monocotyledonous embryo (Coulter and Chamberlain, 1903, pp. 188. 190 ff.).

Instead of the increase of this row of cells from three to foui, or, in other words, to a row of three embryo-cells, as is the case in the above-mentioned genera. Wille, as already stated, finds that two plates of four cells each are formed from the two embryo-cells. These two resulting four-celled segments are, nevertheless, still the representatives of the two small embryo-cells, which are formed first, in Ruppia as well as in Sagittaria and Alisma.

That this condition is the usual one in Ruppia is shown not only by Wille's (1888) observations, which may be correlated so well with my own, but also by the figures and description of Hofmeister (1852, p. 143 and figures 41–46, and 1861, figures 1–7, pl. II)[1].

My twelve-celled embryo (Pl. XIII, fig. 100), composed of three four-celled plates, has obviously arisen by a transverse wall through one of these four-celled segments—which one, it is not possible to state. In the typical embryo of Sagittaria, indeed, it is the lower of the two segments that undergoes a transverse division (Schaffner, 1897. II, p. 262 and Pl. XXIV, figures 46, 48, 49), and this may be the case here.

What has occurred then is simply the formation of three segments, one above the other. comparable to the three upper cells in Sagittaria (Schaffner, 1897, II, Pl. XXIV, figures 48, 49), with the difference that in Ruppia longitudinal divisions precede the transverse ones.

A comparison of this stage with those embryos of related genera which have been worked out, brings to light the following points. The embryos of Zannichellia (Campbell. 1897, p. 48 and cf. fig. 63 Pl. III; Hofmeister, 1861, Pl. I, fig. 18) and Potamogeton (Wiegand, 1900, pp. 37. 38, and Pl. VII, figures 25, &c.) are essentially like that of the typical Sagittaria. consisting of a row of three cells above the suspensor-cell, the terminal one being the first to undergo longitudinal divisions.

As to the embryo of Zostera, it was investigated at a very early period by Hofmeister (1852), and quite recently by Rosenberg (1901. I). Their results are not complete in the early stages, but enough has been shown to indicate that Zostera is more like Ruppia in the early development of its embryo than any of the Potamogetonaceae so far investigated. Hofmeister (1852, p. 139) states that

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[1] Practically the same figures are presented in each of these articles : in the former, the species is given as *rostellata*, in the latter, as *maritima*.

a four-celled embryo is formed from the first embryo-cell by two longitudinal divisions occurring at right angles to each other and these quadrant-like cells are next each divided by a transverse wall. So that. although the divisions here occur in different sequence, the two four-celled segments nevertheless result.

Although these first stages were not followed out by Rosenberg. he presents a figure (1901, I, Pl. I, fig. 20, and P. 12) of an eight-celled embryo similar to that of Wille's (Pl. XIII, fig. 99). It is perhaps possible that later investigation will show that the first wall in Zostera is transverse instead of longitudinal, as Hofmeister has described it.

The task of tracing out the lines of demarcation of the primary segments during the later stages, after the manner of recent embryological investigations, is very difficult, probably from two principal causes. First, the remarkable number of early longitudinal divisions which have already been indicated to a certain extent obscure the segment limits. Second, the form of the mature embryo differs so widely from that of a typical monocotyledon that even in the early developmental stages this influence makes itself felt, and renders comparison with type forms rather unsatisfactory. Wille, indeed (1883, pp. 2. 3), describes his eight-celled stage as forming a sixteen celled structure by transverse divisions in all of the eight cells. But of the subsequent divisions he says (l. c., p. 3), "Herefter gaa Delingerne uregelmæssigt, saa man ikke længere med Bestemthed kan følge de enkelte Cellers Delinger."

Of stages older than the eight-celled embryo of Wille and Hofmeister, I was fortunate in having a fairly large number, and in the following description I shall attempt to show to what extent the segment boundaries may be traced during the embryo development, and how they may be correlated to such a type as Sagittaria.

I have already suggested that such an embryo as is represented in Pl. XIII, fig. 100, composed of twelve cells, or three four-celled segments, arises from Wille's younger form by a transverse wall through one of the segments, and that this is parallel to the case in Sagittaria, where a three-celled row arises by a transverse division in the lower embryo-cell.

Fig. 101 is a slightly older embryo, in which the divisions are somewhat irregular, but still admit of an interpretation which reveals the outlines of the cells and segments in the preceding stage. It is evident that a transverse division has occurred in the terminal segment.

In fig. 102 appears evidently the beginning of the formation of the dermatogen. at least in the terminal segment at the left. Fig. 103 shows the segment lines more irregular, the dermatogen has became more pronounced. and in fig. 104 shows a distinct differentiation. A point of interest at this stage is the considerable increase in size of all the cells, which are, however, only slightly greater in number than in fig. 103. The embryo here measures about 0.075 mm. in diameter, as against the 0.05 mm. of the preceding one.

Up to this point the embryo has exhibited a globular form, but in fig. 105, where it measures about 0.085 mm. it has commenced to elongate. It is at this period that the divisions are initiated in the terminal segment. appearing here at the left, which signalize the approach of cotyledonary development.

It has been shown by Schaffner (1897, II, pp. 263–265) in Sagittaria that of the proembryo of three cells, the uppermost develops the cotyledon. the middle divides transversely, and of the two resulting segments the upper develops the stem apex and the lower the hypocotyl, root, and secondary suspensor. The lowest cell of the three-celled proembryo remains undivided, forming the basal suspensor-cell.

Obviously the primary segments in Ruppia do not all have destinies similar to those of Sagittaria. For, to begin with, no secondary suspensor is here found, and only a minute rudimentary root, as will be shown later. Again, it seems quite probable, although for the reasons stated above no conclusions can be certainly drawn, that the terminal segment produces the stem-apex as well as the cotyledon. Campbell (1897, p. 49) finds such a condition in Zannichellia, and the appearances shown in fig. 106 point to such a situation here. Solms-Laubach (1878), as noted by Campbell, has also reported a terminal origin of the stem-apex in some of the Commelinaceae and Dioscoreaceae. This leaves the second segment (which, as has already been indicated, probably divides transversely to form the second and third segments, as in Sagittaria) for the development of the hypocotyl, extraordinarily large in Ruppia, and the small rudimentary root adjacent to the suspensor-cell.

Pl. XIII, fig. 105, however, represents the oldest embryo in which the primary segments can be determined with accuracy. In the next figure (fig. 106), it is quite impossible to make out definitely the line separating the terminal segment from the rest of the embryo, to say nothing of any other segment-lines. Nevertheless, in this embryo, cell divisions are active in the terminal region

and one can easily see a small prominence ʼfig. 106, *cor* , accompanied interiorly by a number of cell-divisions. which is obviously the ʼʼʼʼʼeginning of the cotyledon. Moreover. at its right is another markedly meristematic region. the future stem apex or epicotyl ʼfig. 106. *st* .

Wille's description of the subsequent development up to the mature embryo agrees essentially with my slides, so that I confirm his observations in the main, adding a few details. For the sake of clearness, each member will be described separately.

*The Cotyledon.* Originating as described above. the cotyledon elongates and the epicotyl develops at its base. In the course of its growth, the cotyledon develops basal sheaths similar to those characteristic of the leaves of a mature plant. which then surround the epicotyl. Concerning this condition Wille (1883. pp. 3, 4) says, "Hos den modne Frugt er Plumulaen ganske omgiven af Cotyledonet (Tavl. I, fig. 25), kun en trang Spalte fører ind til Hulen," but figures an embryo with a large open hollow at the base of the cotyledon, where the epicotyl may be seen. As a matter of fact, in *Ruppia maritima* at least. the cotyledonary sheaths overlap one another for almost the whole of their length, essentially as do the sheaths of the foliage leaves (Pl. XV, fig. 118). Only at the upper end of these sheaths is there a small cleft remaining (Fig. 118, *x*) which may be the " trang Spalte " mentioned by Wille. Thus the sheaths enclose the epicotyl so that it is quite shut off from view. In Pl. XIV, fig. 112, and Pl. XV, fig. 116 these sheaths appear in section in not quite mature embryos, and figures 117 and 118 show them in a cotyledon dissected from a mature embryo. It is interesting to note that the axillary scales may be seen within the sheaths, at their base, as in the mature plant.

Wille (1883, Pl. I, fig. 25) figures the cotyledon as elevated a considerable distance above the hypocotyl, although still extending more or less horizontally, and one might conclude that possibly herein is a specific difference between *Ruppia rostellata* and *R. maritima*, were it not for the fact that Irmisch (1858, fig. 37, pl. I) figures the cotyledon closely appressed to the hypocotyl as I find it in *R. maritima* (Pl. XV, fig. 119). In Text-fig. 28, p. 152, this horizontal position has been slightly disturbed through manipulation.

*The Epicotyl.* Pl. XIV, fig. 112 shows the epicotyl in a nearly mature embryo. It consists always of a second leaf and the growing point, the latter not appearing as prominently here as in other sections. Wille (1883, p. 3, figs. 25 and 26) is in doubt whether this smaller pro-

tuberance is really the growing point or the third leaf, but a comparison with the growing point of the mature stem (Pl. I, fig. 1; Text-fig. 2, p. 75) conclusively proves its nature.

*The Hypocotyl.* The great bulk of the embryo is taken up by the hypocotyl (Pl. XV. fig. 119, Text-fig. 28), which is nothing but a mass of storage tissue, its cells being gorged with large starch grains. Even in such a young stage as in Pl. XIII, fig. 105. the cells in this region are distinctly larger than in the remainder of the embryo. They continue their enlargement and acquire an ever richer content of starch as development proceeds (cf. Pl. XIII, figs. 105, 106 ; Pl. XIV, figs. 110–112, 114: Pl. XV. fig. 115).
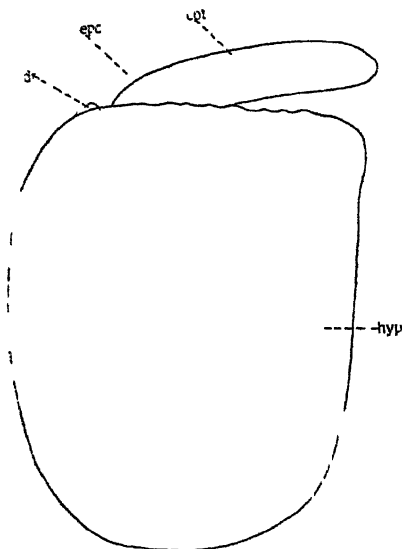


Figure 28.—Mature embryo dissected from ripe seed, the cotyledon slightly elevated during manipulation. *cot*, cotyledon ; *epi*, region of epicotyl ; *adr*, adventitious root ; *hyp*, hypocotyl. × 35.

*The Primary Root.* At a fairly early period (Pl. XIII, fig. 106 ; Pl. XIV, fig. 110) a certain group of cells becomes differentiated at the base of the embryo, by acquiring denser contents and lacking the starch grains of the storage cells above them. Each of these cells later develops (at least those on the periphery of the embryo) into unicellular papilla-like projections, essentially as figured and described by Wille (1888, p. 4 and Pl. I, fig. 19) and Murbeck (1902, pp. 17, 18 and figs. 60 b, 61 b). Like Murbeck, I find also often more than a single layer, but whether arising from periclinal divisions in the

dermatogen. as he asserts I was not able to determine. The whole group is supposed to represent the vestiges of the primary root Murbeck, 1902. p. 18: Wille. 1883. p. 4 , which develops only to this rudimentary degree. and never functions.

*The First Adventitious Root.* At about the period when the coty-ledon and epicotyl have become quite distinct in outline, the form-ation of an adventitious root occurs near the base of the epicotyl. This grows rapidly, forming a noticeable protuberance (Pl. XIV, figs. 112, 113), and in the mature embryo (Text-fig. 28) may be seen pointing almost directly upward, or nearly at right angles to the position of the cotyledon.

The distribution of the meristematic regions at the tip of this adventitious root is not as clearly marked as in the roots of the mature plant. It will be recalled that the roots of the mature plant, as in Zannichellia, contain at their tip four distinct meristematic regions. representing the initial areas of calyptrogen, dermatogen, periblem and plerome.

In the young adventitious root of the embryo an interesting point is the division of the epidermis of the hypocotyl, immediately over the young developing root, by periclinal walls. apparently to form root cap. These divisions continue, and anticlinal as well as periclinal occur. This is a very different condition from that in the roots of the mature plant.

Whether the dermatogen and periblem are each also here re-presented by a single layer at the apex of the growing-point would be impossible to assert definitely, on account of the irregularity of the cells, but such seems to be often the case (Text-fig. 29). How-ever, some embryos show only one layer for both dermatogen and plerome. as Campbell finds most usual in the primary root of Zannichellia (Campbell, 1897, p. 51). The plerome is pretty clearly marked (Text-fig. 29), but also does not have as definite a point of origin as in the roots of the mature plant.
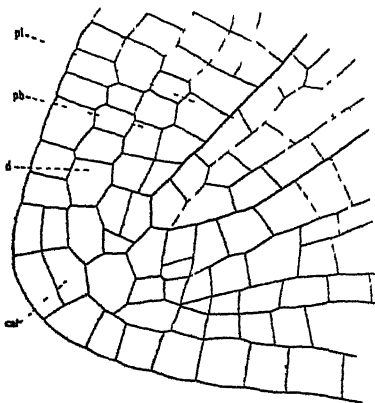
Figure 29.—Longitudinal section through apex of adventitious root of nearly mature embryo; showing *pl,* plerome, *d,* dermatogen, and periclinal divisions initiating development of calyptrogen, *cal.* × 530.

The mature embryos of *Ruppia maritima* measure 1.5–2 mm. in length, by a little more than 1 mm. in width. Although I find no measurements recorded of the embryos of *R. rostellata*, Irmisch's figure (1858, fig. 37. Taf. I) is somewhat longer in proportion to its width.

Murbeck (1902, p. 18 and Pl. III, figs. 62, 64, and 65) has called attention to the resemblance of this curious embryo of Ruppia to those of the related genera, Zannichellia, Halophila and Zostera. The embryo of Phyllospadix is similar, but characterized by a peculiar lobing of the hypocotyl around the base of the cotyledon (Dudley 1893, p. 413 and Pl. II, fig. H).

To the well known controversy regarding the real nature of the adventitious root and of the primary root, I will briefly allude and record my own views here.

In writing of the Potamogetonaceae, Ascherson (1889, p. 200) says, "Meist entwickelt sich eine kräftige Hauptwurzel an dem unteren Ende des Embryos; nur bei Ruppia befindet sich dieselbe seitlich neben der Plumula."

Thus Ascherson, whose view has been adopted by Goebel (1898, pp. 464–466), rejects the conclusions of Wille, who found the peculiar cell-group developed in *R. rostellata*, which I have also reported for *R. maritima*, at the base of the hypocotyl and immediately over the large suspensor-cell. This basal region is, as Wille says, the place for the primary root of the embryo. Wille, therefore, considers this the rudiment of the primary root, and the structure near the base of the plumule, which Ascherson calls the primary root, he terms an adventitious root.

Murbeck agrees with Wille and in further investigations finds that the primary root rudiment "sich eben am Festpunkte des Embryos, mit anderen Worten, eben am Platze der Radicula befindet. Dass dieselbe wirklich die Anlage der Radicula repräsentiert, kann wohl schwerlich bezweifelt werden" (1902, p. 17).

That the adventitious root is of exogenous origin is explained by Wille (1883, p. 5) and defended by Murbeck (l. c. p. 18) by the fact that practically all of the cells of the young embryo are meristematic, so that an adventitious root developing at this period may easily have an exogenous origin.

My own slides testify to the correctness of the interpretations and figures of Wille and Murbeck. Nowhere is there the least evidence that the hypocotyl is the "seitliche Auftreibung" described by Ascherson, and there seems no doubt but that the root near the base of the epicotyl is properly the first adventitious root, which

has assumed the function of the primary root of the embryo. Such an adventitious root. we have seen, may occur in a similar way at any node in the mature plant. just below the point of leaf-insertion.

Just what the group of cells at the base of the hypocotyl does represent is difficult to say definitely. Murbeck thinks that they may represent calyptrogen and calyptra (1902. p. 18). This is. of course. possible, but it would be difficult to prove. We may be sure of this much, however, that from their position. appearance, and development. they bear some relation to the now functionless primary root.

Another interpretation of this whole thickened hypocotyl with its curious basal cells seems plausible. namely, that practically the whole swollen area itself represents the primary root or radicle which has been by degrees metamorphosed into a storage-organ. That this is actually what has happened would be difficult to prove, for it is dangerous to lay much stress on the form and position of the storage organs. The strongest grounds for such an opinion are drawn from a comparative morphological study. The embryo of Zannichellia, e. g., is very similar to that of Ruppia. Although Wille considers (1883, p. 7) that the small caplike body at the base of the hypocotyl comprises the entire root, Campbell's figures and description (1897, pp. 50, 51 and Pl. V, figures 120. 122, 123) go to show that practically the whole hypocotyl is a root structure and the small body at its base is the root cap.

In Zannichellia the primary root functions for a time during germination (Wille, 1883, p. 8). The absence of a distinct plerome and periblem in what has been called the hypocotyl of Ruppia may be accounted for by the degeneration of this tissue. its assumption of the function of storage, and the transfer of its functioning power as a root to the adventitious root.

# FRUIT AND SEED

Pl. IV. fig. 13 shows a cluster of mature fruits, and, as may be seen. the form varies somewhat, but is always bluntly pointed at the apex, with a one-sided base, the whole being slightly oblique with respect to the axis of the stipe. At the maturity of the fruit. the stigma generally drops off, leaving the blunt apex shown in the figure.
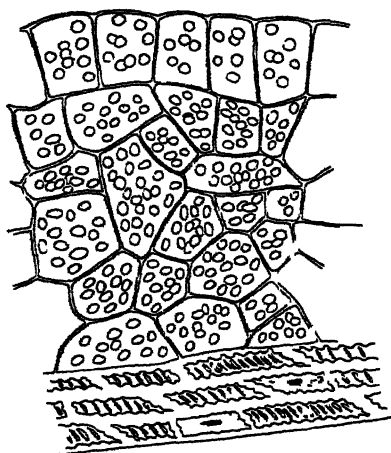


Figure 30.—Section through ovary wall of ripe fruit. showing the three innermost hardened layers, which form the covering of the seed, and the outer soft parts. × 210.
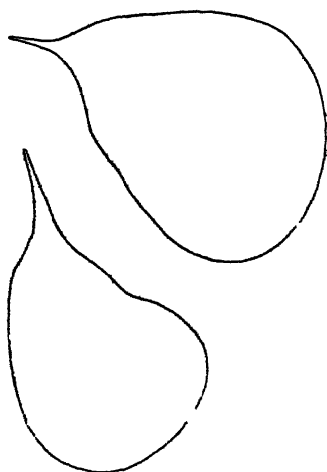
Figure 31.—Two ripe achenes. after disintegration of the outer soft parts of the fruit covering; showing the pronounced beaks. × 35.

During the development of the seed, changes occur in the wall of the ovary, the most important of which is the thickening of the cell-walls of portions contiguous to the seed, with the result that this inner layer of the ovary-wall becomes quite hard. This hardened portion is limited to about three layers of cells. Outside of this hard layer are about five or six layers of thin-walled cells, copiously supplied with starch, and these, in turn, are bounded by the epidermis, also rich in starch (Text-fig. 30).

The ripened fruit has a greater specific gravity than water. which one can easily demonstrate by breaking off the fruit from the stipe. It then sinks immediately to the bottom, and. in most cases, probably passes the winter embedded in the mud. Under such conditions. the outer soft parts of the fruit-covering soon decay, leaving the inner thickened portion of the ovary-wall which surrounds the seed. Since this area extends up into the stylar canal. the result is an appearance as in Text-fig. 31. At the end is a long beak. derived from the stylar region. Such a structure is accordingly an achene. its outer layer being hard, dry and indehiscent, and derived from the ovary.

A prominent beak is supposed to be a specific character of *Ruppia rostellata*. But a comparison with the figures of Irmisch (1858) of *R. rostellata* demonstrates that the beak represented there is not any longer than that of my specimens. although of a slightly different shape.

A similar development of fruit and seed is indicated by Campbell (1897) in Zannichellia, where a seed is formed surrounded by a pericarp from the ovary-wall. Such achenes are also well known to occur in the grasses.

# SEEDLING

*Germination.* A considerable number of seeds were germinated in the laboratory. Seeds gathered in October, at the end of the flowering season, first began to germinate in December, and from that time on seedlings appeared intermittently. It is evident, therefore, that the seed requires a short period of rest before the new growth begins.

Seedlings were grown either in mud from their native habitat, or in clean-washed sand, the latter being preferable, on account of the numerous algae, bacteria, &c. which soon develop in the former. The salt water used was taken from New Haven Harbor.

Irmisch (1858) noticed in the achene of *Ruppia rostellata* a small, slightly swollen, elongated area which he claimed was the place of exit for the cotyledon and root on germination. Such a spot occurs also in *Ruppia maritima*, but I have been unable to ascertain definitely if this is the region which is ruptured at germination. In most cases a more or less triangular area of the hard, dark-brown covering is pushed off and the cotyledon and adventitious root make their appearance (Pl. XV, fig. 120). In figure 121 the whole pericarp has been purposely removed to show the enormous hypocotyl.

Figure 120 shows a seedling about three days old. Here the three vegetative organs are disclosed: the first foliage leaf is developed from the cotyledon, which becomes green; the stem is formed by the elongation of the hypocotyl; and the root grows rapidly downward, becoming abundantly furnished with root-hairs.

In the case of the root, it is interesting to note that it exhibits a general tendency to first grow upwards for a short time, only later turning downward. The proportional growth of roots and leaves seems to vary considerably (Pl. XV, figs. 120, 121).

*Formation of Rootstock.* Text-figs. 32 and 33 are drawings—natural size—of seedlings grown in the laboratory in washed sand and harbor water. The seeds were collected in January, being extracted from mud taken from the bottom of a ditch where *Ruppia maritima* grows and fruits abundantly, many of the parent plants being even at that time in a green, flourishing condition. On being placed in a warm room, germination occurred in a few days.

During their growth the young plants showed very clearly the manner in which the horizontal rootstock is developed. At first the seedling is erect, but soon. as it increases in length reclines
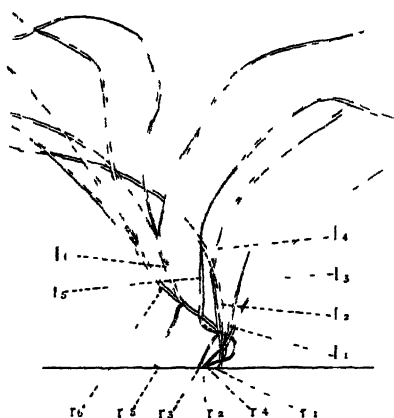


Figure 32.—Young seedling. showing development of horizontal root-stock. *l1. l2. l3.* &c.. successively older leaves ; *r1. r2. r3.* &c.. successively older adventitious roots. ½ natural size
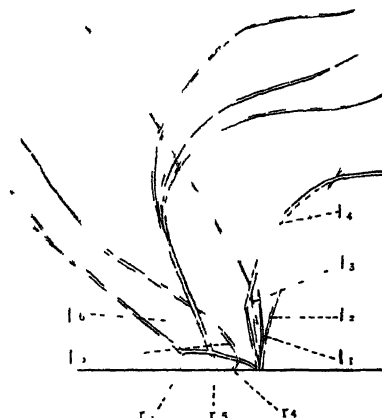
Figure 33.—Young seedling. older than preceding one. showing more advanced development of horizontal rootstock. Abbreviations as in previous figure. ½ natural size.

somewhat, and at the same time an adventitious root (Text-fig. 32, $r_1$) appears at the first node and grows downward. fixing itself in the soil or sand. This is repeated at successive nodes, the shoot reclining more and more, and Text-fig. 32 shows an advanced stage where five adventitious roots have been developed, and a sixth ($r_6$) is just appearing. Three days after the drawing was made, this sixth root had reached the surface of the sand. In this figure the roots have almost the appearance of drawing down the stem to a horizontal position. Whether or not this is actually the case, a horizontal location is ultimately attained, as in Text-fig. 33. Here, indeed, the first roots have apparently drawn the oldest part of the seedling well down into the sand.

From this time on the seedling assumes the habit of the mature plant. From the nodes upright shoots may arise, which may also develop later in the same way into root stocks, thus forming a branched rootstock; or it is conceivable that these upright shoots may continue in an erect position, ultimately reaching the surface of the water and flowering.

The anatomy of the seedling corresponds in the main to that of the mature plant. The central vascular area in the stem is thus essentially the same as in the older plant, so that no ancestral characters were detected in the vascular system in this young stage.

## SUMMARY OF PARTS
## ON REPRODUCTIVE ORGANS, EMBRYO, &c.

*Flower.* The inflorescence is spadix-like, and even a small spathe is present, the latter structure having almost entirely escaped the notice of investigators.

The small scaly outgrowths from the anther-connective probably represent reduced floral leaves, judging from the analogy of Potamogeton, and from the fact that the flower of Ruppia is evidently a reduced one.

The two flowers of the spadix arise laterally from the vegetative cone; they develop practically together in point of time, and the anthers precede the pistils in development.

*Microsporangium.* Appearances indicate that the archesporial initials in the microsporangium are a comparatively large group of cells which originate simultaneously both in plerome and periblem.

A plate of sporogenous cells becomes differentiated to form the future septum dividing the two sacs of the theca.

The tapetum is evidently derived from the sporogenous cells.

The sporophytic number of chromosomes was definitely ascertained to be sixteen.

*Megasporangium.* The megasporangium arises from a hypodermal cell in the usual way. Generally one layer of parietal cells is formed.

An interesting case was found of two megaspore mother-cells in one megasporangium, a condition little known among monocotyledons.

*Female Gametophyte.* Eight chromosomes appear in the first reduction division, and probably in the succeeding ones.

A double megaspore mother-cell was found, which had in each case divided into two daughter-cells.

The four potential megaspores are oriented in such a way that the two outer cells are in contact with the third cell, but separated by it from the innermost cell.

The embryo-sac develops in the usual way and always i⁻ m t¹ e innermost potential megaspore, the outer ones becoming resorbed. The antipodal cells are surrounded by cytoplasm and a thin membrane, and are always three in number. No further divisions among them were observed.

*Male Gametophyte.* The tetrads are oriented to each other like the four quadrants of a sphere.

Eight chromosomes appear in the reduction-division.

The generative cell is formed very early in the development, at the end of the young pollen-grain. No membrane now or later separates it from the rest of the grain.

When the grain has nearly reached maturity, the generative cell divides, the two resulting male cells remaining united.

The mature pollen-grain is of peculiar shape, but similar forms occur in other Potamogetonaceae.

The wall of the pollen-grain is formed inside of the walls of the tetrads, these latter walls becoming apparently dissolved.

The wall is never of more than a single layer. The thickenings on its exterior appear to be the result of depositions. At certain regions these depositions are omitted, and these spots are probably the places where the pollen-tube may commence its formation.

*Pollination.* Pollination is accomplished by means of water, an unusual method.

*Endosperm.* The endosperm is scanty, never more than a thin layer lining the embryo-sac, and containing free nuclei.

*Embryo.* The suspensor is limited to a single large cell, which later develops to an enormous size.

The embryo evidently develops from two or three primary segments as in the typical monocotyledonous embryo.

An unusual state of affairs exists, however, in the large number of early longitudinal divisions.

Apparently both cotyledon and stem-apex arise from the terminal segment as in Zannichellia, but this cannot be definitely proved, on account of the obscurity of the segment boundaries.

The two lower segments form hypocotyl, adventitious root, and primary root.

I agree with Murbeck in locating the vestiges of the primary root at the base of the enlarged hypocotyl.

Another possible interpretation is that practically the whole swollen area is a root, metamorphosed into a storage organ.

*Seed and Fruit.* The fruit is an achene, the hard coat being derived from the inner portion of the ovary wall.

*Seedling.* At germination the cotyledon develops chlorophyll, and the adventitious root of the embryo is the first functioning root.

At first the young seedling is upright, but very soon a horizontal root-stock is developed.

## SUMMARY OF RELATIONSHIPS TO OTHER POTAMOGETONACEAE.

On account of the incomplete knowledge we have at present of the members of the Potamogetonaceae, a thorough comparison with them in all points of gross morphology and of anatomy is of course impossible. In the course of this paper I have from time to time alluded to the points of likeness and difference with respect to the remaining Potamogetonaceae, as far as I have been able to glean such from the various articles cited. It is my purpose now to briefly review these here, touching upon the various characters in the order pursued in my paper.

As regards the methods of branching, Cymodocea, Phyllospadix, and Zostera most resemble Ruppia, with a monopodial system in the rootstock and its branches, and a sympodial inflorescent system.

In stem anatomy, perhaps *Potamogeton pectinatus* and *Zostera marina* resemble Ruppia most closely, while Zannichellia and Althenia are also very similar in all respects except that they lack the cortical bundles. The evidence here seems to point to the conclusion that Zostera and Potamogeton are more primitive genera, while, on the other hand, Zannichellia and Althenia are more reduced, with Ruppia somewhere between.

The leaf of Ruppia shows a great similarity in external form to certain species of Potamogeton and to Zannichellia and Althenia. I do not find the secretion cells of the leaf—or indeed of any part of the plant—reported for any but Cymodocea. Posidonia, and Halodule, besides Ruppia. In other points of anatomy, however, the leaf of *Potamogeton pectinatus* is very similar, while the leaves of Zannichellia and Althenia show a strong resemblance, but with absence of the lateral bundles. The natural conclusion based on this evidence is, as stated before, that Ruppia is descended from some form similar to the present submerged Potamogetons, and also that Zannichellia and Althenia are still further reduced.

In the root system. although Ruppia has usually only one root at each node, Zannichellia and Althenia have two. and the Potamogetons several. even *Potamogeton pectinatus* developing four or more (Irmisch. 1858). In Zostera and Phyllospadix a cluster of roots occurs at the nodes. The coleorrhiza has been reported in Potamogeton, Zannichellia. and Cymodocea, and probably occurs also in Althenia and Zostera. In the root anatomy, *Potamogeton pectinatus*, Zannichellia and Althenia correspond closely with Ruppia, as is the case in the other vegetative organs. Zannichellia showing the four distinct meristematic regions at the growing-point as in Ruppia.

Passing to the reproductive organs. where, owing to our lack of information, the comparison is most unsatisfactory, we have seen that the inflorescence is spadix-like, resembling Potamogeton and Zostera. and that the flowers themselves are apparently reduced from some form like Potamogeton.

The archesporium of the microsporangium probably arises in Zannichellia in much the same way as in Ruppia. i. e., originating simultaneously from a large group of cells. Only one species of Potamogeton has been investigated on this point, and here the archesporium is traceable to a single hypodermal cell. Like Ruppia, Zostera also derives tapetum from the outer sporogenous cells, but *Potamogeton foliosus* differentiates it from the wall.

Parietal tissue exhibits a much greater development in the megasporangium of Potamogeton than in Ruppia, and in this respect Zannichellia probably more closely resembles Ruppia. This consideration again points to Potamogeton as the more primitive form. In *Potamogeton natans* (p. 138) an arrangement of the potential megaspores has been found such as occurs in Ruppia.

The pollen-grains of Zannichellia and Potamogeton are more or less globular. those of Ruppia somewhat elongated, while those of Zostera and Phyllospadix have attained extreme length so that they might be termed filamentous. These differences depend presumably on the different modes of pollination prevailing among these genera.

More than any other member of the Potamogetonaceæ so far investigated, the embryo of Zostera has points in common with Ruppia in the manner of its development, and, as is the case with the mature embryo of Zannichellia, resembles it in its mature form. The primary root of Zostera, however, functions for a time during germination.

It will be seen from these considerations that, on the whole,

Ruppia is closer to Potamogeton, Zannichellia, and Althenia than to any others of the Potamogetonaceae, especially with regard to the vegetative organs; and yet, in its embryo, it most strongly resembles Zostera. Similarities in the vascular structure of the vegetative organs are looked upon generally as comparatively fixed indications of relationship; so that in this case the evidence points to Ruppia as being derived from some form similar to the present submerged Potamogetons, with Zannichellia and Althenia serving as examples of still further reduction.

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# EXPLANATION OF PLATES

Abbreviations. *A* & *B*. primary segments of embryo: *adr*, adventitious root; *ant*, antipodal cells; *as*, axillary scale; *ax*, axillary member; *axb*, axial bundle; *axbr*, axillary branch; *C*, third segment of embryo; *c*, cortex; *cal*. calyptrogen; *cb*. cortical bundle; *cc*, companion cell; *col*. coleorrhiza; *con*, connective; *cot*. cotyledon; *d*, dermatogen; *di*. diaphragm; *e*, egg; *ems*, embryo-sac; *end*, endodermis; *ep*. epidermis; *epc*. epicotyl; *fl*, flower; *fsl*, floral scale leaf; *gen* generative cell; *GP* or *gp*, growing point; *hyp*, hypocotyl; *i*, intercellular space; *ic*, inner cortex; *la*, lacuna; *L* or *l*, leaf; *lb*, lateral bundle; *low*. *sfl*, lower subfloral leaf; *lpn*. lower polar nucleus; *ma*. male cells; *mc*, middle cortex; *oc*, outer cortex; *pa*, parietal layer or layers; *pb*, periblem; *ped*, peduncle; *pist*, pistil; *pl*, plerome; *pm*, pollen mother-cells; *pr*, primary root cells; *r*, root; *rh*, root hairs; *rha*, rhachis; *rs*, root stock; *s*, sieve tube; *se*, secretion cells; *sfl*, subfloral leaf; *sg*, starch-grains; *sh*, sheath; *spo*. sporogenous cells; *spt*, septum; *st*, stem or stem apex; *sta*. stamen; *stc*. stylar canal; *stg*, stigma; *sti*. stipe; *sus*. suspensor; *syn*. synergidae; *ta*. tapetum; *th*, theca; *tn*, tube nucleus; *tra*. tracheae; *tri*, trichoblasts; *u*, undulations; *upn*, upper polar nucleus; *up*. *sfl*. upper subfloral leaf; *ust*, upright stem; *vsl*, vegetative scale leaf.

# PLATE I

**Figure 1.**—Longitudinal section of stem-apex, showing growing-point and manner of development of leaves and axillary buds: $L^I$, $L^{II}$, $L^{III}$, &c.. primary leaves: $l_1^{(I)}$, $l_2^{(I)}$, &c.. $l_1^{II}$, $l_2^{II}$, &c.. leaves of axillary buds of $L^I$, $L^{II}$, &c.: $G\ P$. main growing-point of stem: $gp^{(V)}$, $gp^{(IV)}$, &c.. growing-points of axillary buds : $ab$. line where such a cross section as is represented in Plate VII, figure 43, might be cut perpendicular to plane of paper. $\times$ 30.

**Figure 2.**—Sketch from living specimen, showing vegetative branch system in a young stage: $I$, $II$, $III$, $IV$, &c.. primary leaves. $\frac{1}{2}$ natural size.

**Figure 3.**—Cross section of axial bundle of stem, showing endodermis, sieve tubes, central cavity formed by disorganization of tracheae, &c. $\times$ 300.

**Figure 4.**—Cross section of stem. $\times$ 60.

**Figure 5.**—Cross section of epidermis, cortex and endodermis of stem. $\times$ 212.

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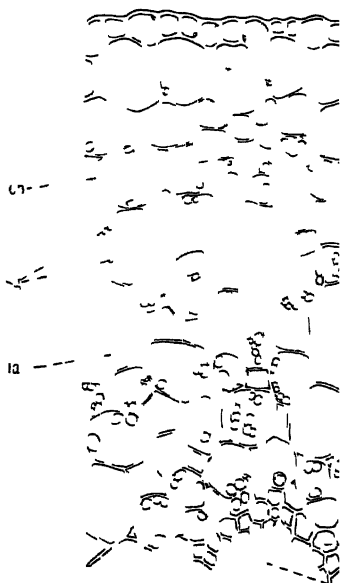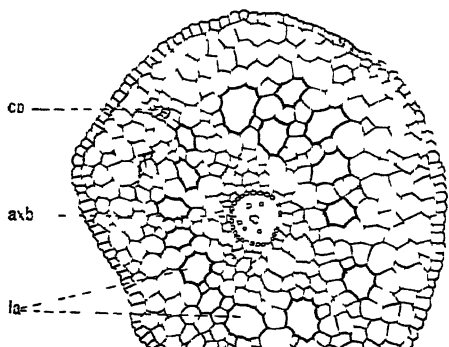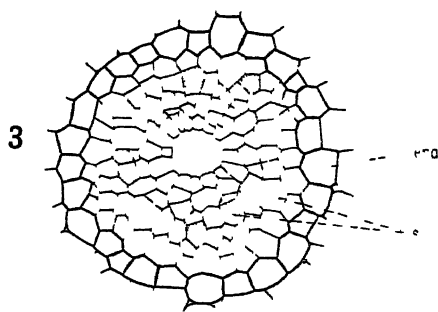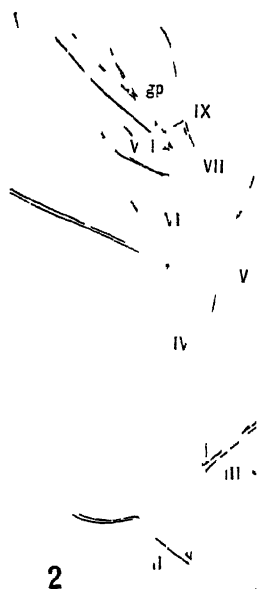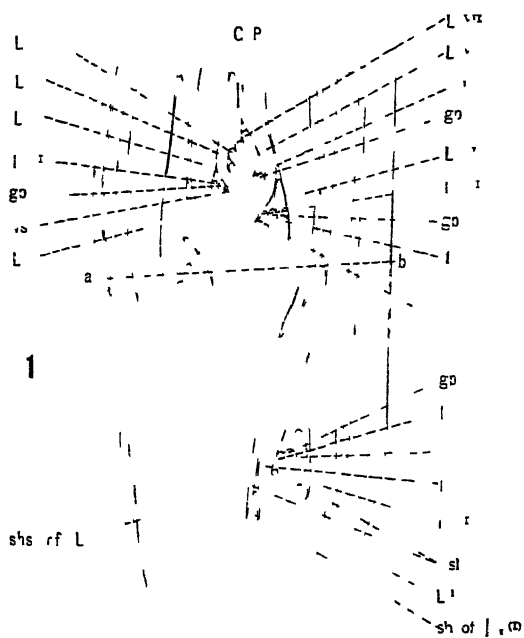
# PLATE II

**Figure 6.**—Sketch from living specimen. showing vegetative branch system in a mature stage, just previous to the appearance of flowers. Abbreviations as in figure 1. $\frac{1}{4}$ natural size.

Plate II



6

# PLATE III

**Figure 7.**—Photomicrograph of cross section of stem, showing axial bundle, endodermis, cortex, lacunae, and epidermis. × 30.

**Figure 8.**—Photomicrograph of cross section of stem, axillary bud and subtending leaf: showing axillary scales, vegetative scale leaf surrounding axillary bud, secretion cells, &c. × 25.

**Figure 9.**—Photomicrograph of cross section of rootstock: showing axial bundle, endodermis, cortex, lacunae, cortical bundles, epidermis, and starch-grains in cortex. × 50.

**Figure 10.**—Photomicrograph of cross section through node of rootstock: showing two roots developing, and epidermis of rootstock over each root becoming meristematic. × 35.

**Figure 11.**—Photomicrograph of cross section through rhachis: showing axial bundle with lacunae, &c., and surrounding stamen, with its two thecae, connective, &c. × 35.

**Figure 12.**—Photomicrograph of longitudinal section through end of rhachis: showing its blunt apex and various sections of pistils and anthers. × 35.

Plate III



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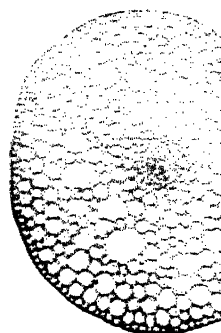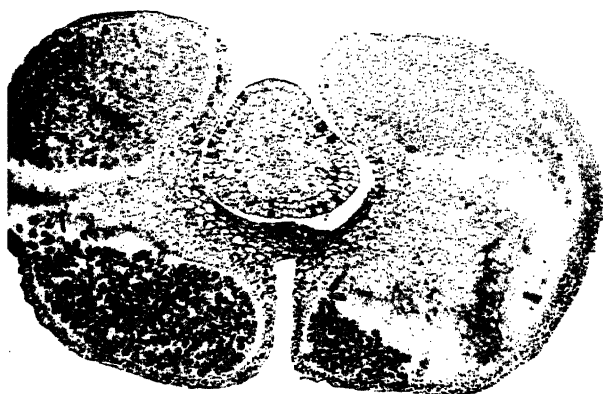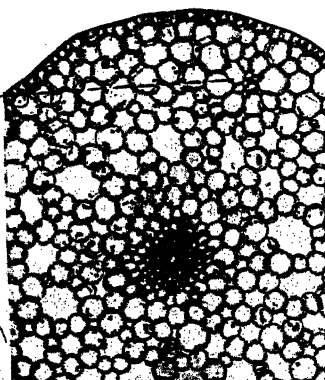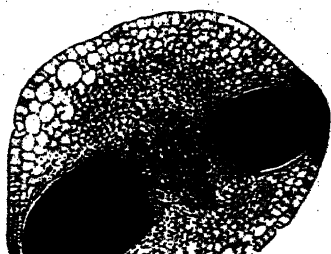
# PLATE IV

**Figure 13.**—Drawing of cluster of fruit about mature. $\succ$ 5.

**Figure 14.**—Surface view of epidermis of stipe young stage : showing secretion cells. $\times$ 212.

**Figure 15.**—Cross section of mature stipe ; showing epidermis. cortex. and axial bundle ; intercellular spaces and lacunae stippled : x = apparently halved epidermal cells. $\times$ 185.

**Figure 16.**—Part of cross section of stipe. showing strengthened epidermis. part of cortex. lacunae and small intercellular spaces. $\times$ 212.
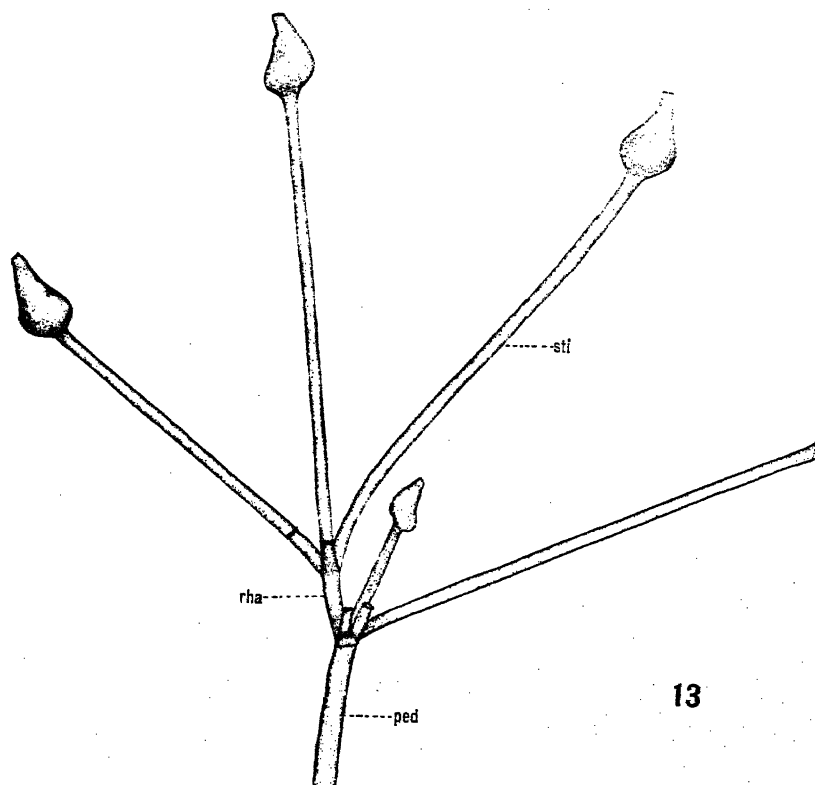
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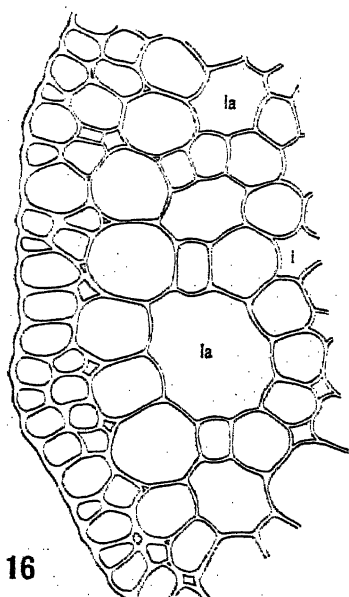


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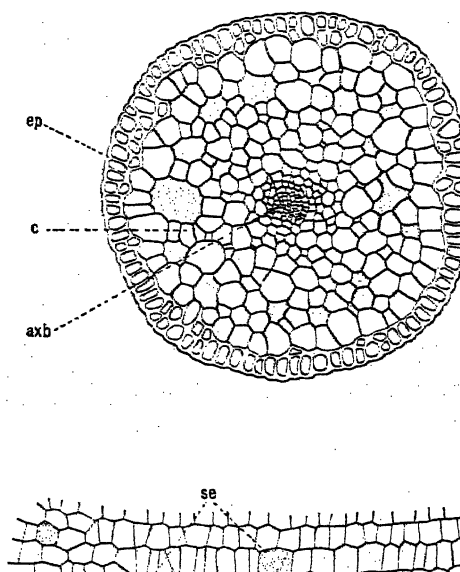
# PLATE V

**Figure 17.**—Diagrammatic representation of median longitudinal section through stem and axillary branch; *L.* subtending leaf; vascular portions stippled. × 212.

**Figure 18.**—Diagrammatic representation of median longitudinal section through stem at base of inflorescent branch system, showing the course of the vascular bundles. Vascular portions stippled. × 212.

**Figures 19–24.**—A series of cross sections through stem beginning just below point of leaf insertion and extending up into leaf, showing course of cortical bundles from stem to leaf and independent origin of new cortical bundles in the stem. × 35.

Plate V



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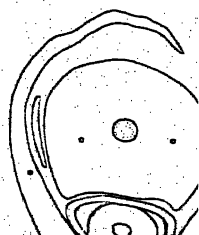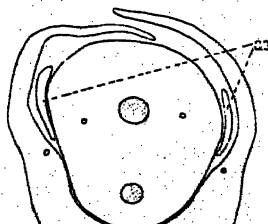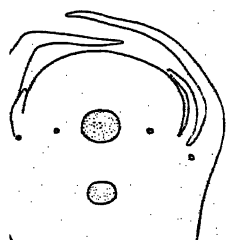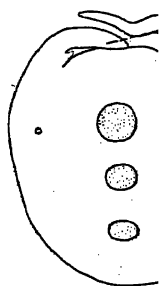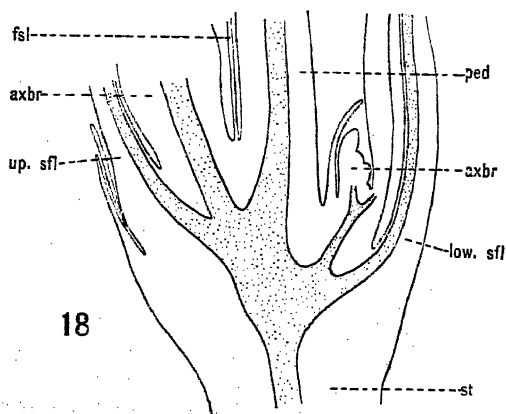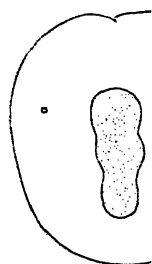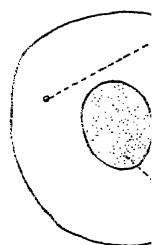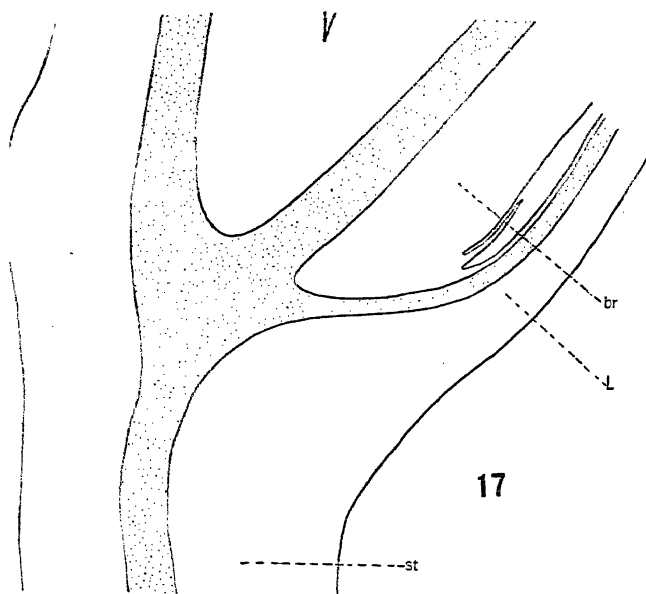
# PLATE VI

**Figure 25.**—Photomicrograph of longitudinal section through stem apex; showing growing point and leaves in successive stages of development, similarly as in Pl. I, fig. 1. ✕ 40.

**Figure 26.**—Photomicrograph of apical portion of leaf, not quite mature; showing marginal teeth. ✕ 35.

**Figure 27.**—Photomicrograph of cross section of root; showing epidermis, exodermis, cortex, endodermis and axial vascular bundle. ✕ 60.

**Figure 28.**—Photomicrograph of young leaves surrounding growing-point, dissected from living specimen. ✕ 35.

**Figure 29.**—Photomicrograph of portion of epidermis near apex of root; showing trichoblasts (the darkly stained cells). ✕ 40.

**Figure 30.**—Photomicrograph of surface of leaf treated with a five per cent solution of potassium hydroxide; showing epidermal secretion cells and central vascular bundle. The marginal teeth have been worn off. ✕ 30.

**Figure 31.**—Photomicrograph of a cross section through a node of the stem showing the origin of a single root—the typical condition. In this figure the meristematic layers at the growing point of the root are clearly defined. ✕ 35.

**Figures 32 and 33.**—Photomicrographs of adjoining portions of a nearly median longitudinal section through the root apex, showing trichoblasts. (The root cap was slightly separated from the growing-point in preparation.) ✕ 50.

**Figure 34.**—Photomicrograph of cross section of young peduncle, showing axial bundle, lacunae in cortex, &c. ✕ 35.

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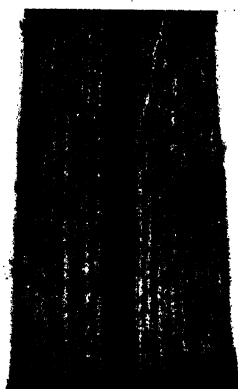
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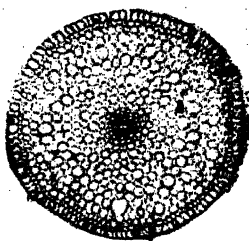
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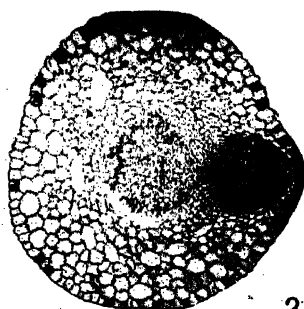
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# PLATE VII

**Figure 35.** Sketch of flower bud; showing basal sheaths of sub-floral leaves enclosing young flower. × 5.

**Figure 36.** Sketch of portion of plant; showing horizontal root-stock and arrangement of roots and upright shoots. ½ natural size.

**Figure 37.** Group of cells from vegetative scale leaf. × 162.

**Figure 38.** Coleorrhiza from which young root has been forced out. × 10.

**Figure 39.** Young leaf; showing teeth limited to extreme apex and comparatively large basal sheath. × 212.

**Figure 40.** The vegetative scale leaf, detached, and flattened out. × 5.

**Figure 41.** Longitudinal section through young inflorescence, showing floral scale leaf developing in connection with the flowers. × 30.

**Figure 42.** Floral scale leaf *in situ*, close to young flowers. × 4.

**Figure 43.** Cross section through vegetative bud, a little below the region of the growing point, cut at about the level shown by the line *ab* in figure 1. Abbreviations as in figure 1. The line *ab* in this figure represents the plane perpendicular to the paper, in which a longitudinal section such as is shown in figure 1 would lie. × 65.

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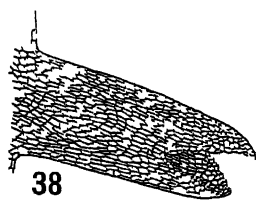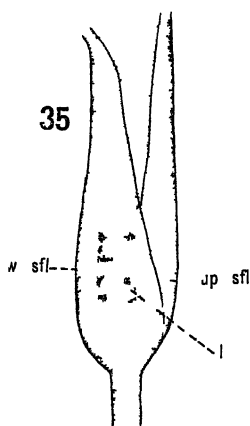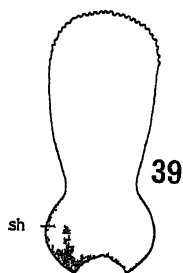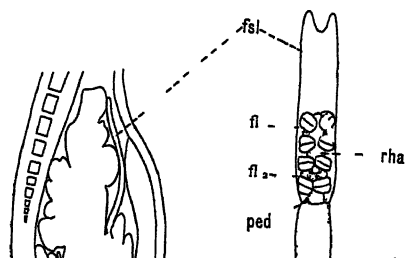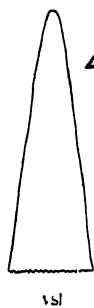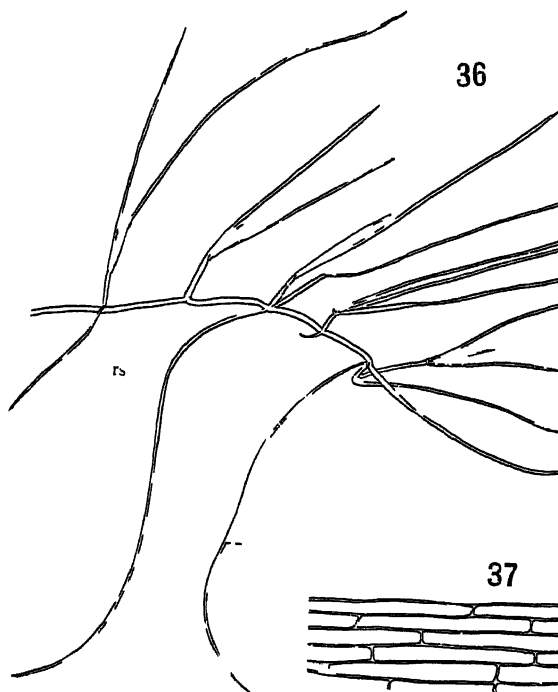
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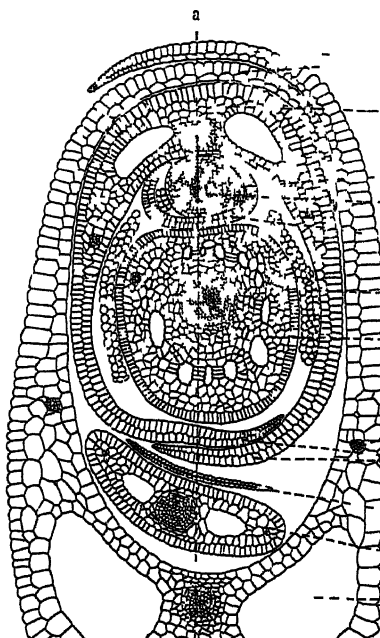
# PLATE VIII

**Figure 44.**—Longitudinal section of young root still enclosed in the stem; showing its origin in the stem, the meristematic epidermis and subepidermis in the stem above root apex and the four meristematic layers in the root. ✕ 210.

**Figure 45.**—Portion of rootstock and root; the latter showing coleorrhiza. ✕ 6.

**Figure 46.**—Cross section of vascular bundle of root with endodermis; showing tracheae and sieve tubes. ✕ 400.

**Figure 47.**—Longitudinal section of vascular bundle of root, also with endodermis and, to the left, three layers of cortical parenchyma; showing tracheae, sieve tube, companion cell and undulations in walls of cortical parenchyma. ✕ 400.

Plate VIII



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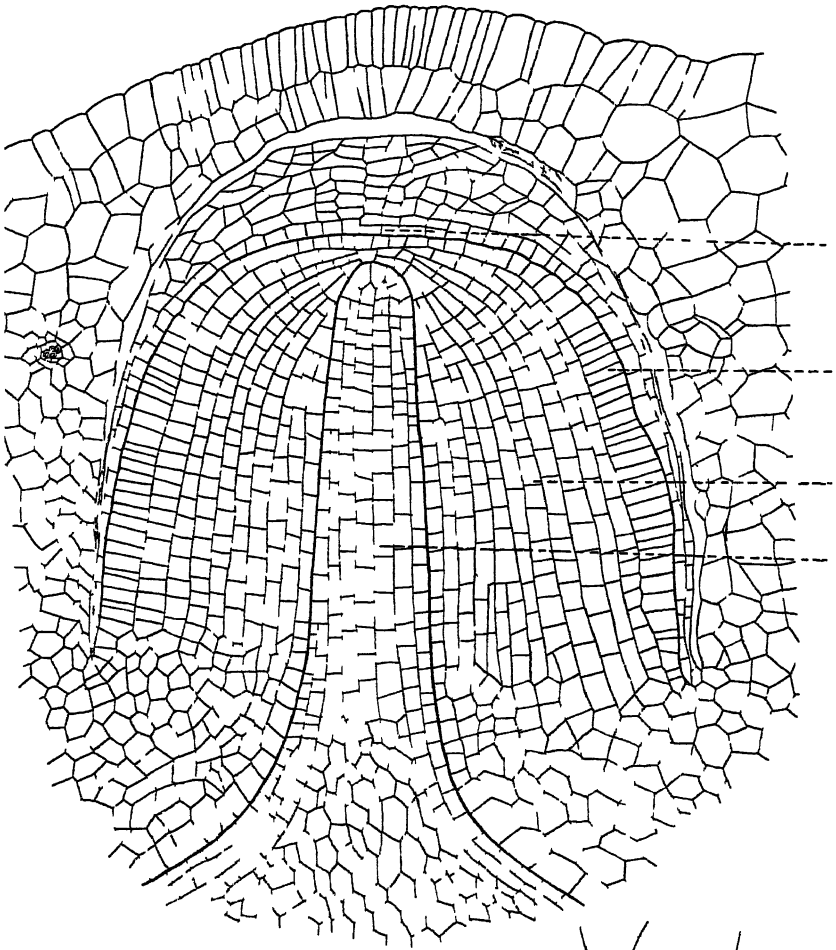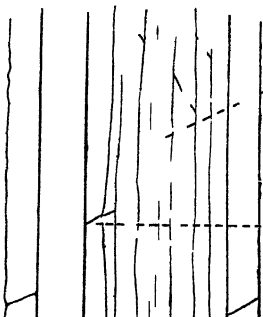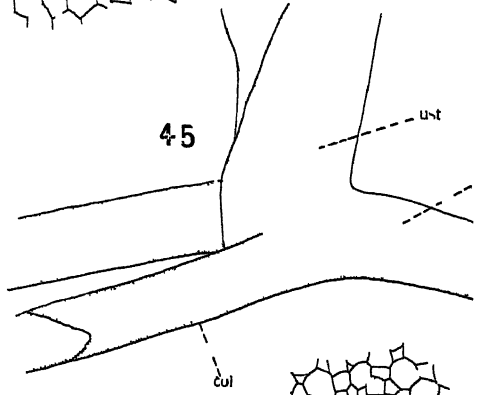
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# PLATE IX

Figure 48.—Surface view of cells from epidermis of root, showing root hairs developing from the trichoblasts. × 210.

Figure 49.—Rhachis with the two flowers; showing arrangement in the adult condition. × 12.

Figure 50.—Rhachis and young flowers; showing arrangement in young stage. × 30.

Figure 51.—Longitudinal section through nearly mature rhachis and flowers. × 20.

Figure 52.—Very young floral primordia. Stippled portions represent regions which will develop the flowers. × 210.

Figure 53.—Slightly older floral primordia. Stippled portions represent regions which will develop the flowers. × 210.

Figure 54.—Older floral primordia. Floral scale leaf appearing. × 210.

Figures 55 and 56.—Successively older stages in floral development. In figure 56 the primordia of pistils and stamens appear well defined. × 60.

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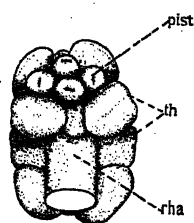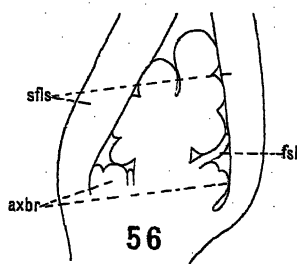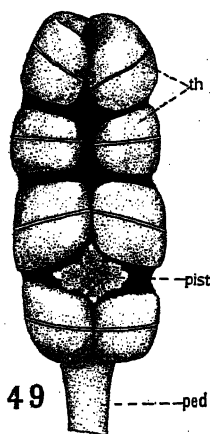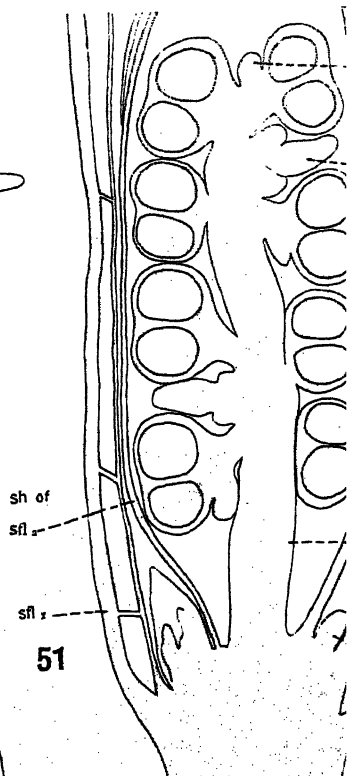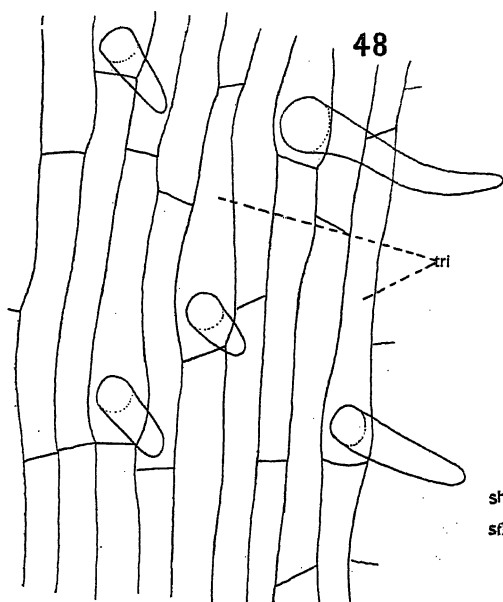
# PLATE X

Figure 57, a and b.—Two views of a mature theca, the dotted triangular region representing the region of attachment to the connective. × 30.

Figures 58–66.—Median longitudinal sections illustrating development of the pistils.

Figure 58.—Section through flower rudiment; showing primordia of two pistils, an anther-connective at each side. × 75.

Figure 59.—Section of older stage; showing the beginning of the development of the stylar canal. × 75.

Figure 60.—Older stage. × 75.

Figure 61.—Older stage. One-sided development of the pistils commences. × 75.

Figures 62 and 63.—Stigma commences to form. Stylar canal well developed. × 75.

Figure 64.—Stigma assumes peltate form. Ovule 'and embryo-sac well developed. × 75.

Figures 65 and 66.—Development to approximately the mature form of pistil; showing the gradual oblique orientation of ovule. Figure 65, × 75; figure 66, × 37.

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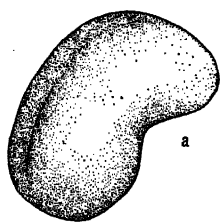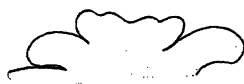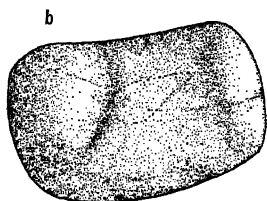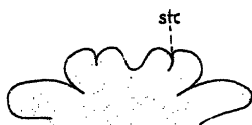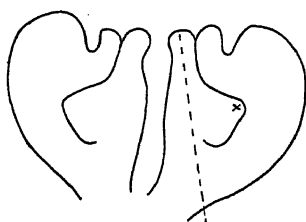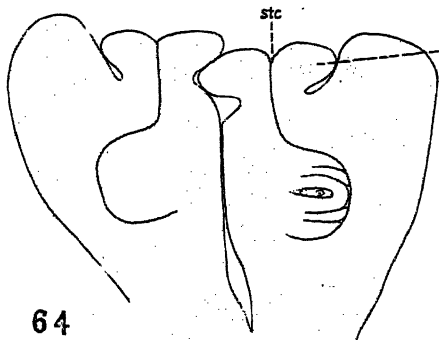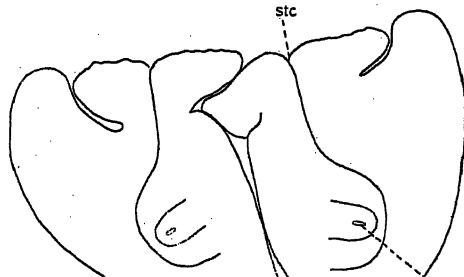
# PLATE XI

Figure 67.—Cross section through young theca of a flower 0.2 mm. long, the measurement being taken in the direction of the dotted line shown in figure 68. Stippled cells represent the archesporial initials. ✕ 600.

Figure 68.—Section of young flowers from which figure 67 is drawn, the actual length of each flower being about 0.2 mm. x marks the region represented in figure 67. Dotted line shows direction for measurement of the length of the flower. ✕ 60.

Figure 69.—Cross section through slightly older theca of a flower 0.25 mm. long; showing division initiating development of septum, and also formation of the primary parietal layer. ✕ 600.

Figure 70.—Section of flowers from which figure 69 is taken. x marks the region in figure 69. ✕ 60.

Figure 71.—Cross section through theca of flower measuring about 0.3 mm. in length; showing sporogenous cells in the two sacs of the theca, two parietal layers, in places, and septum. ✕ 600.

Figure 72.—Section of flowers from which figure 71 is taken. x marks the region shown in figure 71. ✕ 60.

Figure 73.—Cross section through theca of a flower measuring 0.33 mm. in length; showing sporogenous cells in two sacs of theca, two parietal layers, and septum. ✕ 400.

Figure 74.—Cross section through one of the sacs of a theca of a flower measuring about 0.5 mm. in length. Parietal layers often three in number. ✕ 400.

Figure 75.—Group of cells from cross section of pollen-sac; showing pollen mother-cells surrounded by the still intact tapetum. ✕ 600.

Figure 76.—Portion of section through older pollen-sac; showing pollen mother-cells and dissolution of tapetal cells. ✕ 600.

Figure 77.—Section of megasporangium showing two megaspore mother-cells and one parietal layer. ✕ 375.

Figure 78.—Section of megasporangium showing four daughter-cells resulting from a double mother-cell, and also two parietal layers. ✕ 400.

Figure 79.—Section of megasporangium; showing the four potential megaspores, the wall separating the two upper ones being approximately parallel to the plane of the paper. The three upper cells are all being resorbed. ✕ 375.

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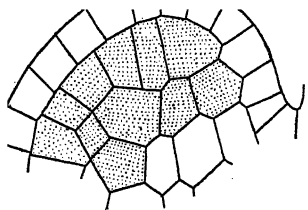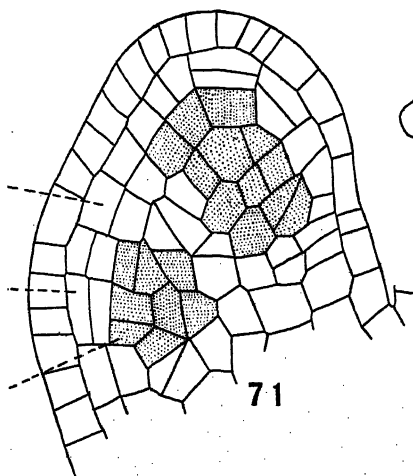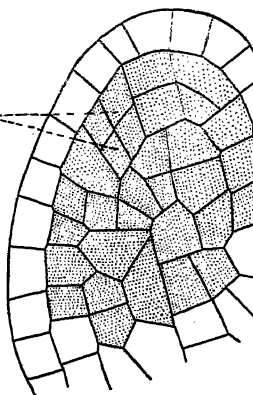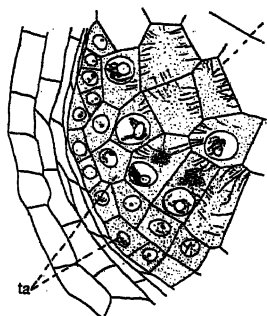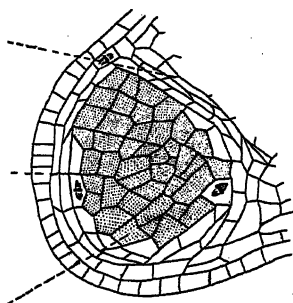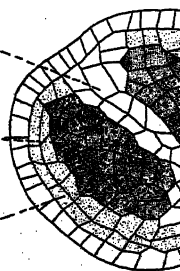
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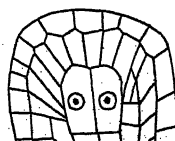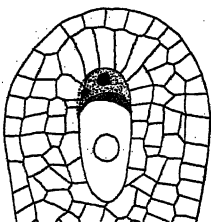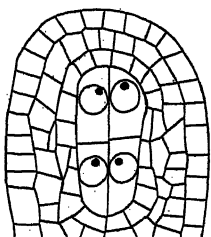
# PLATE XII

**Figure 80.**—Section of megaspore mother-cell with surrounding cells, showing spindle of first reduction-division. Eight chromosomes were counted in this spindle, four of which are shown in the figure. ✕ 800.

**Figure 81.**—Portion of section of megasporangium, showing the unusual condition of the four potential megaspores in a row, the two upper already being resorbed. Or, judging from the very indistinct wall separating the two lower cells and from their size, this may be a case where only three megaspores were formed, and the lowest has undergone the first division. ✕ 800.

**Figure 82.**—Portion of section of megasporangium, showing omission of formation of wall between the two outer potential megaspores, these and the next lower becoming resorbed. ✕ 800.

**Figure 83.**—Portion of section of megasporangium showing young embryo-sac with two nuclei; upper potential megaspores and part of surrounding tissue becoming resorbed. ✕ 800.

**Figure 84.**—Young embryo-sac with four nuclei; showing also remnant of upper potential megaspores. ✕ 800.

**Figure 85.**—Mature embryo-sac, showing synergidae, upper polar nucleus, egg, lower polar nucleus, and antipodals. ✕ 800.

**Figure 86.**—The three antipodal cells lying in pouch at base of nature embryo-sac. ✕ 800.

**Figure 87.**—Pollen mother-cell, synapsis stage. ✕ 800.

**Figure 88.**—Young pollen-grain in tetrad before the dissolution of the tetrad wall. ✕ 800.

**Figure 89.**—Pollen-grain just before formation of generative cell, showing enclosed starch grains and beginning of wall thickenings. ✕ 800.

**Figure 90.**—Pollen-grain showing spindle preparatory to formation of generative cell. ✕ 800.

**Figure 91.**—Cross section through middle of pollen-grain at stage shown in figure 90. ✕ 800.

**Figure 92.**—Pollen-grain just after formation of generative cell. ✕ 800.

**Figure 93.**—Longitudinal section through almost mature pollen-grain, showing tube-nucleus and generative cell just previous to formation of male cells. ✕ 400.

**Figure 94.**—Longitudinal section of mature pollen-grain showing the two male cells still united, and the degenerating tube-nucleus. ✕ 375.

**Figure 95.**—Longitudinal section of mature pollen-grain, showing characteristic dumbbell shape. ✕ 175.

**Figure 96,** a and b.—Surface and profile views, respectively, of the pollen-grain wall thickenings, at mature stage. ✕ 800.  ▌▌

**Figure 97.**—Pollen-grains, during the dissolution of the tetrad walls and formation of the pollen-grain wall. ✕ 800.
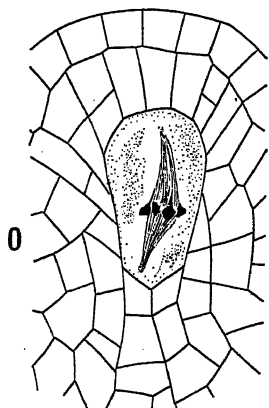
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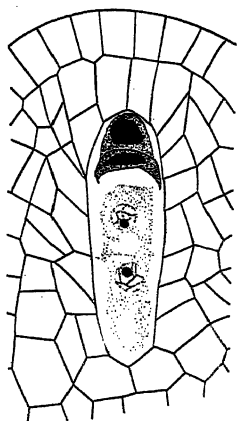
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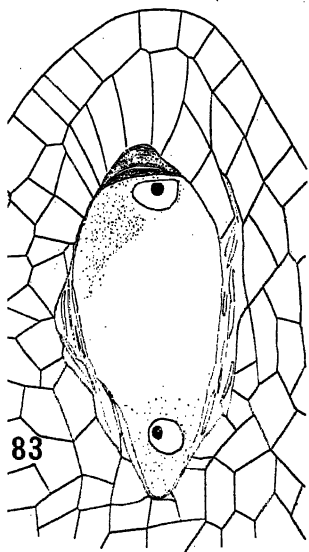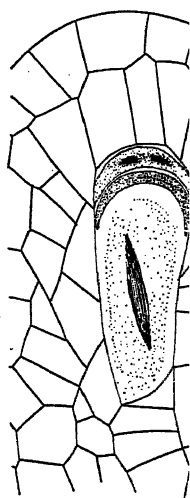
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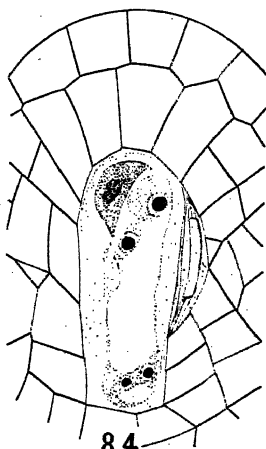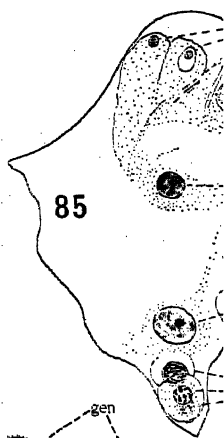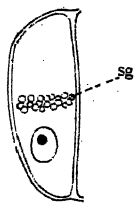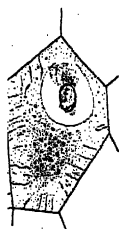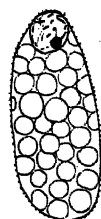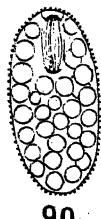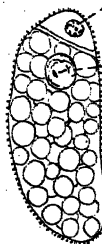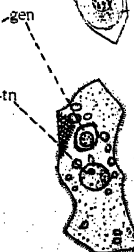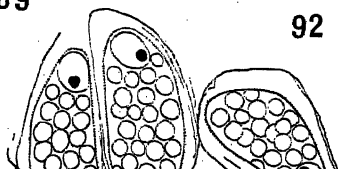
# PLATE XIII

**Figure 98.**—Three-celled proembryo of *Ruppia rostellata*, showing basal suspensor-cell and two smaller embryo-cells. After Murbeck. × 446.

**Figure 99.**—Nine-celled embryo of *Ruppia rostellata*, showing basal suspensor-cell and eight small embryo-cells. After Wille. × 340.

**Figure 100.**—Thirteen-celled embryo, showing division into three transverse segments of four cells each, and a large suspensor-cell with nucleus and cytoplasm in the characteristic position. × 800.

**Figure 101.**—Longitudinal section through older embryo, the heavy lines marking the outlines of the cells shown in figure 100. × 800.

**Figure 102.**—Longitudinal section of older embryo, showing the outlines of the original segments and the beginning of the formation of dermatogen, at least in the terminal segment, at left. × 800.

**Figure 103.**—Longitudinal section of embryo with a diameter of about 0.05 mm.; showing rather irregular segment lines, and globular form of embryo. × 800.

**Figure 104.**—Longitudinal section of embryo of a diameter of about 0.075 mm.; showing increase in size of all cells, but only a slightly greater number than in figure 103. × 800.

**Figure 105.**—Longitudinal section of embryo measuring about 0.085 mm. in length; showing commencement of elongation, and divisions in the terminal segment, at the left, indicating approach of cotyledonary development. × 300.

**Figure 106.**—Longitudinal section of embryo measuring about 0.135 mm. in length; segment limits obscure; apparently the terminal segment is commencing the formation of stem as well as cotyledon. Basal portion shows cells which go to form the rudimentary primary root. × 375.

00 A B C

105

102

99

01 A B C sus

103 sus

cot
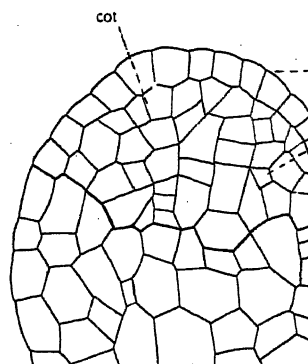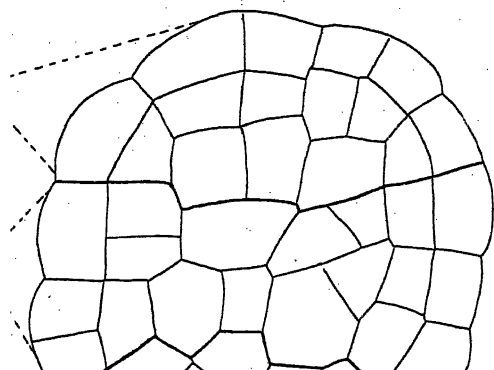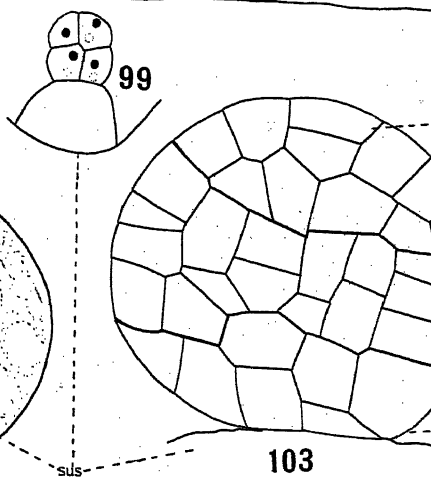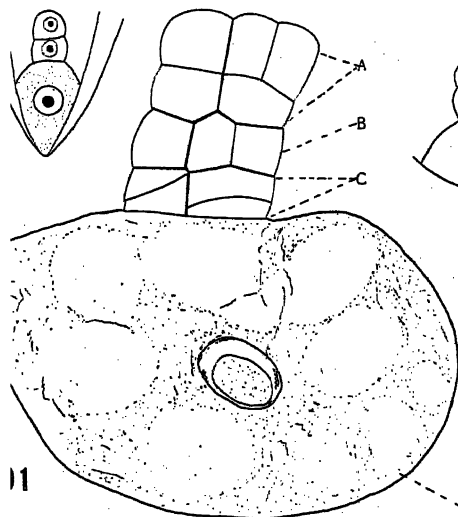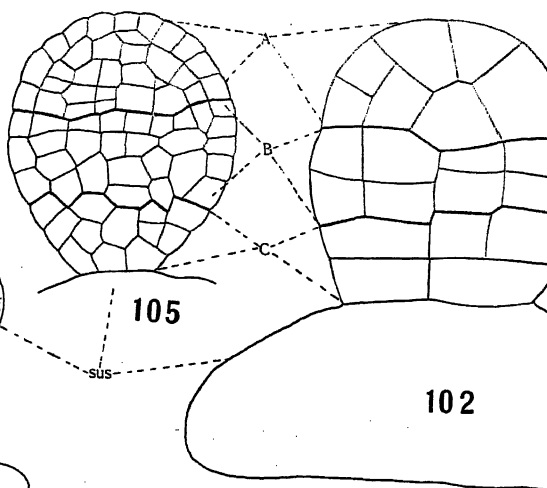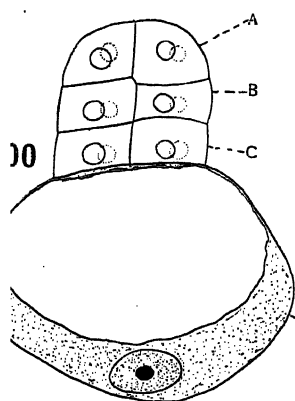
# PLATE XIV

**Figure 107.**—Part of longitudinal section through older embryo, showing origins of cotyledon and stem apex. ✕ 375.

**Figure 108.**—Section of embryo from which figure 107 is taken. x marks the region figured. ✕ 75.

**Figure 109.**—Longitudinal section of embryo measuring about 0.35 mm. in length; showing development of cotyledon and stem-apex or epicotyl. ✕ 50.

**Figure 110.**—Basal region of embryo shown in previous figure, illustrating the group of cells which forms the rudimentary primary root. ✕ 300.

**Figure 111.**—Single storage cell from hypocotyl of embryo shown in figure 109, showing enclosed starch grains. ✕ 600.

**Figure 112.**—Upper part of longitudinal section of nearly mature embryo shown in figure 113; showing stem-apex with growing-point and second leaf rudiment, part of cotyledon with section of basal sheath, and adventitious root. ✕ 212.

**Figure 113.**—Longitudinal section of nearly mature embryo from which figure 112 is drawn; showing hypocotyl, epicotyl, cotyl-edon, and adventitious root. ✕ 35.

**Figure 114.**—One of storage-cells from hypocotyl of embryo shown in figure 113. ✕ 600.

**107**

**114**

sg

**110**

**108**

cot
epc

**109**

**11**

cot
epc
adr

**113**

sh

L ¹¹

G P

**112**

cot

lr
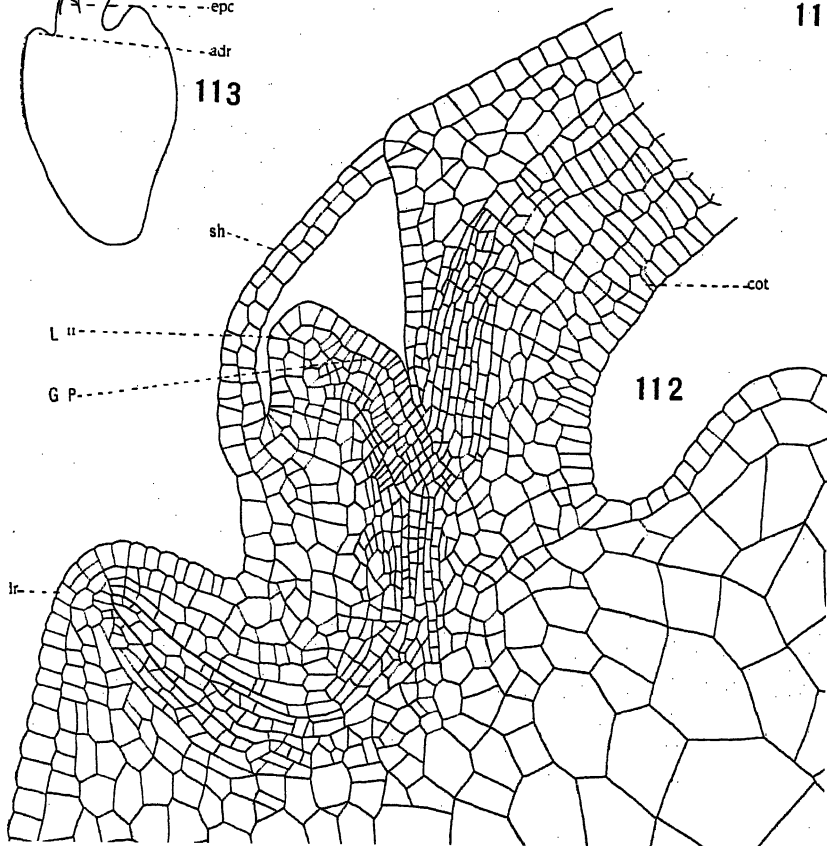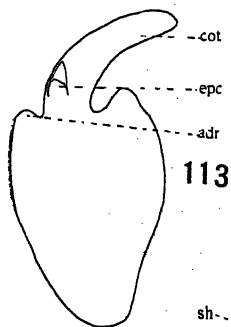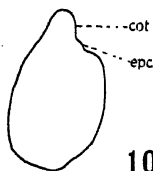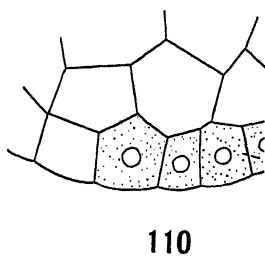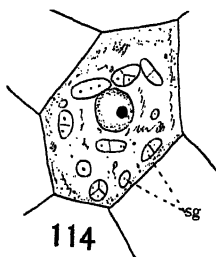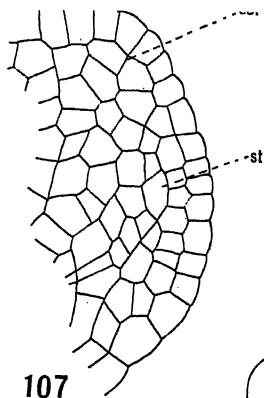
# PLATE XV

**Figure 115.**—One of the storage cells from hypocotyl of embryo represented in figure 116, showing increased size of all parts. × 800.

**Figure 116.**—Nearly mature embryo, older than that shown in figure 113, and showing the parts represented in the latter. × 35.

**Figures 117** and **118.**—Two views of a cotyledon, dissected from a mature embryo, showing position of epicotyl and the overlying cotyledonary sheaths. Figure 118, represents the narrow cleft between the sheaths. × 35.

**Figure 119.**—Mature embryo dissected from ripe seed. × 35.

**Figure 120.**—Seedling about three days old; showing cotyledon and young stem with the first node, adventitious root with root hairs, and the thick, storage part of the hypocotyl still within the hard shell. × 6.

**Figure 121.**—Two views of young seedling; showing long adventitious root with root hairs, cotyledon, epicotyl, &c. The hard shell has been purposely removed from the hypocotyl. × 6.

115

119

adr

hyp

116

cot
epc
adr

hyp

120

cot
st
hyp
rh
adr

121

cot
epc
hyp

rh

adr

sg

# Supplement to the New England Spiders

BY

## J. H. EMERTON

## III.—Supplement to the New England Spiders

### by J. H. Emerton.
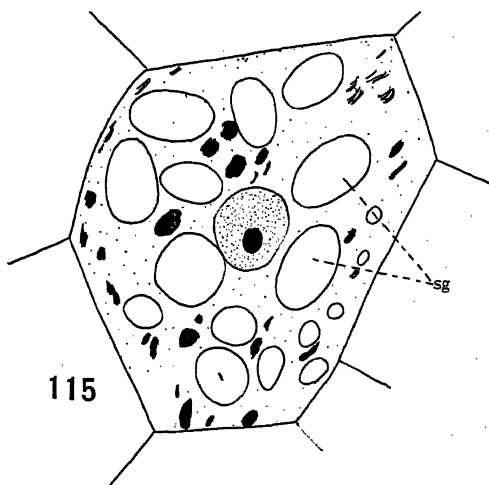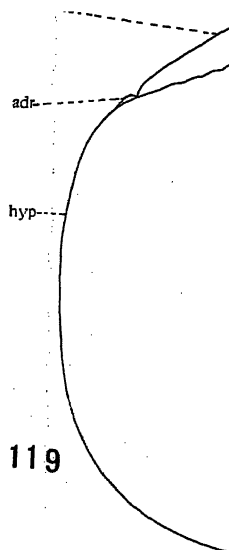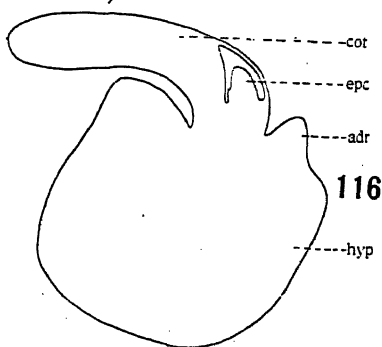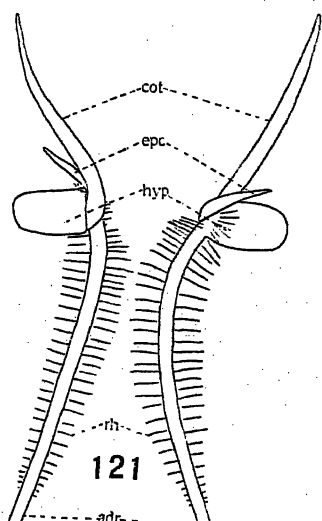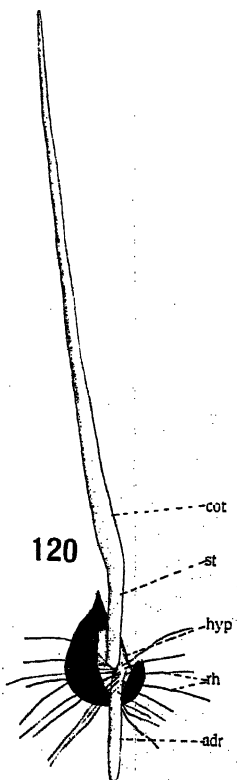
From 1882 to 1892 the writer published in the Transactions of the Connecticut Academy a series of papers containing descriptions and figures of New England Spiders known to him at that time. The present paper contains additional notes, descriptions, and figures of 48 of these species, partial descriptions, figures and references to descriptions of 38 species described by other persons since 1882, and descriptions and figures of 35 apparently new species.

The portion of New England explored is still chiefly eastern Massachusetts and New Hampshire as far north as the White Mountains. In Maine there have been collections on the coast at Portland and Monhegan, and in the north at Bangor and around Moosehead Lake. Explorations in Canada show the extension northward and westward of many Maine and White Mountain species, as *Theridium zelotypum* to Manitoba, and *Epeira patagiata* and *angulata* to the Pacific coast, and *Epeira carbonaria* and *Lycosa greenlandica* in the Rocky Mountains and Labrador. In the south there have been large collections in Connecticut at Simsbury near Hartford, and at New Haven on the coast, and in Rhode Island near Providence.

The distribution of several species has been made clear by collections on Long Island, N. Y., which is the northern limit of *Oxyopes salticus*, *Pellenes cœcatus*, and *Epeira verrucosa*, and where the following species are found in abundance, that extend northward only as far as Connecticut; *Lycosa scutulata*, *Acrosoma rugosa* and *Argyrodes cancellatus*. In the western part of Massachusetts, Connecticut, and Vermont but little has been done, but small collections in the Adirondacks and the observations of Mr. Banks around Ithaca, N. Y., show only slight differences from the spider fauna of New England.

The writer has depended chiefly on his own collections, but gives his thanks to G. W. Peckham, Nathan Banks and Miss E. B. Bryant for the use of their specimens and constant help of all kinds.

For references to publications of New England Spiders, the reader is referred to Miss Bryant's list of New England Spiders lately published by the Boston Society of Natural History.

Among the *Epeiridæ, Zilla atrica* is the only additional species found since the publication of New England Epeiridæ in 1884, but the males of the following species are described : *E. corticaria E. Nordmanni, E. juniperi, E. thaddeus.*

The separation of the *Therididæ* and *Linyphiadæ* as two distinct families seems to me an improvement, and I have adopted it in this paper, but not the union of the *Linyphiadæ* and *Epeiridæ* into one family, which obscures the marked differences between these two groups. Between certain genera of these families the family differences are hard to define, but the same is true of the differences between genera of the *Therididæ* and *Linyphiadæ.*

The *Linyphiadæ* are divided naturally into two subfamilies— *Linyphieæ* and *Erigoneæ*, the former containing the larger long-legged forms, and the latter the smaller forms with short legs and short spines. The genus *Microneta* in the *Linyphieæ* resembles in its form the *Erigoneæ*, and its species are hard to distinguish from those of *Tmeticus*. The most typical species are *viaria, cornupalpis* and *discolor. Microneta* (Bathyphantes) *bihamatus* belongs to this genus rather than *Bathyphantes*. Two new species are described and new specimens have been examined of all the old species except *crassimanus, furcata, longibulbus* and *olivacea.*

In the *Linyphieæ* the principal additions are *Linyphia maculata*, which has been found sparingly in many different localities and described by Banks as *L. conferta Hentz*, and *Tapinopa bilineata Banks*, which has been found singly in several localities.

In the *Erigoneæ* more than in any other group, new species are frequently found, and our descriptions are often made from one or a few individuals. They live for the most part near the ground, hidden in moss and leaves, only small quantities of which can be closely examined, and so little is known about their species and distribution. In their classification they offer many difficulties. Their small size makes their comparison inconvenient, and their uniformity in form and color makes it hard to define their differences. The only characters easy to see and describe are those of the adult males—the organs on the ends of the palpi and the modifications of the head. In consequence of these difficulties, the published classifications consist of a number of ill-defined genera, which have been formed from time to time, as new species were discovered, and the relations of which among themselves have never been satisfactorily explained. In the New England Therididæ I used a classification based upon the genera of Menge in the Spiders of Prussia, and in the present paper follow substantially the same, be-

cause it seems to me to show as well as any other the natural relations of the species which I have been studying.

At the beginning of the series come two species which I have described in N. E. Therididae under the name of *Pholcomma* at the end of the *Therididæ*. *P. hirsuta* belongs to a genus near *Pholcomma*, which Simon in Hist. Nat. Araignées has named *Ancyllorhanis*. It has small mandibles and the pointed maxillæ and the simple male palpi of the *Therididæ*. *P. rostrata* belongs to quite a different genus, which Simon has called *Histagonia*, and I have adopted without having seen *H. deserticola*, the type species Another species of the same genus is the *Exechophysis palustris Banks*. *Histagonia* seems to me most nearly related to *Diplocephalus* rather than to *Pholcomma*. The mandibles and maxillæ are like the *Erigoneæ* rather than the *Therididæ*, and the modifications of the head and complicated form of the tibia of the male palpi resemble those of *Diplocephalus*. The tarsal hook is present, though small, as it is in *Diplocephalus*.

The new genus *Caseola* with two species *herbicola* and *alticeps* resembles in form and habits *Ceratinella*, but does not have hard pieces on the back and at the base of the abdomen, nor any of the orange color of *Ceratinella*. The male palpi are simple in both species, with a peculiar club-shaped process of the palpal organ directed toward the inner side.

*Ceratinella* consists of small round spiders, orange-colored or orange brown, with a hard plate on the back of the abdomen in one sex or both. The palpi of the males vary in length, but are all on the same plan, with the palpal organ furnished with a long slender tube turned backward from the distal end of the tarsus toward the base. I consider this genus to include the European *C. brevis* and the American species which Simon separates as the genus *Ceraticelus*, the principal difference being in the sinuous claw of the mandible of *C. brevis*. *Ceratinopsis* consists also of small and brightly colored spiders with usually distinct black markings on the head and sometimes on the palpi and feet. The palpi resemble those of *Ceratinella*, with large and more variable tibiæ. There are no hard plates on the abdomen.

*Cornicularia* includes species resembling *Ceratinopsis*, but with usually more elongated cephalothorax, and in the males a horn on the front of the head between the upper and lower middle eyes. The male palpi have the tibia enlarged and extended over the back of the tarsus in a long flat process, partly divided into two branches. I include those species which have a double horn on the head,

which Simon refers to *Prosopotheca*, and also, as suggested by Simon, *Spiropalpus spiralis* which, though its male has no horns, resembles this genus.

*Grammonata* includes, besides the three species before described, *Erigonoplus gigas Banks*, which has lately been found in Massachusetts. All the species resemble *Amaurobius* in form and markings, having an indistinct pattern of light spots on the abdomen. In the males the head is a little elevated behind the eyes, and in *pictilis* and *gigas* there is a conspicuous hump. The males of *gigas* have the first metatarsus white and much thicker than the other joints. The male palpi resemble those of *Ceratinella*, having a long tube turned abruptly backward from the end of the tarsus. In *pictilis* the tube is very long and coiled in a double spiral.

*Diplocephalus Bertkau*, 1883, is *Lophomma* Em. of N. E. Therididae, in which the males have two humps on the head, each carrying one pair of the middle eyes. The male palpi have the tibia very large, covering the back of the tarsus nearly its whole length.

*Lophocarenum* consists of those spiders, the males of which, except *rugosum*, have holes in the head behind the eyes, and the middle of the head elevated, sometimes into large humps. The male palpi have the patella longer than the tibia, and the latter usually longer than wide, with small hooks and processes of various shapes. Where the enlargement of the head of the male is extreme, the female has a slight elevation of the head as in *montiferum* and *alpinum*. The unusually large size of the front lateral eyes in *quadricristatum* occurs in a less degree in the female.

There is no better example of the difficulty of classifying the *Erigoneæ* than the attempt of Simon to distribute the American species of this genus, without seeing the spiders themselves, among eight different genera. For *florens* he makes a new genus *Hypselistes*, while *decemoculatum*, the females of which cannot be distinguished from those of *florens*, is placed in *Neriene*, which corresponds in part to my *Tmeticus*. *L. pallidum* and *L. longitubus*, which resemble each other as closely as any other two species, are placed one in *Typhocræstes* and the other in *Pocodicnemis*. *L. sco-puliferum* is placed in *Minyriolus*, *L. quadricristatum* in *Pana-momops*, *L. longitarsus* in *Lophomma*, *L. rostratum* in *Trachelo-camptus* and *L. decemoculatum*, *montiferum* and *spiniferum* in *Neriene*. I see no reason to follow any of these changes; they only obscure the relations of the spiders.

*Tmeticus* is still a heterogeneous group. The more typical species,

such as *probatus* and *trilobatus*, approach *Erigone* by their wide maxillæ and long palpi and the tibia widened toward the tarsus, and the males have a strong single spine on the front of the mandibles. *Maximus, tibialis* and *brunneus* resemble each other in size and proportions, but differ in their mandibles and palpi. The other species have little in common except their general size and color, arrangement of eyes and form of mandibles and maxillæ.

*Erigone* now includes four species; *longipalpis, dentigera, autumnalis* and the new *brevidentatus* with wide maxillæ, large mandibles and long male palpi with widened tibia, and a spur directed downward on the patella.

In the *Therididæ* there are but few additions. *Theridium kentuckyense* has been found in a few places. The male of *T. zelotypum* is described and the species found to be common in Maine and New Hampshire. *Latrodectus mactans* has been found in several localities, but is nowhere common. The new *Enoplognatha rugosa* has been found rarely but in localities far apart. The same is true of the new *Pedanostethus pumilus*, and *P. spiniferus*.

In the *Agalenidæ*, *Hahnia brunnea* is described from a single specimen. but there is a second one in the collection of Mr. Banks. *Cryphœca montana* appears to be common in northern New Hampshire, and from description is very near the *C. peckhamii* Simon of the Pacific coast.

The larger *Clubionas* have been better defined and new figures are given of the epigynum of several species. The two new species are one from a single specimen *C. spiralis* and the other *C. prematura* a common species from the summit of Mt. Washington, the female of which has long been known as a variety of *C. ornata* Em.

The North American *Lycosidæ* and *Pisauridæ* have been described and their classification much improved by T. H. Montgomery in Proceedings of Philadelphia Acad., 1902-3 and 4. *Lycosa relucens* Montg. is a common species in New England. *Dolomedes idoneus* Montg. and *D. fontanus Em.* have both been described as *D. tenebrosus* Hentz, which agrees equally well with either. The new *D. vernalis* appears to be common in Maine and New Hampshire. *Pirata* remains a difficult group and each author has his own species. *P. minuta* is the most distinct, *montana* and *insularis* have been again identified and three new species are described. In N. E. Lycosidae 1885 I have described under the name of *L. nidifex* what I now consider as two species named by Marx in the Am. Naturalist, 1881. *L. nidifex* and *L. Pikei. Nidifex* is the inland species which ordinarily makes a ring or turret at the mouth of its burrow; *Pikei*

is the seashore and sand dune species described by Scudder as
*L. arenicola* in Psyche 1877.   *L. avara* Keys, *L. baltimoriana* Keys,
and *Pardosa littoralis* Banks been have found in New England and
new figures and descriptions of them are given.

Since the publication of N. E. Attidae in 1891 the number of
species of that family known in New England has been largely
increased, but nearly all the species have been described from other
parts of the country and appear to have very wide distribution.
Some of the most common species are so variable and their dif-
ferences so hard to define that they are still very imperfectly known,
especially in *Phidippus* and *Dendryphantes*.   The *Icius* which I
described as a dark variety of *elegans* now appears to be a distinct
species, *Icius similis*, Bks.

*Dendryphantes flavipes* Pkm. has been found in small numbers
through Maine and New Hampshire.   The male is fairly distinct
from that of *capitatus* but I cannot distinguish the females.   A new
species *D. Jeffersoni* is described from very few specimens found
on the Mt. Washington range at an elevation of 5,000 feet living
in the moss and lichens.

## List of New Species.

| | |
|---|---|
| *Enoplognatha rugosa.* | *Pirata arenicola.* |
| *Pedanostethus pumilus.* | „      *maculata.* |
| „          *spiniferus.* | „      *sylvestris.* |
| *Ceratinopsis auriculatus.* | *Dolomedes vernalis.* |
| „          *alternatus.* | *Amaurobius borealis.* |
| *Caseola herbicola.* | *Micaria laticeps.* |
| „      *alticeps.* | „      *quinquenotata.* |
| *Lophocarenum cuneatum.* | *Castaneira lineata.* |
| „              *abruptum.* | *Drassus hiemalis.* |
| „              *minutum.* | „      *bicornis.* |
| „              *rugosum.* | *Clubiona spiralis.* |
| *Erigone brevidentatus.* | „      *prematura.* |
| *Linyphia maculata.* | *Apostenus acutus.* |
| *Bathyphantes calcaratus.* | *Cryphœca montana.* |
| *Microneta denticulata.* | *Hahnia brunnea.* |
| „      *serrata.* | *Phidippus Whitmani.* |
| *Lycosa crassipalpis.* | *Dendryphantes Jeffersoni.* |
| *Pardosa diffusa.* | |

*List of described species found in
New England since 1882–1892.*

*Latrodectus mactans.*
*Theridium kentuckyense Keys.*
*Pedanostethus riparius Keys.*
*Ceratinella formosa Banks.*
*Grammonota gigas Banks.*
*Histagonia (Exechophysis) palus-
tris Banks.*
*Lophocarenum (Dismodicus) al-
pinum Banks.*
„ *(Dicyphus) trilo-
batus Banks.*
*Tmeticus flaveolus Banks.*
„ *debilis Banks.*
*Tapinopa bilineata Banks.*
*Zilla atrica Koch.*
*Pachygnatha tristriata Keys.*
*Lycosa avara Keys.*
*Lycosa baltimoreana Keys.*
„ *Pikei Marx.*
„ *relucens Montgomery.*
*Pardosa littoralis Banks.*
*Dolomedes idoneus Montgomery.*
*Dolomedes urinator (Hentz) Mont-
gomery.*

*Œcobius (Thalamia) parietalis
(Hentz).*
*Scotolathys (neophanes) pallidus
Marx.*
*Orchestina saltitans Banks.*
*Micaria gentilis Banks.*
*Prosthesima rufula Banks.*
*Gnaphosa parvula Banks.*
*Cicurina pallida Keys.*
*Phidippus insignarius Koch.*
*Dendryphantes flavipes Pkm.*
*Icius similis Banks.*
*Hyctia Pikei Pkm.*
*Mœvia tibialis Koch.*
*Pellenes (attus) viridipes Hentz.*
„ *(attus) roseus Hentz.*
„ *agilis Banks.*
„ *borealis Banks.*
*Homalattus cyaneus Pkm.*
*Peckhamia (Synemosyna) scorpi-
onea Hentz.*

**Theridium differens,** Em. Trans. Conn. Acad. 1892. (Plate I, figure 7.)

The epigynum of this species is wrongly described and figured in N. E. Therididae. The openings are really on the outer side, as they are in *Theridium spirale*, and differ only in being a little smaller and farther apart. See fig. 7.

**Theridium zelotypum,** Em. Trans. Conn. Acad. 1892. (Plate I, figure 5.)

This species has been found in the White Mountains and all over Maine, as far north as Moosehead Lake, but not south of Portland, Me., and westward as far as Winnipeg, Manitoba. At Monhegan, Me., July 1, 1901 adult males were abundant in webs with the females under spruce branches. The males are as large as the females, and have the abdomen similarly marked. The cephalothorax, legs and palpi are bright orange color, and the legs only slightly darker at the ends of the joints. The dark middle stripe of the cephalothorax is usually shorter than in the female, and does not extend forward to the eyes. The male palpus resembles that of *murarium* with all the appendages more elongated, Pl. 1, fig. 5. At pairing time the webs do not contain the characteristic tents covered with spruce leaves and scales; these are made later and in the last of July and first of August are found in nearly all the webs, hiding the females and eggs. The females remain in the nests with the young as late as September.

**Theridium kentuckyense,** Keys. Spinnen Amerikas, 1884. (Plate I, figures 6, 6a.)

The same size and general form as *differens* and *murarium*. The colors are less bright than in those species and more like *T. tepidariorum*. The legs are pale, with light yellowbrown, wide rings at the ends of the joints, and less distinctly in the middle. The cephalothorax is brown, darker at the sides, and lightest between the eyes and the dorsal groove, without any stripes. The sternum is brown, without markings, and the coxæ and base of the femora pale.

The markings of the abdomen are distinct at the anterior end and also over the spinnerets, while in the middle they consist of small and indistinct spots in irregular transverse rows extending down the sides. At the anterior end is a bright white spot with a larger black spot on each side sharply defined toward the middle line, and irregular and indistinct at the sides. At the hinder end over the spinnerets is a white spot with short black stripes at the sides. On each side of the abdomen is a short, dark, vertical stripe that in some individuals is deep black.

In the male the markings of the abdomen unite into a more distinct middle light stripe, bordered by two dark ones, but the white spots at the ends are distinct as in the female.

The epigynum is small and rounded behind with two openings twice their diameter apart. The skin is so transparent that the tubes of the epigynum are seen through it and obscure the openings.

At the end of the palpal organ is a long thin appendage, widened and twisted at the end, partly enclosing and supporting the tube. Pl. 1, fig. 6 a.

Found at New Haven, Conn., Jaffrey, N. H., and by Dr. Fox at Hollis, N. H.

**Latrodectus mactans,** Fabr.
**Theridion verecundum,** Hentz.

This is the largest and most conspicuous species of the family. The abdomen is round, sometimes a centimeter in diameter, and the whole body is deep black except a bright red spot under the abdomen and one or a row of red spots on the upper side. In alcohol the spots fade to white or yellow. In young individuals there is a white line around the front of the abdomen and three rows of spots partly white and partly red along the back, and the legs are brown in the middle of the joints and black at the ends. The adult males are marked much like the young with the lateral spots elongated and with a red line in the middle of each. The males are much smaller than the females, some of them only three or four millimeters long but with long legs. The lateral eyes, which in most Therididae are close together, are in *Latrodectus* as far apart as they are from the middle eyes. The epigynum is of the usual Theridion pattern with a single, wide, oval opening partly divided on the front edge. The palpal organ has a very large and long tube coiled in two flat turns across the end of the bulb. In alcohol this tube often becomes displaced and coils around the bulb in any direction.

The nest is usually near the ground under a stone or in a hole in the sod. The web extends among surrounding objects sometimes for a foot from the nest in all directions. It consists mainly of large irregular meshes, but includes usually a distinct flat or curved sheet of smaller meshes like the webs of *Steatoda* or *Pholcus*. The cocoons are half on inch in diameter brownish white in color.

This species is found all over the country as far west as the Rocky Mountains and north to southern New Hampshire. In the South it is common, but in New England occurs only occasionally in scattered localities.

**Enoplognatha rugosa,** new. (Plate I, figures 8 to 8c.)

Two males sifted from leaves in a swamp in the Blue Hills, May 6, 1905, are about half as large as *marmorata*, measuring 3.5 mm. in length. The cephalothorax is flat and the head wide and low as it is in *marmorata*, but the legs are longer and more slender. The sternum is widest in front, and less indented around the coxæ than in *marmorata*. The mandibles are more slender than in *marmorata*, and the claw is slender and nearly as long as the basal joint. The process on the under side is near the middle, and has a single pointed tip and below it two small teeth, Pl. I, fig. 8 c. The legs and mandibles are slightly roughened by little elevations at the bases of the hairs, which are shorter and fewer than in *marmorata*. There are similar elevations on the middle of the cephalothorax and around the edges of maxillæ and sternum. The palpi resemble those of *marmorata*. The color in the specimens examined is pale and less yellow than in *marmorata*, in alcohol inclining to red as in *Steatoda triangulosa*. The abdomen has an indistinct pattern consisting of a broken middle line and two rows of spots.

The female found May 30, 1906, under leaves at Three-mile Island, Lake Winnipesaukee, N. H., resembles the males in color and markings and in size. The abdomen is larger, and the legs and mandibles short as in females of related species. The head, sternum and mouth parts are less roughened, but have longer hairs than in the male. The epigynum has a transverse narrow opening behind, covered by a short brown plate.

**Pedanostethus riparius,** Keysl. Spinnen Amerikas, Therididae, 1886. (Plate I, figures 1 to 1 d.)

This species described by Keyserling from Lake Superior, is one of the most common spiders under leaves all over New England.

The length is about 4 mm., the sexes differing little in size. The cephalothorax and abdomen are about equal in length. The cephalothorax is wide in front; about two-thirds as wide as it is at the widest part, and the rows of eyes are almost straight, the upper row only slightly longer than the lower. The palpi of the female are as long as the cephalothorax, and those of the male longer. The maxillæ have the ends straight and nearly parallel, as in *Enoplognatha marmorata,* not oblique as in *Steatoda borealis.* The sternum is as wide as long, widest between the first and second legs and slightly pointed behind. The colors are dull brown and gray, without any markings. The cephalothorax is smooth and shining and darkened a little toward the head. The legs are brown like the cephalothorax, darkened toward the tips and covered with fine hairs. The abdomen is gray, generally lighter than the cephalothorax, and covered with dark gray hairs. The epigynum has a characteristic pear-shaped piece in front, Pl. 1, fig. 1c. but in some individuals this piece is oblong, Pl. 1, fig. 1d.

The male palpi are stout and three-fourths as long as the rest of the spider. The tibia and patella are both short and together equal in length to the tarsus. The tibia is a little narrowed at the base and widened at the end around the base of the tarsus on the outer side. The tarsus is narrow, only partly covering the palpal organ. Near the tip it has a notch on the upper side, and two curved stiff hairs, Pl. I, fig. 1a.

**Pedanostethus pumilus,** new. (Plate I, figures 2, 2a.)
In the maple swamp at Clarendon Hills, south of Boston, three males have been found of this small species. It is 2.5 mm. long, colored like very light individuals of *riparius,* and resembling it in every respect except in the palpi. These are proportionally shorter than in *riparius,* being not much longer than the cephalothorax. The tibia is more contracted at the base than in *riparius,* and the tarsus is shorter, rounder and thicker. The notch near the tip is wider and there are no special hairs. The female is the same size and color as the male. The epigynum is short like that of *riparius* but has the front piece wider than long instead of pear shaped fig. 2a.

One male also found near the Carter notch, White Mountains, Aug., 1906, and another at Three-mile Island, Lake Winnipesaukee.

**Pedanostethus spiniferus,** new. (Plate I, figures 3, 3a.)
The male is 2.5 mm. long, and pale like *pumilus,* and resembles it except that the lateral eyes of the upper row are a little farther back. The palpi have the tibia shaped much as in *riparius,* not

as narrow at the base as in *P. pumilus.* The tarsus is oval and less pointed than in *riparius,* with the notch smaller and not as near the tip. At the base of the palpal organ is a long hook turning out at right angle to the tarsus, Pl. I, fig. 13. The female is of the same size as the male. The epigynum is elongated, a third as long as the abdomen. At the front end close behind the fourth coxæ is a small, dark colored, sharp point directed forward a little behind which the round spermathecæ show through the skin, and behind these two parallel dark lines extend backward and meet at the base of a short, pale, blunt appendage directed backward.

This species is found under leaves in company with *riparius* and *pumilus.* Clarendon Hills and Waltham, Mass.

**Argyrodes cancellatus.** (Plate I, figures 10 to 10c.)
**Theridion cancellatum,** Hentz.
**Lasæola cancellata,** Emerton, N. E. Therididae. Trans. Conn. Acad.
  1882.
**Argyrodes larvatus,** Keyserling. Spinnen Amerikas.

This species, found in Connecticut, is abundant on Long Island and farther south. It is sometimes found in webs of its own and often in webs of larger spiders, especially in those of *Epeira strix.* The colors are light gray and brown, with silvery spots on the abdomen, and when it is motionless with the feet drawn up, it is hard to distinguish from a piece of leaf or bark dropped by accident into the web.

The female is 2.5 mm. long, with the cephalothorax 1 mm. long. The head is higher, and more vertical in front than in *trigonum* and the front middle eyes project slightly on the front of the head. The lower part of the head is rounded and extends forward a little beyond the mandibles. The abdomen is as high as wide, rounded above and narrowed toward the spinnerets, which are in the middle of the under side. At the end of the abdomen is a double tubercle with the lower half largest, and on each side of the abdomen a little farther forward another tubercle. The epigynum has a wide oval opening, partly covered and divided by a projection of the front edge.

The male is 3 mm. long and the cephalothorax 1.5 mm. The lower part of the front of the head extends forward and downward in front of the mandibles in a nose-shaped process, above which there is a round pit on each side of the head. The abdomen is smaller and narrower than that of the female, and covered above with silvery spots mixed with gray and black. The male palpi are

shorter and have the tarsus larger and rounder than in *trigonum*, Pl. I. fig. 10 e. The descriptions and figures arc from specimens taken at Cold Spring Harbor on the north side of Long Island, N. Y.

**Ceratinella formosa,** Banks. Ithaca, 1892. (Plate II, figures 5 to 5d.)

This species was found by Miss E. B. Bryant at Long Island in Portland Harbor, Me., Sept. 11, 1904. It was in great numbers on the stones on the beach and flying by threads in the air. In size and color it resembles *C. lœta.* The males have the whole upper surface of the abdomen hard, while the females have only a hard spot across the anterior end. The cephalothorax and abdomen are both longer and less rounded than in *lœta*, and the sternum is narrower behind, measuring between the fourth legs one-third its length, while the sternum of *lœta* measures half its length. The epigynum has a triangular opening somewhat like that of *lœta.* The male palpus resembles that of *C. brunnea*; the process of the tibia is long and hooked, and the tube of the palpal organ is simple, with no tooth at the bend. This species lives among the small stones above high water on the beach, and runs much faster than the other Ceratinellas. Found at Gloucester, Mass. on beaches and one specimen in the Carter notch, White Mountains, N. H.

**Ceratinopsis auriculatus,** new. (Plate II, figures 9, 9a, 9b.)

1.5 mm. long and much like *C. laticeps.* The colors are yellow and orange like the other species, with a little black on the head and ends of the palpi. The upper middle eyes are more than their diameter apart, and the lateral eyes are farther from them than they are from each other. Each pair of lateral eyes is raised on a little horn turned forward and projecting in a point beyond the eyes. The tibia of the male palpus projects upward and hooks forward. Seen from above it has three indistinct teeth in place of the two long ones of *C. laticeps.*

One male from Three-mile Island, Lake Winnipesaukee, N. H., May 29, 1906, Miss E. B. Bryant; one from Fitzwilliam, July 1907.

**Ceratinopsis alternatus,** new. (Plate II, figures 6, 6a.)

In general appearance this resembles the female *C. interpres.* The length is 2.5 mm. and the color is light orange brown with black between the eyes, but no other markings. The arrangement of the eyes is the same as in *interpres*, but the head is not quite as high and the back not as straight. The sternum is convex and large, and as wide as long, extending between the fourth coxæ as wide as the coxæ are long.

The epigynum is simple, with a middle lobe a little longer than wide.

The male palpi have the patella long and widened at the end, with a large tooth on the under sider. The tibia is very short and has a tooth on the upper side as long as that on the patella but more slender. The palpal organ has a slender pointed process at the end, and a short and flat basal hook, Pl. II, fig. 6.

Three-mile Island and Jaffrey, N. H., Mt. Tom, Mass., Simsbury, Conn., Balsam, North Carolina.

**Caseola, new genus.**

General appearance like *Ceratinella*, but without any hard plate on the abdomen. The cephalothorax is as wide as long, in the male *alticeps*, elevated in front. The two rows of eyes are of equal length the upper middle pair as far from the front middle pair as they are from each other. The abdomen is oval, not much larger than the cephalothorax and covered with scattered hairs. The legs are short and differ little in length. The mandibles have three very small teeth on the inner side of the claw groove and two or three larger ones on the front. The maxillæ are longer than wide and a little pointed at the inner corners. The sternum is as wide as long, widest between the first and second legs and extends backward between the fourth legs, where it is as wide as the coxæ. The male palpi have the patella and tibia both short and the palpal organ simple with a slender curved tube, at the base of which is a flat process widened and oval at the end directed outward.

**Caseola herbicola, new.** (Plate II, figures 1 to 1 d.)

1.5 mm. long and resembling *Ceratinella*, but without any hard spots on the abdomen, which is covered with scattered stiff hairs. The color is pale and whitish, without the orange which is usual in *Ceratinella*. The cephalothorax is darkened a little toward the head, and in the male is browner than in the female. The cephalothorax is nearly as wide as long, very little narrowed or raised toward the head in either sex. There is nothing peculiar in the arrangement of the eyes. The front middle pair are as usual smallest and about two-thirds the diameter of the upper middle eyes, with which they make a quadrangle slightly higher than wide. The front row of eyes is almost as long as the upper row, with the lateral eyes a little raised above the head. The mandibles have four or five very small teeth each side of the claw. The sternum

is as wide as long, and extends backward between the fourth coxa. as wide as the coxæ themselves. The epigynum has a wide transparent lobe in the middle, at each side of which the brown spermathecæ show through the skin, and from which two dark bands curve in half circles toward the middle.

The male palpi are simple with a slender tube curving forward toward a small terminal process with two teeth. Near the base of the tube there is a small dark tooth, and under it, directed toward the inner side, is a pale club-shaped process. The tarsal hook is very small and hard to see. The tibia is widened at the end with no processes or branches, except a slight raised and straight edge on the upper side.

The females have been found in small numbers at several places near Boston under leaves in early spring. Adult males and several females were swept from low plants on Mt. Holyoke, Mass., on June 20th.

**Caseola alticeps,** new. (Plate II, figures 2 to 2 c.)

1.5 mm. long with the general appearance of *Lophocarenum* rather than *Ceratinella*. The males only are known, and they have the head narrow and elevated, somewhat as in *Ceratinopsis interpres*. The eyes are all on the elevation and so are closer together than in *herbicola*. The front middle eyes are only a little smaller than the upper middle pair. The cephalothorax is nearly as wide as long. The abdomen is oval and covered with scattered hairs, which are finer and more numerous than in *herbicola*.

The male palpi have the tibia widened up and down with a tooth on the outer side. The palpal organ is simple, having on the inner side a club-shaped appendage like *herbicola*. The tube ends between two processes at the tip of the organ, one flat and transparent, and the other short and fine, with a peculiar curve at the end. The tarsal hook is very small and easily concealed.

One from Three-mile Island in May, dark colored, and one from Waltham, Mass. in November, which is pale.

**Grammonota gigas.** (Plate II, figures 8 to 8 b.)
**Erigonoplus gigas,** Banks. Canadian Entomologist, 1896.

Two males of this species were found under a board at Ipswich Bluff, Plum Island, Mass. by Miss Mary T. Palmer, June, 1906. They are 2.5 mm. long and resemble in size and color *G. pictilis*. There are markings on the back of the abdomen as in *pictilis*, but the front half is stained with yellow over the other markings. The

front legs have the metatarsus white and twice as thick as the other joints. The end of the tibia is also slightly thickened. The two middle pairs of eyes are nearly as far apart as the lateral pairs, and the head is slightly elevated between them, and covered with hairs directed backward and upward as in the other species. Behind the eyes is a large hump rising abruptly in front and divided into five lobes. The male palpi resemble closely those of *G. inornata*. The tibia has, on the upper side, a large, simple hook turned forward, and the tube of the palpal organ is short and stiff and turned backward at the tip. This was first found by Banks at Ithaca, N. Y. and described by him in 1896.

Another male was found at Fitzwilliam, N. H., July, 1807, in the rhododendron woods.

**Histagonia palustris,**
**Exechophysis palustris,** Banks. Ent. Soc., Wash.. 1905. (Plate II, figures 4 to 4 f.)

This is another species resembling the *Pholcomma rostrata* described in 1882. It is a little over 1 mm. long, short and rounded like *rostrata*, with the abdomen of the male hard on the back and covered with scattered stiff hairs. The head is elevated and extends forward below the eyes in a blunt protuberance, covered on the end with stiff hairs directed upward and backward.

The male palpi have the tube of the palpal organ coiled once around the end of the bulb. The tibia is flattened and, seen from the side, as wide as long, with a recurved black spine on the distal corner, and a smaller black spine near the basal end, the edge between the two spines irregular and cut into several notches. Seen from above with the palpi in their usual position, the tibia is wedge-shaped with the point directed forward. On the outer side of the tibia near the upper edge are two long hairs, which appear to correspond to the two hairs on the tibia of rostrata.

Three-mile Island. Lake Winnipesaukee, N. H., May 25, 1905. Sifted from leaves. Ithaca, N. Y., N. Banks.

**Lophocarenum cuneatum,** new. (Plate III, figures 6 to 6c.)

2 mm. long, the cephalothorax dark brown, the abdomen as dark but grayer in color, and the legs distinctly lighter, pale when freshly molted, and light yellow when mature. The cephalothorax is nearly as wide as long, extended in front under the eyes in a blunt point. The head is elevated into a distinct hump, with long oval grooves at the sides, in the front ends of which, close to the

eyes are the lateral pits. The upper middle eyes are on the front and nearly at the top of the hump, the lower middle eyes half way between them and the mandibles. The lateral eyes are wide apart, just outside the lateral grooves, each pair on a slight elevation. The hairs between the eyes are long and pointed outward. The male palpi have the patella nearly twice as long as wide. The tibia is very much widened toward the tarsus and partly covers it on the upper side, where it has a large sickle-shaped hook turned outward. The tarsal hook is flat and broad, with a small notch. The tarsus is short and rounded and the parts of the palpal organ small, with a short tube curved around the end.

A single freshly molted female has the head slightly elevated behind the eyes, and the middle eyes as far from the front pair as they are from each other. The epigynum is very far forward, and has two pointed lobes directed backward and close together with only a narrow groove between them.

Fitzwilliam, N. H. under leaves near the rhododendrons, May 25, 1907.

**Lophocarenum abruptum,** new. (Plate III, figures 5 to 5c.)

A male a little over 2 mm. long from under leaves on Mt. Holyoke, Mass., June 20. The cephalothorax is depressed in the middle, and the head rises abruptly, carrying the upper middle eyes on the front and upper side. Just above the lateral eyes are wide grooves, with a small round pit a little farther back than the lateral eyes. Between the upper and lower middle eyes are a few fine hairs turned toward the sides. Below the front middle eyes the head projects forward over the mandibles. The tarsus of the male palpus is about half as long as the patella and widened toward the tarsus, and has on the upper side two processes directed forward, the inner one twice as long as the outer and as long as the body of the tibia. The tarsus is rounded and the palpal organ large and thick from above downward. The tube is small and coiled in one turn on the outer side. The colors in this individual are pale, the abdomen darkest.

**Lophocarenum quadricristatum.** (Plate III, figures 4, 4a.)

This has been found again on the summit of Mt. Washington in August, 1906. The female has in a less degree the same peculiar arrangement of the eyes as the male. The middle pairs are unusually far apart, and the lateral pairs have the front eye one and

a half times as large as the other. The head is a little elevated, highest just behind the upper middle eyes. The epigynum is very simple, showing a straight edge behind, with a wide middle lobe separated only by slight grooves. The sternum in both sexes extends backward between the fourth coxæ, where it is wider than the diameter of the coxæ.

**Lophocarenum alpinum,** Banks.
**Dismodicus alpinus,** Banks. Can. Ent., 1896. (Plate III, figures 3 to 3f.)

An adult male and female were found in a thin web under a stone near the summit of Mt. Washington, N. H., and another female and a male not yet molted for the last time under other stones in the same neighborhood. The male is 2 mm. long. The cephalo-thorax is half longer than wide, narrow in front and extended a little beyond the mandibles. The hump is rounded above and rises between the eyes and the middle of the cephalothorax; it is nearly as wide as the front of the head, and inclines forward a little over the eyes. The front of the hump is covered with short hairs, longest below and turned outward toward each side. On each side of the hump at the level of the eyes is a groove with a round pit at the front end. The eyes are spread over the whole width of the head, the lateral pairs largest, the front middle pair very small and near together. The eyes of the upper row are equal distances apart. The palpi are longer than the cephalothorax. The tibia is shorter than the patella, and extended only a little over the upper side of the tarsus, where it is divided into two teeth, the inner one longest but slender and hooked inward at the end. The palpal organ resembles that of the last species and of *L. montiferum.*

A young male almost ready for the final moult, shows a small hump behind the eyes and a slight extension of the front of the head. The palpi are much enlarged, and show the form of the male tibia and palpal organ indistinctly through the skin.

In the female which is about the same size as the male, the front of the head is not extended forward, but there is a slight hump one-fourth as high as that of the male, in the same place between the eyes and the middle of the cephalothorax. The epigynum has a wide middle lobe curved on the edge and shows through the skin the spermathecæ and two irregularly coiled tubes at the sides of the middle lobe.

**Lophocarenum trilobatum.** (Plate III. figures 1, 1a.)
**Dicyphus trilobatus,** Banks. Canadian Entomologist, 1896.

One male only from the maple swamp at Clarendon Hills, about the same size as *L. montiferum*, with a hump as high as that species, but differently shaped. The cephalothorax is not quite as wide as long, and a little narrower in front. The eyes are grouped together as in most species, the hinder middle pair a little farther apart than they are from the lateral and the lateral pairs almost horizontal. The hump is half as wide as the cephalothorax and nearly of the same height. It is rounded behind and in front divided into three lobes, the middle one extending forward nearly to the eyes. The palpi are longer than the cephalothorax, the tibia a little shorter than the patella, but elongated over the tarsus on the upper side, so that it appears longer. This process of the tibia is divided into two teeth. the outer one longer and larger than the inner. The palpal organ has some resemblance to that of *montiferum*, with a small tarsal hook and the tube curled once around the end.

**Lophocarenum minutum,** new. (Plate III, figures 8, 8a, 8b.)

1 mm. long and light yellow brown. The cephalothorax is one-fourth longer than wide and rounded in front. The head is only slightly elevated, and the lateral grooves are behind the eyes, with the pits showing from above through the skin one-third the length of the cephalothorax from the front. The sternum is as broad as long, extending backward between the fourth legs, where it is as wide as one of the coxæ. The palpi have the patella and tibia both short, about as wide as long. The tibia is a little widened toward the tarsus and has on the upper and outer side a short, fine and slightly curved tooth. The tarsal hook is long and slender, and in my specimens turns outward so that it shows from above. The palpal organ is small and simple, and there is a short and blunt black process that extends beyond the end of the tarsus. The tarsus is slightly angular on the outer side. The female has the head slightly lower, with the upper and lower middle eyes closer together. The epigynum resembles that of several other species of the genus having a distinct middle lobe, widened at the end in front of which are two openings.

Fitzwilliam, N. H. under leaves near the rhododendrons, May 25, 1907.

**Lophocarenum rugosum,** new. (Plate II, figures 3 to 3g.)

2 mm. long. The cephalothorax is oval, widest across the middle and highest behind the eyes. The surface is slightly roughened all

over. The sternum is also rough. It extends backward between the fourth legs, where it is wider than the coxæ. It extends also between the first and second, and between the second and third legs. The maxillæ are wider than long, and the mandibles stout, with four teeth on the front of the claw, and three small and one large one on the inner side. The eyes spread across the whole front of the head. The front row is straight, with the middle pair smallest, and the middle quadrangle is higher than wide. The abdomen is round and a little pointed at the spinnerets as in Erigone. The abdomen is covered with short and fine scattered hairs. The coxæ are long, extending beyond the border of cephalothorax, so that all are visible from above, and the legs are long and stout and covered with coarse hairs.

The epigynum is very far forward and has a light colored middle lobe, longer than wide, at the sides of which the spermathecæ show through the skin.

The palpal organs are very simple; the tube and two short appendages showing only at the distal end. The tarsal hook is small and the tarsus short and round. The tibia is widened a little across the middle, and has a flat extension with a straight edge against the upper side of the tarsus.

The relations of this species are doubtful as the male does not have the grooves and pits in the sides of the head which are characteristic of the males of most species of this genus. In other respects, however, its resemblance is close to *L. latum* and *L. crenatum* and still more to an undescribed species from Long Island, N. Y., for which the females are easily mistaken. The sternum in all of these is wide and convex and roughened all over the surface. The extension between the legs occurs in the same way in *L. crenatum*. The form of the epigynum is the same in all four species. The resemblance of the male palpi is equally close, all the species having the tarsus nearly of the same shape and the parts of the palpal organ small and with only slight variations among the species.

Grafton, Mass. Three-mile Island, Lake Winnipesaukee, N. H. under leaves.

**Tmeticus longisetosus,** Emerton. Trans. Conn. Acad. 1892. (Plate IV, figure 9.)

This species has been found again in March, 1907, under leaves in Allston, near Boston. The male and female are of the same size and much alike. They are pale in color, the legs and cephalo-

thorax light yellow tinged, when fresh, with light red on the head and mandibles. There is a row of four or five hairs directed forward on the middle line of the head. The abdomen is covered above and below with scattered hairs about their length apart. The mandibles of the male have a strong tooth in front. The sternum extends backward between the fourth coxæ and is truncated at the hinder end, where it is about half as wide as the fourth coxa. The epigynum appears very simple externally, showing two small spermathecæ through the skin, over which the short scattered hairs are arranged in two clusters. The male palpi have been sufficiently figured in N. E. Therididæ.

**Tmeticus flaveolus,** Banks. Proc. Acad. Nat. Sci. Phila., 1892. (Plate IV, figures 8, 8 a, 8 b.)

This species resembles *T. longisetosus* in size and color. It is 1.5 mm. long. When fresh the cephalothorax is light orange color and the rest of the body pale. The sternum extends backward beyond the fourth coxæ, where it is as wide as one of the coxae. The mandibles of the male have a small spine on the front near the end. The epigynum is much like that of *longisetosus*, with a transverse slit with the spermathecæ showing through the skin. The male palpus resembles that of *longisetosus*, but the tibia and tarsus are both slightly shorter. The tibia is widened at the end as it is in *longitarsus*, with several projections and shallow curves on the edge. The tarsus has two large spines near the base, one much thicker than the other and both about half as long as the corresponding spines in *longisetosus*.

Coffin's beach, Gloucester, Mass., in straw on the shore. Hanover, N. H. and Ithaca, N. Y. in Mr. Banks' collection.

**Tmeticus debilis,** Banks. Proc. Acad. Nat. Sci. Phila., 1902. (Plate IV, figures 3, 3 a, 3 b.)

2 mm. long, and pale yellow, brown and gray, with some individuals almost white. The group of eyes is rather narrow, not more than half the width of the thorax. The sternum is widest at the second legs, and extends between the fourth coxæ in a narrow piece not more than half the diameter of one of the coxæ. The male palpi are large and the palpal organs of a very distinct form. The tibia is small, but widened at the end, where it is about twice as wide as at the base. The tarsus is long and oval, with a smooth strip without hairs near the outer edge. The tarsal hook is slender and has a short rounded process near the base on the outer side.

All the appendages of the palpal organ are long and slender. The tube itself starts at the base under the tarsal hook and extends more than half around the tarsus, and is supported through nearly its whole length by a stouter process with a long hook at the end. usually dark-colored and having a short tooth near its base on the inner side of its curve. At the base of this stout process is another about half as long ; which is soft and white and ends in a blunt point near the tip of the tube.

Hammond's Pond woods, Brookline, Carlisle Pines.

**Tmeticus corticarius,** new.  (Plate VI, figures 4, 4a, 4b.)

This species had only been found singly in Cambridge and in New Haven, Conn., until trees around Boston and Providence were banded with cloths to trap the Gypsy moth caterpillars in 1905. It then appeared in considerable numbers under these cloths in both places from July until October.

The length is 2.5 mm., the males and females being of the same size, the males having only slightly longer legs and smaller abdomen. The color is dull gray, the legs and cephalothorax yellowish, and the abdomen almost black.  The front of the head is narrow and rounded, and the eyes not far apart.  The epigynum is three-lobed, the outer lobes forming part of a semicircular plate a third as wide as the abdomen.  The male palpus has the tibia very short and extended upward and downward.  The upper process is very conspicuous when the palpus is seen from the side.  It is half as long as the tarsus, and curves slightly forward so as to fit the tarsus if both are brought together.

**Tmeticus brunneus,** Em. Trans. Conn. Acad. 1882.  (Plate IV, figures 7, 7a, 7b.)

This has been found again on Mt. Washington by Mrs. Slosson, and is in Mr. Banks's collection.  It is closely related to *T. tarsalis* and *T. maximus,* especially the latter ; the upper projection of the tibia, however, is distinctly more pointed and larger than in *maximus,* and the tarsal hook is longer.  The epigynum is also longer and projects more from the surface of the abdomen than in *maximus.*

**Erigone brevidentatus,** new.  (Plate II, figures 10b, 10c.)

A small species not much over 1 mm. in length.  The colors are the usual brown and gray, rather pale in all three specimens.  The cephalothorax is only a little elevated behind the eyes, and there are no spines around the edge.  The mandibles have one long

spine on the inner side and seven on the outer side, the longest one being opposite the one on the inner side. The male palpi have the patella and tibia of nearly the same length. The usual tooth on the under side of the patella is very small and short.

Mt. Holyoke, sifted from leaves June 20, 1906; Fitzwilliam, N. H., July 20, 1907.

**Linyphia maculata,** new.
**Linyphia conferta** (Hentz) Banks, 1892. (Plate IV, figures 10 to, 10 g.)

This species is related to *clathrata* and *mandibulata*. The abdomen is high behind as in those species, and sometimes extends backward beyond the spinnerets. The cephalothorax of the female is somewhat shorter and the legs longer than in *clathrata* and *mandibulata*. The hinder middle eyes are farther apart than they are from the lateral eyes. The front middle eyes are small and less than their diameter apart.

The cephalothorax and legs are light orange yellow, the cephalothorax a little darker, and the eyes are surrounded by black. The abdomen is pale in front and marked with several dark spots, the front ones in pairs, which toward the hinder end are sometimes almost black. Around the sides of the abdomen are gray spots and a row of irregular opaque white spots. In the male all the colors are darker, and the abdomen sometimes almost black. The sternum and under side of the abdomen are brown without any markings.

The epigynum is widened toward the hinder end, Pl. IV, fig. 109. The palpi of the males have the tarsi and palpal organs black, the palpal organs large and complicated and resembling those of *L. marginata*.

At the time of publication of the N. E. Theridida I had seen only the young of this species at New Haven, Conn. In 1883 an adult male was found at the same place, and one near Boston in 1890. More lately they have been found to be common near Boston, at Ipswich, in the Blue Hills, and at Sharon, living in webs near the ground like *L. mandibulata* but preferring more shady situations under the trees and bushes along paths through the woods rather than open meadows. The webs are large and nearly flat, but the part on which the spider usually stands is sometimes a little raised by tighter threads from above.

**Tapinopa bilineata,** Banks. Journal New York Entomological Soc., 1893, p. 128. (Plate XII, figures 8 to 8 f.)

This species has been found twice, at Woods Hole in 1883, and at Clarendon Hills, south of Boston, in 1904, under leaves in winter in a maple swamp, both specimens females. The male was found in 1906 at Portland, Me.

The length is 5 mm. and the length of the cephalothorax 2.5 mm. The cephalothorax is one half longer than wide, and the projecting middle eyes and the black bands narrowing toward the front make it appear longer and more pointed at the head than in the nearly related species. The middle eyes of the front row are as large as those of the upper row, which is unusual in this family, and the four middle eyes form a quadrangle longer than wide and nearly as wide in front as behind. The front middle eyes project forward over the mandibles. The mandibles are wide in front, with long claws and have seven teeth in front, the middle one-half the diameter of the mandible in length. On the under side of the mandibles are five or six shorter teeth, Pl. XII, fig. 8 d.

The abdomen is shaped as in *Linyphia phrygiana* and *Bathyphantes nebulosa*, high in front and low and pointed behind.

The colors are translucent, white and black or dark gray, all becoming yellow in alcohol. The cephalothorax has two wide black bands at the side that cover more than half its surface, leaving a middle light band narrowing behind and toward the front. The dark bands do not quite extend to the sides of the head or much below the eyes in front. The back of the abdomen is marked with a series of pairs of dark spots, in one specimen united on the posterior half, so that half of the back is entirely black. The legs have wide dark bands around the ends and middle of the longer joints. The sternum is gray, darkest at the sides and the coxæ are gray at the outer ends.

The epigynum is curved downward in a half circle and widened at the end, Pl. XII, fig. 8f. At the base it is as wide as long, with an opening at each side and a thin partition in the middle, Pl. XII, fig. 8 c.

The markings are more distinct, and darker than in the European *longidens,* of which there are specimens from Germany sent by A. Menge of Danzig in the Museum of Zoology at Cambridge.

The male resembles the female, except that the legs are longer, and the top of the head above the eyes more hairy. The male palpus resembles that of *T. longidens*: the tarsus has a long tooth near the base on the upper and inner side which is curved backward, but is not divided at the end into two teeth as it is in *longidens.*

**Bathyphantes calcaratus,** new. (Plate IV, figures 13, 13a.)

This species has been found at Portland, Maine, Moosehead Lake, and the lower part of Mt. Washington. The largest measures 3 mm. long. All the specimens are distinctly marked with gray, the darker one resembling *Drapetisca socialis.* The legs are long and slender, the femur darker toward the tip, and the tibia and metatarsus dark at the end and in the middle. The cephalothorax has a dark spot in the middle, wide in front and tapering to a line behind. The abdomen is white and gray, the markings of the front half united into a middle stripe with broken edges and two narrow lateral stripes. On the hinder half the markings are in pairs, slightly connected in the middle. The male palpi are as long as the cephalothorax. The patella and tibia are both short, but the tarsus is elongated with a short and sharp spur at the base. The tarsal hook is very large, recurved and widened at the end, where it has a short point above, and a longer one below, as shown in the figure. The tarsal hook resembles that of *Microneta crassimanus,* a larger and shorter legged spider.

**Microneta persoluta.** (Plate IV, figures 11, 11a.)

The old figures in N. E. Therididæ do not give a correct idea of the form of the tarsal hook, though they do show its characteristic sinuous lower edge. The tarsal hook is turned outward and thickened at the end, where it has several blunt irregular teeth as shown in the figure. It has been found at several new localities and seems to be a common species.

**Microneta denticulata,** new. (Plate IV, figure 14.)

This species resembles closely *M. persoluta* in size and color, and is found in company with it, but is easily distinguished by the palpi, Pl. IV, fig. 14. The tarsal hook is nearly horseshoe shaped and has a thick edge on which are six or moreprominent teeth, those near the base partly united. The parts of the palpal organ are longer and more separate than in *persoluta.* The mandibles are without a prominent tooth on the front.

**Microneta latidens,** Emerton. Trans. Conn. Acad., 1892. (Plate IV, figures 12 to 12c.)

The male of this species was described in 1882 from New Haven, Connecticut. Since that time both sexes have been found at several places and in large numbers. It is 2 mm. to 2.5 mm. in length, the females being usually a little smaller than the males. The

general color is gray, paler on the legs, and there is great difference in the depth of color in different individuals. In alcohol the wetting of the hairs makes them paler and more translucent, and they soon become yellow. The abdomen is marked with four longitudinal lighter lines partly broken into spots. There are no markings on the cephalothorax, except a little black around the eyes.

The epigynum is not folded, but extends backward half way to the spinnerets, curved slightly inward toward the body and outward again at the tip. Pl. IV. fig. 12 d.

The male palpi have the tibial hook large and wide, turning outward with three teeth on the thickened edge. The base of the tarsus has a slight horn, shorter than in *viaria*. The end of the palpal organ has two small black processes, one twice as long as the other. Pl. IV. fig. 12a, which show from below when the palpi are held in the usual position.

**Microneta serrata.** (Plate IV, figures 15, 15a, 15b.)
One male from a fence in Boston, Nov. 20, 1900, during the autumn flight. Length 1.5 mm. The cephalothorax is a third longer than wide, and narrowed toward the front. The eyes cover the whole front of the head and are large for the size of the spider. The front middle eyes are only slightly smaller and closer than the upper middle pair. The cephalothorax is highest in the middle where it is more than half as high as wide. The sternum is large and convex, widest in front, and ending in a blunt point between the fourth coxæ.

The male palpi are very peculiar. The patella is as long as wide; the tibia is twice as long as the patella and a little widened at the end, with a thin projection on the outer upper corner, extending forward and turned a little inward. There is a little ridge on the back of the tarsus parallel to this process. The tarsus has a slight spur at the base. The tarsal hook is slender as in several small *Bathyphantes*. The middle appendage of the palpal organ is larger than in *Microneta viaria* and has on the outer side a line of short black spines, Pl. IV, fig. 15a.

**Epeira angulata,** Clerck.
**E. silvatica,** Em. N. E. Epeiridæ. Trans. Conn. Acad., 1884.
**E. solitaria.** N. E. Epeiridæ. Trans. Conn. Acad., 1884.
**E. nigra.** Canadian Spiders. Trans. Conn. Acad., 1894.
Comparison of several specimens from western Canada leads me to think that *silvatica, solitaria* and *nigra* are all varieties of *angu-*

*lata.* In New England this species continues to be rare, but in the Rocky Mountains and in Oregon and California it is common on fences and outside of houses. On the piazza of hotels through the Canadian Rocky Mountains, the males vary in size from that of *solitaria* with the cephalothorax 5 mm. in length, to the smallest *silvatica* only 3.5 mm. The length of the first femur varies in these specimens from 5.5 to 4 mm. Four males from the hotel at Glacier varied among themselves nearly as much. The palpi of the larger specimens resemble *solitaria* and *nigra*, with the tube curved upward at the base and strongly curved toward the end, while in smaller, light-colored individuals, the tube is less curved, lies closer to the bulb and tapers more regularly toward the point, as in the smaller *silvatica.* The shape of the second tibia is the same in all the varieties, the spines being somewhat longer and stouter in larger individuals.

The females vary but little, except in color, most of the western specimens being darker than those from New England. The shape of the epigynum is very uniform, with the finger very long and slender.

In August, 1906, Mrs. Annie Trumbull Slosson found a male on the hotel at the summit of Mt. Washington that resembles very closely the original *E. solitaria* from Massachusetts.

**Epeira corticaria,** Em. New Engl. Epeiridæ, 1884. (Plate V, figures 3, 3 a.)

Mature males and females are found on the lower part of Mt. Washington, N. H., in the early part of August. The females have the finger of the epigynum broken off or shrivelled. The males are marked and colored like the females, except that in the males the dark stripes at the sides of the cephalothorax are wider, and the dark rings of the legs more distinct. The second tibiæ are slightly thickened and curved, and five spines on the upper side and two on the inner side are thickened and dark-colored. There are no spines on the coxæ.

**Epeira Nordmanni,** Thorell.

A male from The Glen at the base of Mt. Washington, N. H. is 9 mm. long, the cephalothorax 5 mm. The dark stripes at the sides of the cephalothorax are wider and more definite than in the female. The markings of the abdomen are like those of the female, but are less distinct. The second tibiæ are slightly thickened and

curved, and the spines on the upper and inner sides stout and dark-colored. There are no spines on the coxæ.

**Epeira thaddeus,** Hentz. (Plate V, figures 2, 2 a.)

The males I have seen, from Sharon and Waltham, Mass., are a little smaller than females from the same places. The front leg is much elongated, the patella and tibia together being as long as the spider from eyes to spinnerets. The usual little process on the anterior end of the first coxa is lengthened into a spine directed forward about half the diameter of the coxa in length. The second leg is slightly thicker than the first; the tibia is a little curved, and the four spines on the inner side are stouter but not shorter than the others on this joint. The color is pale, without any bright orange on the legs or dark brown around the abdomen common in females. The first and second legs have brown rings at the ends of the joints, while the third and fourth have the dark ends of the joints less strongly marked than in females. The cephalothorax is pale, with a pale gray stripe in the middle. The abdomen does not have the brown band around the sides which is so character-istic of females, and on the back it is marked with pairs of bright yellow spots, the two anterior pairs larger than the others, some-what as in *E. globosa*. Some females have similar markings on the back of the abdomen.

**Epeira juniperi,** Em. (Plate V, figures 1, 1 a.)

Two males swept from bushes at Ponemah, N. H., were slightly greenish on the abdomen, which is striped with white at the sides and across the front. The rest of the body was pale and yellowish. The ends of the tibiæ of first and second legs were light orange, covering nearly half the joint, but not forming a definite ring. The cephalothorax is nearly as wide as long and 2 mm. in length. The legs are long and slender, the tibia and patella of the first pair measuring 3 mm. The spines of the legs are dark colored and very long, especially on the tibial joints; those on the first tibiæ being half as long as the whole tibia. There are no modifications of the second tibia.

**Epeira labyrinthea,** Hentz.

Hentz. in his description of this species, says that a tube, similar to that of Agalena, leads from the web to the nest. I have never seen such a tube; but often there are several threads, as in *Zilla atrica*. leading from the center of the round web to the nest, and

the center is drawn tight by them, giving the appearance of a funnel-shaped opening to a tube. There is, however, no hole in the center of the web, and the cluster of threads may be flat or slightly depressed in the form of a gutter.

**Zilla montana.** (Plate V, figure 4b.)

This is a common house spider at Deer Island and at northern end of Moosehead Lake, Maine, making its nests like *Z. atrica* under the edges of clapboards. In North Carolina it lives on houses and in bushes at the summit of Roan Mountain, and in houses and barns at the base of the mountain, near the railroad.

**Zilla atrica, Koch.**
**Eucharia atrica, Koch. 1845.** (Plate V, figures 4 to 4d.)

In size and color this resembles the other species. The markings of the back of the abdomen resemble closely those of *x-notata*, but the middle of the back is usually lighter, and the two diverging dark marks near the anterior end are longer and narrower than in *x-notata*. The cephalothorax has a more distinct dark middle stripe than in the other species. In the males the palpi (fig. 4a) are twice as long as the cephalothorax, and about twice as long as those of *x-notata*. The front legs of the male are, however, one-eighth shorter than those of *x-notata*, the front tibia and patella measuring a little less than twice the length of the cephalothorax. The form of the epigynum is shown in fig. 4b in comparison with those of *x-notata* and *montana*.

The webs are like those of other species with a large central spiral from which a strong thread extends to the nest. A large segment opposite this thread is usually left open, but is often partly or entirely closed. Adults are found from August until winter.

First noticed by McCook at Annisquam, Mass., about 1885, and, now found abundantly at Ipswich, Gloucester, Salem, and Lynn, where it lives in hedges and on the outside of houses, making tubular nests open at both ends under the edges of the clapboards. At Ipswich, I first noticed them on a new cottage near the shore far from any other house, in 1900. At that time there were none of them on other cottages in the neighborhood or on the old farmhouse at Lakeman's beach. In 1904 they were on all the neighboring houses and barns and in the liliac bushes around them.

**Tetragnatha vermiformis, Em. N. E. Epeiridae.**

Positions of male and female while pairing. Fresh Pond marshes

Cambride, Mass. Sept. 3. 1901, 8 a. m. in irregular net on marsh-grass (Pl. V, fig. 5.) Position of mandibles while pairing (fig. 5a.).

**Pachygnatha tristriata,** Keysl. 1882. (Plate V, figures 6 to 6d.)

This species is not the same as *brevis*. The size is about the same as *brevis*, but both the cephalothorax and abdomen are slightly longer and narrower. The cephalothorax has three stripes in both species, but the abdomen of *tristriata* has the dorsal marking with straight black edges instead of scolloped as in *brevis*. The four middle eyes are raised above the head with the upper pair higher than the top of the cephalothorax, while in *brevis* the eyes are lower than the highest part of the cephalothorax. The cluster of middle eyes is as far above the mandibles as it is high. In *autumnalis* the upper middle eyes are larger than the others and farther apart, and the cluster of middle eyes is much higher than it is distant from the mandibles. In males the differences are more distinct than in females. The male palpi of *tristriata* have the tarsus and palpal organ longer and more slender than in *brevis*, the bulb is narrower, and the tube and narrow end of the tarsus are twice as long as they are in *brevis*. The tarsal hook of *tristriata* is straighter and more slender than in *brevis*.

Orono, Me., Salem, and Gloucester, Mass.

**Lycosa avara,** Keys. Zool. bot. Ges. Wien, 1876.

**L. rufiventris,** Banks. (Plate VII, figures 2, 2a.)

This spider resembles very closely *L. pratensis*. The light stripe on the cephalothorax widens behind the eyes, and has a middle dark line and a broken dark line each side of it as in *pratensis*. The front row of eyes, which in *pratensis* is straight, has in *avara* the lateral eyes a little lower than the middle pair. The eyes of the second row are a little larger than in *pratensis*, so that it appears slightly longer than the first row, while in *pratensis* it is slightly shorter; the difference is, however, too small to measure and cannot be seen in all specimens of *pratensis*. The two specimens of *avara* examined vary in size as does *pratensis*. The most distinctive character of *avara* is the form of the epigynum as shown in fig. 2. At first sight it shows a pair of round holes, and it is only by rubbing away the hairs that the shape of the middle lobe can be seen. This is anchor-shaped with the pointed ends curved around so that they point directly forward. There is a slight projection in the middle. At the front end the middle lobe widens, and its edges are continuous with the anterior borders of the two large holes.

Two females were found by Miss E. B. Bryant one in Allston. Mass., and the other at Long Island, Portland, Maine.

**Lycosa frondicola,** Em. N. E. Lycosidæ.
**L. nigroventris,** Em. is the male of this species.

This species and *L. Kochii* are often found in the same localities. They both mature late in autumn and carry their cocoons of eggs in May. *Frondicola* is darker brown and less mottled than *Kochii*. The middle stripe of the cephalothorax is straight in *frondicola* and notched at the sides in *Kochii*. The young of *frondicola* are more mottled on the legs than the adult and resemble the young of *L. cinerea*. The *L. nigroventris* described in N. E. Spiders is an unusually large male *frondicola*. The male is usually two thirds the size of the female with the under side darker. The legs are lighter and the markings on back of abdomen more distinct.

**Lycosa carolinensis,** Hentz.

Mr. W. L. W. Field of Milton, Mass., has watched for many seasons a large number of these spiders in a pasture on a gravelly hillside, where they make holes six or eight inches deep, sometimes straight and sometimes curved irregularly, to avoid large stones. Sometimes the mouth of the hole is funnel-shaped, spreading to twice the diameter of the lower part of the tube. The males appear only in the late summer, and the fertilized females winter in the tubes which are closed partly by the wheather, and lay their eggs in the last of May or June. In the summer the half-grown spiders are sometimes found without holes, and they have been known to abandon their holes and make new ones.

**Lycosa baltimoriana,** Keys. Zool. bot. Ges. Wien, 1876. (Plate VII, figures 1, 1a, 1b.)

This is a large and distinctly marked species, the female 15 mm. long, the cephalothorax 8 mm. long, and 5.5 mm. wide. The eye area is small, occupying one-third the width of the head and one-sixth the length of the cephalothorax. The front and second rows of eyes are of the same length. The legs are of moderate length, as in *carolinensis* and *tigrina*. The general color is gray like *carolinensis* with black markings. The cephalothorax has indistinct dark radiating lines. The back of the abdomen has a dark spot following the shape of the heart, and behind it two or three irregular triangular spots, and along the sides are other irregular markings. On the under side of the abdomen is a square black

spot extending from the lung openings back nearly to the spinnerets. The sternum is black. The legs are marked with broken dark rings.

The epigynum is narrow in front with two small openings; it is widened in the middle and has a small T-shaped end behind. Pl. VII, fig. 1 b.

The male palpus is much like that of *L. nidicola* fig. 1 e, which is from a specimen from Providence, R. I. belonging to Mr. Banks.

From Woods Hole, Mass., and Simsbury, Conn.

**Lycosa Pikei,** Marx.   American Naturalist, 1881.

**L. nidifex,** Em.   N. E. Lycosidae.

**L. arenicola,** Scudder.   Psyche, Vol. II, 1877, name preoccupied by Cambridge in Spiders of Dorset.   (Plate VII, figures 3 d, 3 e.)

The burrows of this species do not usually have a tube of straw or other rubbish around the mouth. The edge of the tube is thickly covered with silk, which extends out sometimes an inch around it on the surface of the sand. In digging, the surface of the sand is first covered thinly with silk. A ball of sand held together by the silk is then gathered up and carried to the mouth of the burrow in the mandibles; there, without the spider coming out of the hole, it is placed on the ends of the front legs, and thrown as far away as possible. In full grown spiders this is about two inches, and the balls of sand may sometimes be seen in a circle of this radius around the hole. When looking for prey, the spider sits with the cephalothorax and front of the abdomen out of the hole and the feet turned under the body as if dead. A step on the sand within ten feet will alarm them and they disappear down the burrow, but by creeping slowly without jarring the ground or throwing a shadow over the hole, one may get within two feet of the spider without disturbing it. The spider will notice an insect moving six or eight inches away and will rush out and catch one at that distance, returning quickly with it to the burrow. The adult males live part of the time in holes like females. and lie out at the top and wait for insects in the same way, but in August and September they are often found wandering. A male confined over night and then turned loose near the burrow of a female at once looked into it, reaching down its whole body except the tip of the abdomen and the fourth legs. It quickly came out, followed to the mouth of the burrow by the female who at once went down again, and returning in a few seconds, seated herself in the usual position over the edge of the hole. The male then approached slowly with the front of

the body raised, alternately reaching forward the front legs and jerking them quickly back until almost near enough to touch the female. She then came toward him and struck at him weakly with her front legs, but he turned them aside, jumped on her back and tried to place his palpus under her. She then attacked him in earnest and drove him away, afterward going down in her burrow and remaining there, and the male soon wandered away.

Young an eighth of an inch in length are found in small burrows of their own from June to August, and in holes with adult females as late as Aug. 10.

**Lycosa nidifex,** Marx. American Naturalist, 1881. (Plate VII. figures 3 to 3e.)

In N. E. Lycosidae I have confounded this species with *L. pikei,* under the name of *nidifex.*

This inland species differs distinctly from *Pikei* and approaches *L. missouriensis* Banks of the South and West. The epigynum and palpal organs of these three species are so much alike that they cannot be used to distinguish them. In *nidifex* the black color of the under side of the first leg does not extend inward beyond the patella, and the coxæ are all light-colored, while in *Pikei* the whole of the first leg, including the coxa, is black, and in some individuals the whole of the second leg. In *nidifex* the whole upper surface of the body is a nearly uniform gray color with indistinct stripes on the abdomen, while in *Pikei* the color of both upper and under sides is darkest at the head, and gradually lighter backward with a distinct pattern on the abdomen. In *nidifex* the pads on the *tibia* and *metatarsus* are composed of shorter hairs, so that these legs look but two thirds as thick as they do in *Pikei.*

*L. nidifex* usually makes a turret at the opening of its burrow, sometimes only a slight ring, but often a tube of sticks or grass rising more than its diameter above the surface of the ground. Like *Pikei* the spider sits at the mouth of its burrow with the feet turned under and the head high enough to see the surrounding country. The burrows are often not more than eight or ten inches deep, sometimes curved to avoid stones. The turrets are most conspicuous in October and November, after the surrounding grass has withered. The burrows remain open all winter, the immature spiders lying partly torpid at the bottom. Freshly matured males and females are found in May.

**Lycosa punctulata,** Hentz. (Plate VII, figures 4. 4a.)

The legs of both sexes are shorter than in *scutulata*. In the male the first and second legs are not as much elongated as in *scutulata*, and the first legs are not darker than the others. The stripe on the abdomen is straight in both sexes, without light spots along the edges as in *scutulata*. The under side of the abdomen has irregular black spots which are wanting in *scutulata*. The palpal organs are shaped much as in *scutulata*, but the tarsi and all the joints of the palpi are a little shorter and stouter than in that species.

Framingham, Mass.. Sept. 29, 1906.

**Lycosa relucens,** Montgomery. Proc. Phil. Acad. Nat. Sci., 1902. (Plate VI. figures 1, 1a, 1b.)

This species matures early in the season and is common around Boston in April in open woods. Its general color is that of dried leaves. and it resembles small individuals of *L. frondicola.* The length is 8 mm., the cephalothorax 4 mm. or a little less. The cephalothorax has a straight white middle stripe, the width of the middle eyes extending from them backward and slightly narrowed behind. There is a narrower white stripe near the edge each side, sometimes broken and indistinct in females, and straighter and more distinct in males. The legs are pale yellowish brown, with the femora faintly marked with gray rings that are sometimes absent, especially in males. The abdomen is indistinctly marked with pairs of gray spots and oblique lines. The epigynum has the common T-shape as wide as long, and a single arched opening in front. Fig. 1b. The male palpus has the tibia thickened so that it is nearly as wide as the tarsus. The tube of the palpal organ is abruptly curved forward, and a thin supporting appendage lies along the side of the tarsal cavity without extending beyond its edge. At the base of the tube is a large thick appendage extending forward, Fig. 1a.

New Haven, Conn.. Mass., Lake Champlain, Vt.

**Lycosa crassipalpis,** new (Plate VI, figures 3, 3a.)

Three small males from Ipswich, Mass., and one from Portland, Me., are only 6 mm. long and the cephalothorax 3 mm. The male palpi have the tibia thickened as in *relucens*, but the tarsus and palpal organ are proportionally smaller and not wider than the tibia. The legs are pale without any gray rings on the femora. The lateral white lines on the cephalothorax are well defined and removed more than their width from the edge as they are in *biline-*

*ata.* The sternum has a light middle line for half its length, which shows indistinctly in the darker specimens.

**Lycosa bilineata.**

**Pardosa bilineata,** Em. N. E. Lycosidæ.

**Lycosa ocreata, pulchra,** Montgomery. Proc. Phil. Acad. Nat. Sci., 1902. (Plate VI, figures 4, 4a, 4b.)

The female of this species was described in N. E. Lycosidæ, from New Haven, Conn., without the male being known. This was later found at Cold Spring Harbor on Long Island. The female resembles in color and markings *Pardosa pallida* more than it does its nearest relative, *Lycosa ocreata*. It is 6 mm. long, with the cephalothorax 3.5 mm. The colors are light yellow and brown, with gray hairs on the legs and abdomen. The cephalothorax has three pale stripes, the middle one as wide as second row of eyes, the lateral half as wide and a little above the edge. The legs are pale yellow without any markings except faint traces of rings on the femora. The markings on the abdomen are like those of *ocreata*: a dark pointed stripe in the middle bordered by light stripes, outside of which are rows of dark spots. The colors of the male are the same except the tibia and end of the metatarsus of the first leg, which are deep black and surrounded by stiff black hairs, Fig. 4a. The epigynum is much like that of *relucens*, T-shaped, and as wide as long. The male palpi have the tibia slightly enlarged, but not as much as in *relucens* or *ocreata*. The palpal organ is like that of *relucens*, with the appendage supporting the end of the tube longer, so that it projects out over the edge of the tarsus, and the large thick terminal appendage is wanting.

**Pardosa littoralis,** Banks. (Plate VI, figures 5, 5a, 5b.)

This species described by Banks from Long Island, N. Y., where it is common, has now been found at Ipswich and Plum Island, Mass. The females are 7 mm. long, with the cephalothorax 3 mm. It is not as slender as *pallida* and *nigropalpis*, but has the proportions of *glacialis*, the young of which it much resembles, Fig. 5.

The color is pale yellow with gray markings. The legs are yellow without markings. The cephalothorax has a narrow black line each side and two wide dark stripes leaving a light stripe on each side and a less defined one in the middle. The abdomen has a middle light stripe with indented edges, and the sides are marked with light mixed with gray. In the male all the dark markings are darker than in the female.

The epigynum resembles that of *nigropalpis* but is shorter and stouter, Fig. 5 b.

The male palpus also resembles that of *nigropalpis*, Fig. 5 a, which I have figured from a Long Island specimen belonging to Mr. Banks.

**Pardosa diffusa,** new. (Plate VI, figures 6, 6 a. 6 b.)

Two males from Ipswich and Hyde Park, Mass. are distinguished from the ordinary male *nigropalpis*, even when running on the ground, by the darker color of the cephalothorax. The middle light band is narrow, and hardly shows in front of the dorsal groove. The light bands at the sides are very narrow and close to the edge. The legs are marked on the femora with broken rings darker and closer together from behind forward, the first femora being almost black. In the palpal organs the basal process is shorter and does not have the long curved hook which crosses the tube in *nigropalpis*, Fig. 6 a. No mature females have been found in company with this, but females found in August without males in Massachusetts and Maine are supposed to belong to the same species.

The epigynum differs plainly from that of *nigropalpis* and *albopatella*. The anterior pit is rounder and wider, and the transverse posterior end is much wider than in the other species. The females differ in markings from *nigropalpis* and *albopatella* in the same way as the males.

Males from Ipswich, Hyde Park, and Sharon, Mass.

Females from Medford, Mass., Northern Maine, and Long Island, N. Y.

**Pirata insularis,** Em. N. E. Lycosidae (Plate VI, fig. 7).

A new figure is given of the markings of this species from a specimen from Danvers, Mass.

**Pirata arenicola,** new. (Plate VI, figures 9 to 9 c.)

Female 6 mm. and male 4 mm. long. In the female the lateral light stripes are wide and extend over the edge of the cephalothorax, but in the male the edge of the cephalothorax is marked with a broken dark band. The legs are pale and faintly ringed with gray. On the under side the female is entirely pale, and the male has three gray lines on the abdomen.

The epigynum has two oblique lobes behind slightly pointed on the inner ends.

The male palpi have the tarsi shorter than in *P. sylvestris* more as in *piraticus*. The appendages of the palpal organ are all small,

the terminal process as usual divided into two branches, the outer straight and opaque, the inner thin and transparent and turned across the tarsus.

Ipswich, Mass., June 6, 1903.

**Pirata maculatus,** new. (Plate VI, figures 10, 10a, 10b.)

6 mm. long, the same size and much like *P. montanus*. The markings are the same as in *montanus*, but the dark portions are much darker, and the rings on the legs more distinct than in any other species. The dark markings of the under side are also more prominent than usual; there is a distinct light middle stripe on the sternum, and a light area in the middle of the abdomen, bordered at the sides with black and partly divided by a middle dark stripe, narrow in front and widened behind. The hinder part of the epigynum is divided into two lobes, slightly pointed in the middle, and showing no opening on the outer side.

Moosehead Lake, Me., Aug. 7. Females with eggs.

**Pirata sylvestris,** new. (Plate VI, figures 8 to 8c.)

Female 8 mm. long; male 5 mm. long. In the female the usual three light marks behind the eyes are very narrow, but the light marks at the sides are wide and extend to the edge of the cephalothorax. In the male the edges of the cephalothorax are dark and the lateral light markings narrow. The abdomen has the usual gray color with a light middle stripe in the anterior half, and four pairs of bright white spots covered with white hairs and indistinct white lines on the sides and along the sides of the middle stripe. The sternum is pale without stripes. The under side of the abdomen is in some individuals pale, while in others there are traces of three dark stripes. The legs are pale without rings. The epigynum has the usual two lobes behind bluntly pointed on the inner corners where they are partly covered by a middle bunch of fine white hairs. In some light colored females the spermathecæ show through the skin near the outer corners of the lobes.

The tarsus of the male palpus is slender. The palpal organ has the usual two small appendages in the middle, one slender and the other a short and stout tooth. The terminal process is long and curved in quarter of a circle, with the transparent inner branch showing beyond the outer which is thicker and darker.

**Dolomedes sexpunctatus,** Hentz. (Plate VII, figures 6, 6a, 6b.)

A male from Wellesley, Mass. has the cephalothorax 5 mm. long and the same in width. The hind leg 23 mm. The spider had been put in alcohol very soon after moulting and the legs and palpi are probably not fully extended. The markings are like those of the female and the colors like a young and pale female preserved in the same way.

The male palpus has a long process on the outside of the tibia nearly as long as the joint itself. It is thin and flat. widened and rounded at the end, and has a small tooth on the under side near the base. The end of the tibia is shrunken and should no doubt be wider at the end than at the base, as it is in a Tennessee specimen apparently of the same species. The palpal organ is like that of *D. fontanus.*

A nest of this species was found at Amherst, Mass., Sept. 5, 1905 on golden rod two feet above the ground. The nest was about three inches in diameter, and the young spiders, early in the morning, were gathered in the lower part of it. The female was on the plants a short distance below the nest.

**Dolomedes fontanus,** Em. New Eng. Lycosidæ.

The male of this species was described and figured in New England Lycosidæ in 1885. The female was described in the same paper under the name of *D. tenebrosus.*

Marx in a note in his catalogue of N. American Spiders in 1890 gave his opinion that these were male and female of the same species, which a study of more specimens has shown to be correct.

The female has the cephalothorax 9 mm. long and 8 mm. wide, and the abdomen varies from 10 mm. to 15 mm. The eyes of the front row are small and the middle pair only slightly larger than the lateral, while in *idoneus* the middle pair are twice as large. The epigynum, which is correctly figured in N. E. Lycosidæ, has a narrow middle lobe bluntly pointed behind. The color in alcohol inclines to be olive, while in *idoneus* it is reddish brown. The marginal white stripes on the cephalothorax in life connect together in front of the head. The light middle stripe, which is distinct on the cephalothorax of *fontanus*, is less so in *idoneus*. The sternum of *fontanus* has a distinct light middle stripe which is absent or very indistinct in *idoneus.*

The male is smaller than the female, with the legs more slender but as long as those of the female. The cephalothorax is as wide as long, measuring 7 mm. The first and fourth legs are of the same

length. 36 mm. The palpi are 9 mm. long with the tibia straight and with a forked process in the middle of the outer side. The tarsus and palpal organ have been correctly figured in N. E. Lycosidæ.

This species seems to be common as far south as the mountains of North Carolina. On Lake Champlain, Vt, and Lake Winnipesaukee, N. H., it matures about July 1, when it is common along the shore under loose stones and the floats of boat landings. It runs on the surface of the water and on the bottom, carrying a large amount of air adhering to its hairs. It remains under only a short time, coming quickly to the shore as soon as it has escaped pursuit.

**Dolomedes idoneus,** Montgomery. Proc. Phila. Acad. Nat. Sci., Nov. 1902. (Plate VII, figure 8.)

The female of this species is of the same size as *fontanus* and has similar markings, but the color in alcohol is reddish brown instead of olive gray, which is usual in *fontanus*. The shape of the head is the same as in *fontanus*, and the arrangement of the eyes is the same, the only difference being in the size of the front middle eyes, which in this species are twice as large as the laterals of the same row. The shape of the epigynum is characteristic of this species, even when half grown. The middle lobe is round and swells out beyond the surface of the abdomen, and there is a distinct opening on each side between it and the lateral lobes.

The male has not been described.

Females have been found at Lake Champlain, Vt., and at Simsbury and New Haven, Conn.

**Dolomedes urinator,** Hentz. Montgomery, Proc. Acad. Nat. Sci., Philadelphia, 1904.

The male spider described by me in 1885 as the male of *D. tenebrosus* appears to be *urinator* or *lanceolatus*, Hentz. I have not found females but have one from Pennsylvania sent me by Mr. Montgomery.

An immature male from Milton, Mass., resembles closely the drawing of *D. lanceolatus* by Hentz. It has the tuft of stiff hairs on the femur of the fourth leg, like the mature males that I have described.

**Dolomedes vernalis,** new. (Plate VII, figures 7 to 7d.)

Males from Falmouth, Me., and Three Mile Island, Lake Winnipesaukee, N. H. Cephalothorax 3 to 4 mm. in length and a little less in width. Fourth and first legs 16 to 18 mm. The colors are

pale yellow and gray. The cephalothorax is dark in the middle
and light at the sides, with light gray spots over the coxæ. The
dark middle area extends forward between the eyes to the front
edge of the head, dividing into two below the eyes. The mandibles
are striped on the front with black. The abdomen is light at the
sides, and the middle dark marks are united into a broad stripe
with irregularly indented edges. The legs are marked with broken
dark rings, the femur and the tibia having parts of four rings each.
The sternum is dark around the edges, and the whole under side
of the abdomen is gray, darkest at the sides, with two indistinct
light lines converging toward the spinnerets. The tibia of the male
palpus is as short as the patella. The process of the tibia is as
long as the diameter of the joint. It is flat and widened at the
end, hollowed in at the middle, and with the corners sharp, and
sometimes two little teeth in the hollow. The palpal organ resem-
bles that of the other species.

A female just moulted, from Three Mile Island, Lake Winni-
pesaukee, N. H., May 25, 1905. Cephalothorax 6.5 mm. and abdomen
the same length; fourth and first legs 24 mm. Colors and markings
like those of male. The epigynum resembles that of *D. idoneus*
with the middle portion not as prominent, and the pockets at the
sides more open.

At Three Mile Island, between May 20 and 27, 1905, one female
and several males made their last moult. They were under stones
and loose boards lying on the ground near the shore.

**Oxyopes scalaris** (Hentz) Em.
This species was found again at Durham. N. H., in June 1904. It
resembles closely a species found commonly on the Pacific Coast
from British Columbia to California.

**Œcobius parietalis.**
**Thalamia parietalis,** Hentz. Journal Boston Soc. Nat. Hist. (Plate VIII,
    figures 1 to 1 e.)
2.5 mm. long, pale and translucent, with black spots on the head
and legs and around the sides of cephalothorax and abdomen. The
cephalothorax is as wide as long, and almost circular. The eyes
are on the top of the head in two nearly straight rows, the front
row shorter than the upper, and the front middle eyes farther apart
than they are from the lateral eyes. The upper middle eyes are
not round but irregularly oval, largest from front to back. The
head extends forward a little beyond the eyes, and under this

projection are the mandibles which are very small. The maxillæ are inclined toward each other, over the short and rounded labium. The sternum is as wide as long. The abdomen is oval, as wide as the cephalothorax, and one-half longer. It is marked with irregular opaque white spots, a black line around the front end and several pairs of black spots, Fig 1. The shape of the end of the abdomen and the arrangement of the spinnerest are very peculiar in this genus. At the end of the abdomen behind the anus is an oval appendage surrounded by a single row of curved hairs of the same thickness throughout their length, and rounded at the end. The hinder pair of the spinnerets are their length apart, and extend backward so as to be seen for nearly their whole length from above. The spinning tubes extend along the under side. The cribellum is slightly divided by a notch in the middle. The calamistrum consists of two parallel rows of ten or twelve sligthly curved hairs, extending half the length of the fourth metatarsus. The legs are all about the same length. and the feet have three claws. The epigynum has a double tube directed backward and resting in a shallow groove on the under side of the abdomen. The male has the legs longer and the abdomen smaller, but otherwise resembles the female. The male palpus has very short patella and tibia, and the tarsus is wide and oval, and the palpal organ thick and furnished with a cluster of short appendages near the base.

This *Œcobius* lives in houses and on walls and fences. It makes a flat web one to two inches long and half as wide, fastened at several points around the edges, leaving open spaces through which the spider can run in and out. The spider stands on the wall behind and not on the web.

It has been known since the time of Hentz in the Southern States, but has lately been found in a house in Roxbury, a suburb of Boston, where it seems to be well established around window frames and behind furniture.

**Scotolathys pallidus** (Marx) Simon.
**Neophanes pallidus,** Marx. Proc. Ent. Soc. Wash., 1891. (Plate VIII, figures 2 to 2 d.)

1.5 mm. to 2 mm. long and pale and translucent without any markings. The cephalothorax is shaped like that of *amaurobius.* The abdomen is slightly larger and wider than the cephalothorax and a little wider behind than in front. The eyes are six in number, large for the size of the spider, all about the same size and arranged in two groups. The cribellum is small, about as wide as one of

the anterior spinnerets. The calamistrum consists of seven or eight pairs of hairs about the diameter of the leg in length, Fig. 2 c. The coxæ of the fourth legs are more than their diameter apart, and the end of the sternum extends backward beyond them in a blunt point, Fig. 2 d. The epigynum shows externally two round spermathecæ, each crossed by an opaque spot, and in some specimens spiral tubes can be seen connecting with them. The male palpi resemble those of *Dictyna.* The tube of the palpal organ coils around the edge of the tarsus, where it is supported by a wide thin appendage; it curves around the base of the tarsus to the upper end, where it is twisted and rests against a blunt process of the tibia.

These little spiders live under leaves and are found by sifting in company with various Erigoneæ. They have been fond in various places around New Haven Connecticut and at Three mile Island, Lake Winnipesaukee, and Fitzwilliam, N. H.

**Amaurobius borealis,** new. (Plate VIII, figures 3 to 3 d.)

Female 5 mm. and male 4 mm. long. The cephalothorax and legs are yellow brown, the legs darker toward the tips and the cephalothorax darker around the edges, but little toward the head. The abdomen is reddish brown with an indistinct pattern. The cribellum is small, not wider than the length of the first spinnerets and is indistinctly divided in the middle. It is on a slight elevation just back of the spiracle. The calamistrum occupies half the length of the metatarsus. The epigynum has a wide middle lobe, covered at the ends by the lateral lobes. The male palpus has the patella as wide as long, with one stout spine projecting over the tibia. The tibia is curved and has the usual complications shown in the figures.

Fitzwilliam, N. H., abundant near the Rhododendrons. Portland, Me., under leaves on the ground. Mature in May; females with eggs in July.

**Orchestina saltitans,** Banks. Ent. News, 1894, p. 300. (Plate I, figures 4, 4a, 4b.)

Cellar of Boston Soc. Nat. Hist. building, March 6, 1889. Found by Banks in house at Sea Cliff, Long Island, N. Y.

**Micaria laticeps,** new. (Plate X, figures 4 to 4c.)

One male of this species was found under a stone at New Haven, Conn. The length is 3 mm. The cephalothorax is a little higher than in the other species, and the head nearly as wide as the widest part of the thorax. The eyes of the upper row are at equal dis-

tances apart, and the whole group of eyes wide in proportion to the width of the head. The abdomen is oval and slightly indented in the middle. The colors are all dark, and were not noticed when the specimen was fresh. The cephalothorax is of the usual brown. and the legs the same color with the ends of the first and second pairs lighter. The abdomen seems to have been lighter in front of the depression, but there are no distinct markings to be seen in its present condition. The male palpi have no process on the tibia. The palpal organ is prominent as in *quinquenotata*.

**Micaria quinquenotata,** new. (Plate X. figures 1 to 1 e.)

This species lives in sandy places, sometimes in company with *longipes*, which it resembles in color and habits. It is smaller than *longipes*, measuring 4 mm. in length, the cephalothorax between 1¾ and 2 mm. in length. The cephalothorax is shorter and the head narrower than in *longipes* and the lateral eyes are nearer the middle pairs, Fig. 1 a. The sternum and the legs are slightly shorter than in *longipes*.

The legs and cephalothorax are light orange brown, with scattered shining hairs of the same color. The abdomen is covered with iridescent scales, yellow in front, and darker toward the hinder end. The colors vary in different individuals and some are greenish gray as in *longipes*. There are two pairs of white spots on the abdomen, one pair in the middle and another at the front end, and just behind the front pair is a middle white spot of about the same size, Fig. 1.

The epigynum differs little from that of *longipes*, but is usually less regular in shape. The male palpi also are like those of *longipes* with a similar process on the tibia, Fig. 1 a.

This species is common on the sand dunes at Ipswich, Mass., among the roots of sand grass. It matures about the first of June, when both sexes are active, running about on the sand from one bunch of grass to another, or hiding under any loose object lying on the ground. The cocoons which are found early in June are white and thin, and contain about eight eggs.

In pairing the male holds the female by the first and second legs around the thorax between her third and fourth, reaches his head under her and inserts the palpus of the same side as the clasping legs, Fig. 1.

**Micaria gentilis,** Banks. Canadian Entomologist, 1896. (Plate X, figures 3 to 3 d.)

Mature males and females have been found from the middle of May to the first of July at Portland, Me., and at Monhegan, Me.

The cephalothorax is shaped as in *quinquenotata*, a little narrowed in front and not much elongated. In some individuals the cephalothorax is unusually narrowed behind, so that the widest part is in front of the middle. The cephalothorax is a little less than 2 mm. long. The abdomen is oval without any constriction in the middle, sometimes in females twice as long as it is wide, in males not much longer than the cephalothorax.

The color of the cephalothorax varies from light brown to black, covered with light shining hairs not very close together. The first and second legs have the femora dark like the cephalothorax, and the other joints light yellow. The third and fourth legs are brown, the femora darker. The sternum is dark brown and the front coxæ are the same color; the other coxæ are partly light colored; the fourth pair almost entirely light yellow. The abdomen is covered with dark green iridescent scales, with a narrow white band across tho middle, and in some individuals another transverse white band near the front end, but this is oftener broken into two short white streaks at the side.

The epigynum has a large opening in front, covered by a wide rim with a dark colored edge, Fig. 3 d.

The male palpi have no process on the tibia. The palpal organ is flatter than it is in *longipes* and *quinquenotata*, but has a hook in the middle as in those species.

**Castaneira lineata,** new. (Plate X, figures 5, 5a, 5b.)

This small species has the general appearance of a *Micaria*. It measures 6 mm. in length, the cephalothorax nearly 3 mm. The cephalothorax is twice as long as wide, widest in the middle and narrower behind than in front. It is slightly indented at the sides between the second and third, and between the third and fourth legs, Fig. 5. The head is three-fourths as wide as the thorax, wider than in the other species, and the eyes are farther apart. The upper eyes are equidistant and cover three-fifths of the width of the head. The sternum is narrowed and pointed behind, more than it is in the other species. The abdomen is a little longer than the cephalothorax, widest behind and a little constricted in the middle. The pedicel is as long as wide and can be seen from above between cephalothorax and abdomen.

The color of the cephalothorax is dull orange as in *Micaria longipes.* The femora are marked with two longitudinal dark stripes as in *M. longipes* and *C. bivittata.* The other joints of the legs are orange

yellow, except the ends of the fourth legs, which are somewhat darker.

The abdomen is lighter in front and has two white spots at the sides, nearly meeting in the middle.

The epigynum resembles that of *C. pinnata* with two small holes wide apart.

One female from low bushes in Sharon, Mass., July 7, 1902.

**Prosthesima rufula,** Banks. Phil. Acad. 1892. (Plate IX, figures 6 to 6h.)

7 to 8 mm. long; cephalothorax 3 mm. A little smaller than *P. atra* and more slender. The cephalothorax is narrower across the middle and less pointed in front. and the legs are more slender and the front pair less distinctly larger than the others. The sternum is narrower than in *P. atra.*

The color is light reddish brown without markings, the abdomen paler than the rest.

The epigynum varies in shape, the edge in front varying from nearly straight to the shapes shown in the figures.

The male palpus has a process on the outer side of the tibia, that lies along the edge of the tarsus for about a third its length and is slightly twisted at the tip. The tube of the palpal organ is on the outer side and extends nearly straight the whole length of the tarsus.

New Haven, Conn., and Cold Spring Harbor, L. I. N. Y.

**Pœcilochroa montana,** Em. N. E. Drassidæ, etc., Trans. Conn. Acad. 1890. (Plate IX, figures 4, 4a, 4b.)

The female of this species was described in 1890 from the White Mountains, but only lately the male has been found on the Blue Hills near Boston. The individual is probably a small one and measures only 5 mm. in length. The cephalothorax is shorter and rounder than in *variegata,* and the legs proportionally a little longer. The difference in shape of the cephalothorax in these two species is shown in the sternum, which in *montana* is distinctly wider than in *variegata,* Fig. 4a. The color is less brilliant than in *variegata,* the orange of the latter species being absent. The cephalothorax is dark brown, covered with white hairs. The femora and basal joints of all the legs are dark brown or black and the other joints light yellow. The abdomen is black with a narrow white band across the middle, a wider white band across the front end, with a little black showing in front of it, and a white band at the hinder

end over the spinnerets. The male palpi resemble those of *varie-gata*, but have the process of the tarsus a little stouter and more curved at the tip.

**Gnaphosa parvula,** Banks. Proc. Am. Ent. Soc.. 1896. (Plate IX, figures 3, 3 a, 3 b.)

This species is a little smaller than *brumalis,* the largest female measuring 8 mm. long, and the cephalothorax 3.5 mm. The color is the same rusty black as in the other two species. The lateral eyes of the upper row are placed as in *brumalis,* not as far from the middle eyes as in *conspersa.* The epigynum resembles that of *conspersa* more than *brumalis,* Fig. 3. The male palpi have the process of the tibia half as long as the tarsus, with the tip sharply pointed and a little curved, Fig. 3a. The palpal organ resembles that of *brumalis,* but the tube does not have a tooth at its base as in *brumalis,* Fig. 3b.

Ipswich, Mass. mature male and females, May 20. Described by Banks from Hanover, N. H.

**Drassus hiemalis,** new. (Plate IX, figures 1 to 1d.)

This species is a little smaller than *robustus.* The cephalothorax is 3 mm. long and a little narrower at the head than in *robustus,* and the lateral eyes are a trifle nearer together than in that species. The abdomen is a little more elongated than in *robustus,* and the epigynum farther back.

The epigynum is shaped somewhat as in *robustus,* but the lateral ridges are much thinner and lower, and in front of them is a trans-verse depression with a hard and dark colored rim. Fig. 1 d. The colors are the same as in *robustus,* but lighter than most specimens of the latter species.

The males are the same size as the females. The male palpus has a process on the upper side of the tibia which is nearly straight, not curved as in *robustus,* and extends over the tarsus one-third its length. The palpal organ has several hard brown processes that cover the surface and nearly conceal the end of the tube.

From Blue Hill and from Hammonds Pond, Brookline, under leaves in winter. Three Mile Island, May 25, adult males and females.

**Drassus bicornis,** new. (Plate IX, figures 2, 2a, 2b.)

Slightly smaller than *D. hiemalis.* The cephalothorax 2.5 mm. long, but form and color are the same, and there is nothing to distinguish these two species except the epigynum and palpi. The

epigynum has a large oval opening divided at the posterior end into two. The hard brown part around the hole extends forward on each side like a pair of horns turning toward each other at the ends.

The male palpi have a long process on the upper side of the tibia that extends over the tarsus for a third of its length. It is narrowed in the middle and obliquely truncated at the end. The palpal organ is hard and brown, smooth around the base, and divided at the end into a complicated group of processes, Pl. IX, fig. 2a.

Three Mile Island, Lake Winnipesaukee, N. H.

**Clubiona spiralis.** (Plate X, figures 10, 10a, 10b, 10c.)

6 mm. long, fourth leg, 9 mm. Larger than *C. rubra* and longer legged, but resembling it in the short mandibles and the arrangement of the eyes with the upper middle pair farther apart than they are from the lateral eyes. The male palpi have a general resemblance to those of *rubra*, but the double lateral process is differently shaped, round at the base and with the tip sharp and curved upward. The tarsus and palpal organ are more elongated than in *rubra*, and the large black process more slender. The only specimen found is pale, even to the mandibles.

Magnolia, Mass.

Two females. one from Ipswich, Mass. and one from the Blue Hills appear to belong to this species. They are the same size and color and have the same eye arrangement, with the legs shorter and stouter, as usual in females of this genus. The epigynum is shown in Fig. 10c. It has a partly divided transverse opening turned forward.

**Clubiona præmatura,** new. (Plate X, figures 7, 7a, 7b.)

In N. E. Spiders of the Family Drassidæ, etc. this species is confounded with *C. ornata* (Americana Bks.), on account of the distinct dorsal markings of the female which until recently was the only sex known. It is a little smaller than *ornata*, the cephalothorax of the female being 2.2 mm long, and the abdomen from 4 mm. when filled with eggs, to 3 mm. after the eggs have been laid. The color is pale, with the cephalothorax slightly darkened on the head and mandibles. The abdomen has a pattern similar to that of *ornata*, but less distinct. The arrangement of the eyes is similar to that of *ornata*, the upper middle pair being only slightly farther apart than they are from the lateral eyes. The shape of the border of

the epigynum is constant and characteristic. It does not extend backward in a point as in *ornata* and *rubra*, but is transverse with a deep notch in the middle, Fig. 7 b.

The male has the cephalothorax narrower in front than the female, and the palpi short, with little resemblance to those of *ornata*. The tibia is widened into a large process on the outer side, without any sharp teeth. The tube of the palpal organ is short and turned backward, and the other appendages are short and blunt, Fig. 7.

This species is very abundant under stones all over the top of the Mt. Washington range. The females make thin silk nests and lay their eggs about the first of July, by which time the males are scarce.

**Agroeca pratensis,** Em. Trans. Conn. Acad., 1890.

Females with epigynum like *A. repens* Em. Trans. Conn. Acad., 1894, have been fond in several New England localities at the same time with males of *pratensis* which makes it probable that *pratensis* and *repens* are one species with two forms of epigynum.

**Anyphæna rubra,** Em. N. E. Drassidæ. Trans. Conn. Acad., 1890.
(Plate IX, figures 8 to 8 c.)

The males of this species as well as the adult females are rarely found, because they mature very early in the season. A young male that had wintered under leaves was taken in Franklin Park, Boston, April 17. and moulted April 22.

The males differ but little from the females in size and color, but as usual are a little more slender and have longer legs and longer and straighter mandibles. The male palpi have a long process on the outer side of the tarsus, curved outward and slightly notched at the end, and in some individuals sharply pointed. The palpal organ swells out from the tarsus at the base. The tube begins on the inner side and curves around the base of the palpal organ and along the outer side of the tarsus nearly to the tip, Fig. 8.

**Apostenus acutus,** new. (Plate IX, figures 7 to 7 c.)

Immature males 4 mm. long. An adult male, which is dried and shrunken is of the same size. The cephalothorax is oval and much narrowed in front, so that the head is only one-third as wide as the widest part of the thorax. The eyes are low and arranged as in *Agroeca pratensis*, except that the front middle pair are much smaller. The front row is slightly curved upward, the middle eyes less than

half as large as the lateral. The upper row is more curved; the eyes all about equal in size, and the same distances apart. The lateral eyes of the two rows are near each other, but do not touch. The legs are long, with long spines, the fourth pair longest. The tibiæ of the first and second legs are thickened and have on the under side two pairs of long spines under the metatarsi. The sternum is almost circular with a slight point behind between the fourth coxæ.

The colors are translucent white and dark gray, like *Phrurolithus alarius*, but usually darker. The cephalothorax is light in the middle, with black edges and radiating dark lines. The abdomen is dark, with a series of pairs of light spots down the back. On the under side the sternum and coxæ are light and the abdomen spotted irregularly with dark gray. The male palpi in an individual that has been dried have the tibia and patella of about the same length. The tibia has a stout process on the outer side that turns inward against the base of the tarsus. The tarsus is oval, and the palpal organ long and thick. The tube seems to start near the outer end and curve around toward the inner side.

Adults were found at New Haven, Conn., May 1, and young males at Cold Spring Harbor, April 10.

**Cœlotes calcaratus,** Keys. Zool. bot. Ges. Wien, 1887.
**Cœlotes longitarsus,** Em. Trans. Conn. Acad., 1890.

On Plate VII, Vol. VIII, fig. 2 a is not the epigynum of this species but that of *Cicurina arcuata*. A correct figure of the epigynum of *C. calcaratus* is given in Common Spiders of the U. S. by J. H. Emerton 1902, page 104, fig. 242.

**Cicurina arcuata, pallida** and **brevis.** (Plate VIII, figures 6 and 7e.)

The three species of *Cicurina* live under dead leaves on the ground at all seasons, all three being sometimes found in the same locality. *C. arcuata* Keys. = *complicata* Em. is the largest and most deeply colored, with the abdomen covered with gray oblique marks. *C. pallida* is of the same shape and a little smaller, without markings. It is less common than the other two. *C. brevis* = *Tegenaria brevis* Em. = *C. creber* Banks, is smaller than the others and pale, with two rows of gray spots on the abdomen. The cephalothorax of the male is rounder and the head narrower than in the female, and more so than in the males of other species. All the species have very complicated palpal organs and a large appendage of the tibia of the palpus which lies against the tarsus and is not easily

distinguished from parts of the palpal organ. In *P. arcuata* this appendage is as long as the palpal organ and nearly as wide. In *C. brevis* it is narrow but longer than the rest of the tibia. This appendage was not noticed in my description of *Teg. brevis* but is correctly described by Banks under *C. creber* in the Spiders of Ithica. In *C. pallida*, although it is larger than *brevis*, the palpal organ is smaller, and the appendage of the tibia reaches only to its base.

**Cicurina arcuata,** Keys. Zool. bot. ges. Wien, 1887.
**Cicurina complicata,** Em. Trans. Conn. Acad., 1890.
    In New England Agalenidæ &c. Trans. Conn. Acad., 1890, Pl. VII, fig. 2a is the epigynum of this species, not of *Coelotes longitarsus.*

**Cicurina pallida,** Keys. Zool. Botan. Ges. Wien, 1887. (Plate VIII, figures 7 to 7c.)
    5 mm. long and pale and without markings. The cephalothorax is 2.5 mm. in length and 1.5 mm. wide, the head only a little narrower in the male than in the female. The epigynum is smaller than that of *C. brevis* and the parts seen through the skin rounder. The tarsi of the male palpus are as long as those of *brevis,* but more pointed and the palpal organ is smaller and more simple, though resembling in its general structure that of *brevis.* The process of the tibia which is so long and conspicuous im *complicata* and in *brevis,* is in *pallida* but little longer than the rest of the tibia, Pl. VIII, fig. 7.
    Found under leaves at Sharon and Northfield, Mass.. in company with *brevis* and *complicata.*

**Cryphoeca montana,** new. (Plate VIII, figures 4 to 4i.)
**Cryphoeca peckhamii,** Simon, from Washington territory, resembles this species.
    Males 4 mm. long, females 3 mm. General appearance like a small *Coelotes* or *Amaurobius.* The cephalothorax is narrowed in front of the first legs and at that point is as high as wide, curving downward toward the eyes. The eyes cover half the width of the front of the head, both rows arched upward. The upper row is largest, the eyes of equal size, and equal distances apart. In the lower row the middle eyes are half the size of the lateral. The lateral eyes of both rows touch each other. The sternum extends in a long blunt point between the coxæ of the fourth legs. The legs are of moderate length, the fourth longest in females, and the first in males. The first and second legs have two spines on the

outer side of the tibia, and four on the inner side, and three pairs of spines on the metatarsus. In females these spines are long, more than half the length of the tibia; in males they are short like the spines of the other legs. The abdomen is oval, not much longer than wide, resembling in shape as well as in markings that of *Amaurobius sylvestris*. The lower spinnerets are wide apart, and there is a wide opening to the tracheæ between and in front. The edge of the tracheal opening is thickened and colored on the inner side so that it resembles a small cribellum.

The colors are translucent white and gray. The legs are marked with broken dark rings at the ends and middle of the joints. The cephalothorax has a narrow black edge and broken radiating dark marks like *Cœlotes medicinalis*. The abdomen is marked with a series of oblique light spots in pairs like *Amaurobius*. On the under side the abdomen is light in the middle; the coxæ are light, and the sternum is light in the middle and dark at the sides. The light color turns yellow by long keeping in alcohol.

The male palpi have two processes on the tibia—one on the upper side turned outward and sharp pointed, the other on the outer side about half as long, stout, and directed forward. The palpal organ is large, extending backward beyond the base of the tarsus. The tube begins at the hinder end, extends around the inner side and ends in the groove of a thick process on the outer side.

Adult males and females half-way up Mt. Washington, June 10. Females Stow, Vt., July 29, Miss Bryant. Young males under leaves Jackson, N. H., in February.

**Hahnia brunnea,** new. (Plate VIII, figure 5.)

A single female from Clarendon Hills maple swamp is 3 mm. long, three-fourths the size of *agilis*. The proportions of the body, the eye arrangement and the shape of the sternum and maxillæ are the same as in *agilis*. The opening of the trachea is midway between the epigynum and spinnerets, not as far forward as in *agilis*. The spinnerets are in a line, with the lateral pair slightly larger than the others as in *agilis*, but the spinnerets are closer together, the middle pair almost touching. The lateral spinnerets are shorter than in *agilis*, being a third the length of the abdomen, while in *agilis* they are half as long as the abdomen. The epigynum is shaped much as in *agilis*, but on each side there is a brown loop under the skin that does not show in *agilis*. The color is light brown, the legs without rings or markings. The cephalo-

thorax is light brown, a little lighter than the abdomen. The abdomen is marked by a middle row of five pairs of oblique light spots, and the front pair of muscular spots is not conspicuous as it is in *agilis*.

Hanover, N. H., C. M. Weed in N. Banks' collection.

**Phidippus albomaculatus,** Keys. Zool. bot. ges. Wien, 1885.
**P. mystaceus,** Em. Trans. Conn. Acad., 1891.
**P. incertus,** Pkm., 1901 from Texas is thought to be the *Attus mystaceus* of Hentz.

**Phidippus brunneus,** Em. Trans. Conn. Acad.. 1891. (Plate XI, figure 1.)
Male a little smaller than the female, and the same general color. The cephalothorax is darker than in the female, and the abdomen covered on the upper side with dull yellow hairs. The legs are darker than those of the female. The mandibles are iridescent green. The male palpal organ is short and wide at the base, and the tube is stout and with a double bend: Pl. XI. fig. 1.

Found at the same time with females at Hyde Park. Mass.. May 2, 1903.

**Phidippus Whitmani.** (Plate XI. figure 5 and Plate XII. figure 1.)
The male of this species is very distinctly marked. It is about 8 mm. in length, larger than most males of *multiformis*, Pl. XII. fig. 1. The cephalothorax and abdomen are red, in some individuals inclining to orange. There is a distinct black band across the front of the head behind the eyes and as wide as the largest eyes. There is a narrow yellow band around the front of the abdomen, and in some individuals two pairs of indistinct yellow spots near the hinder end, but in others the whole back of the abdomen is red without any spots. In alcohol the spots are more distinct, and another pair of spots often shows in front of the others. The legs and palpi are gray with irregular dark and light spots obscured by long hairs. In alcohol the femora are dark and the other joints have dark rings at the end. The palpal organ is long and narrow, the bulb extending backward the whole length of the tibia. Pl. XI, fig. 5. The writer does not know the female.

Sharon, Mass. Three Mile Island, N. H.

**Phidippus insignarius,** Koch. 1846. (Plate XI, fig. 2 and 2 d.)

The male is described by Peckham as the male of **Phidippus comatus** in Trans. Wisconsin Acad., April, 1901.

Male 8 mm. long; cephalothorax black with two wide white stripes beginning below the lateral eyes in front, and turning upward behind, where they nearly meet under the front of the abdomen. There are two pairs of tufts of long black hairs at the sides of the head. The abdomen is orange red with black and white markings; there is a white stripe around the front, and a scolloped black middle band including a middle orange spot. and two smaller orange spots in front of it. The ornamentation of the face and front legs is striking and complicated. The lateral white stripes extend around under the front eyes as far as the middle pair, but do not meet under them, and below these are long white hairs that cross each other and nearly cover the mandibles, so that their iridescent blue color is concealed. The palpi are white, with a little mixture of brown. The first legs are covered on the under side with long white hairs; the hairs of the coxæ point downward, nearly to the ground; the femur has a row of stiff white hairs as long as its diameter along the outer side, and the other joints have hairs extending more than their diameter each side to the ends of the tarsi. When the first legs are pointed upward, the whole front appears white except the upper part of the head, which is black, extending outward at the sides in four black tufts. When the first legs are down in walking position, the upper side becomes visible in front, and this is covered with black hairs at the sides and, as far back as the patella, with a middle stripe of orange. The second leg is striped in the same way, but not as brightly, and has shorter white hairs.

The female is a little larger than the male, and marked on the back less distinctly in the same way. The cephalothorax is brown with lateral white stripes and tufts of long hairs on the head as in the male. The abdomen is light and dark brown with gray hairs; there is a white stripe around the front end and a square white spot in the middle. The dark middle band is broken into two pairs of black spots in the front half. The epigynum has a small notch in the hinder edge and two anterior openings close together separated only by a narrow ridge.

**Dendryphantes Jeffersoni,** new. (Plate XI, figures 3 and 3 e.)

Males 4 mm. long. Color brown mixed with white and yellow. The cephalothorax has the usual white stripes at the sides that

connect in front with a large white patch extending backward in the middle nearly as far as the dorsal eyes. The abdomen is marked with a front white band and five or six pairs of white spots extending forward on their inner corners. The legs are ringed with white at the ends of the joints. In alcohol the white disappears and the abdomen appears marked with a series of black spots on a light ground. The first legs are 5.5 mm. long, with the tibia a little thickened. The palpus of the male differs but little from that of *capitatus* and *flavipedes*. The bulb is wide at the base and more nearly square than in *capitatus*. The tube resembles that of *flavipedes* in having a long process parallel to it, but both are curved in a half circle, fig. 3.

Two males were found in the moss near Spalding's Spring on the Mt. Washington range at a height of 5000 ft., July 6, 1904, and a female at the same place, July 4, 1907.

A female found in the same locality several years later is 7 mm. long and dark brown with light gray hairs without any distinct white or yellow marks. In alcohol the abdomen shows indistinctly light marks similar to those at *militaris*. The epigynum has the notch shallow and truncate and the two openings a little farther apart and more angular than in *militaris*.

**Dendryphantes flavipedes,** Pkm. Trans. Wisc. Acad., 1888. (Pl. XI, figures 4, 4 a.)

The males do not differ from the females as much as in *capitatus* and *militaris*. My specimens are 4 mm. in length. The cephalothorax is light brown as in female *capitatus*, with white longitudinal bands at the sides below the eyes widening behind. The abdomen has the dark middle area broken by three pairs of spots in the front half and three or four light chevrons behind. The dark area is less sharply defined than in the male *capitatus* and connects with several oblique rows of dark spots. The legs are not ringed as in the other species but pale and translucent with longitudinal dark lines on the inner side. One of the the males from Portland, Me., and others from Fitzwilliam, N. H., are light gray, almost as light as *Drassus saccatus* without any distinct markings on the back, but with fine distinct longitudinal black stripes on the legs. The male palpi are a little darker than the legs and the tarsi and the palpal organs resemble those of *D. capitatus*, except that the tip of the palpal organ is double, the tube having a slightly curved process longer than itself parallel with it on the outer side. The process

of the tibia is short and transverse, differing little from the same
part in the other species.

Long Island, Portland, Me., Sept. and Crawford Notch, N H,
July 4. Fitzwilliam, N. H. in July.

**Eris nervosus,** Pkm.  Wisconsin Academy, 1888.
**Zygoballus terrestris,** Emerton.  N. E. Attidae, 1891.

**Icius similis,** Bks.  1895.  Colorado.
**I. elegans,** dark variety, Em.  Conn. Acad., 1891.

This species is described in New England Attidae as a variety
of *Icius elegans*.  The colors are not as brilliant, and it does not
have the tufts on the front legs or such large tufts over the eyes.
The palpal organs also differ slightly from those of *elegans* as figured
in N. E. Attidae.

**Icius formicarius,** Em.  New Eng. Attidae, Trans. Conn. Acad., 1891.
    (Plate XI. figures 8, 8 a.)

The male of this species has been found by Miss E. B. Bryant,
July 3, 1904, at Allston, Mass., near Boston.  It resembles the female
in form and color, and has no tufts on the head and no peculiar
modifications of the front legs.  It is 4.5 mm. long.  The male palpi
resemble those of the other species of *Icius*; the patella and tibia
are both very short, the tibia shorter than it is wide, and having
a process on the outer side longer than the rest of the tibia.  The
palpal organ has the same general shape as in *elegans* and *Hartii*,
but is a little more elongated, and the tube is a little more slender.
In the same neighborhood with this male, a female 6 mm. long
was found under a stone with a cocoon of eggs.

**Mævia tibialis,** Koch.  1848.  XIV, p. 78.
**Admestina Wheeleri,** Pkm.  Trans. Wisconsin Acad., 1888.  (Plate
    XI, figures 6, 6 a.)

The female is 4 mm. long, the cephalothorax 1.5 mm.  The
cephalothorax is one-half longer than wide,—a little the widest across
the hinder half and flat on the top.  The abdomen is oval,—widest
across the middle.  The spinnerets are long, the third pair extending
their whole length behind the abdomen.  The legs are short, the
first pair a little thickened, and as long as the cephalothorax.  The
sternum is one-half longer than wide and pointed at the posterior
end: it is narrow in front, but does not extend beyond the first
coxæ.  The cephalothorax is covered with white hair but in alcohol

appears black.    The abdomen is white with a middle gray band
broken at the edges by spots and indentations.    The legs are white
with black spots at the ends of the joints.

The epigynum is large for so small a spider and is at the end
of the first third of the abdomen.    It has two large spermathecæ
that show through the skin, and two small openings in front of them.

**Hyctia Pikei,** Pkm.    Trans. Wisconsin Acad., 1888.    (Plate XI,
    figures 7, 7 c.)

Cephalothorax and abdomen both elongated and narrow, whole
length 6 to 8 mm., cephalothorax 2.5 to 3 mm.    Abdomen 1.5 to
2 mm. wide; cephalothorax two-thirds as wide as long, a little
wider in males than in females.    The second, third and fourth legs
are short and slender, but the first pair are thickened in both sexes,
in the females twice as long as the cephalothorax, and in the males
longer.    The color is light gray with brown markings.    In females
the cephalothorax has three light brown longitudinal stripes, two
extending the whole length from the lateral eyes and a middle
stripe on the hinder half only.    The abdomen has three fine stripes
or rows of spots, sometimes forming a broken wide middle stripe.
In males the whole middle of the abdomen has a wide brown
middle band partly divided into triangular spots.    Young individuals
sometimes have no markings at all and are greenish in color like
the sand grass in which they live.    When approaching the female
the male raises his front legs stiffly upward at an angle of sixty
degrees with each other, and lifts the abdomen slightly, walking
on the six short legs.

The sternum is half as wide as long and pointed at both ends
and the first and fourth coxæ are close together and may touch
each other.    The epigynum has a simple oval opening with a
thickened edge in front.    The male palpi are very short; the patella
is as long as wide, and the tarsus is shorter, but with a thick
pointed process on the outer side. as long as the rest of the tibia.
The tarsus is curved downward and has a ridge along the outer
side, the part below which is smooth, with few and short hairs.
The bulb of the palpal organ projects at the base in a long blunt point.

Common on sand grass along the sea shore.

**Pellenes viridipes,** Hentz.
**Pellenes Howardi,** Pkm.    Bull. Wisconsin Nat. Hist. Soc., Oct., 1900.
**Attus viridipes,** Hentz.    Boston Journal Nat. Hist. 1846.    (Plate XII.
    figures 5, 5 a.)

The male is 5 mm. long with the cephalothorax 3 mm. long. The colors are bright and the markings of the back sharply defined. The first legs are light, transparent green, and the other light portions pale fawn color. The green fades entirely in alcohol. The top of the head covering the whole area between the eyes is orange brown, and the dark markings are dark brown, almost black. The pattern can best be seen in the figures. The front legs have a narrow stripe of fawn color on the upper side dividing the green, the other legs and palpi are fawn color, broken along the sides by dark scales, forming parts of rings at the ends of the joints. The three inner spines of the front tibia are dark colored and flattened and two of them are long and spatulate, showing distinctly beyond the hairs. The patella of the third leg is widened and flattened and has a black and white eye spot and a black border under the eye spot, and just over the joint is a spine slightly turned up at the end. When the third legs are drawn up in the usual standing position, the modified patellæ show in front over the head. The face below the eyes is for a short distance dark brown and below this white. The mandibles are also white on the front.

The female is slightly larger than the male, but the cephalothorax smaller. The color is dull orange brown, at first sight uniform, but showing indistinctly the same light and dark pattern as the male.

The males mature about the first of May and the females a little later. The females are usually found under stones and the males moving about in dry paths in the woods in Hyde Park and Sharon, Mass. It has been found at several places across the country as far as California.

**Pellenes roseus.**
**Attus roseus,** Hentz. Journal Boston Soc. Nat. Hist. 1846. (Plate
   XII. figure 4.)

Male 4 mm. long, cephalothorax 2 mm. Neither the first or third legs are modified or ornamented. The cephalothorax and the front of the abdomen are bluish white and covered with fine short scales. The rest of the back of the abdomen is light pink, with a metallic lustre. The legs and palpi are thinly covered with white scales, and the color is modified by dark hairs and the yellow of the skin. The face and mandibles are covered with white scales, the mandibles indistinctly striped with black.

The female is the same size as the male and resembles the female of *splendens.* The cephalothorax is covered with light gray scales mixed with darker hairs. The abdomen is light fawn color and

black.   There is a light band each side and one across the front
of the abdomen.   There is also a light middle band indented at
the sides, extending forward from the spinnerets two-thirds the
length of the abdomen.   The legs are light gray without any mark-
ings.   On the under side of the abdomen there are three dark lines.

Ipswich, May 20, 1893, in an open field near the shore.   Specim-
ens from New York State were found and sent to Mr. Peckham
at about the same time.


**Pellenes agilis,** Banks.   Ent. Soc. N. Y. 1892.
**Pellenes auratus,** Pkm.   Bull. Wisc. Nat. Hist. Soc., Oct., 1900.
    (Plate XII, figures 3, 3 a, 3 b.)
5—6 mm. long, the cephalothorax 3 mm. long.   The female
is covered with bluish gray hairs, through which can be seen indis-
tinct white markings on the abdomen and dark gray at the ends
of the joints of the legs.   In alcohol the light gray color disappears,
and dingy gray and brown take its place on which the white and
dark markings show more distinctly.   The male is brightly marked
with black and white.   The cephalothorax has a pair of white stripes
at the sides and another pair just above the lateral eyes extending
its whole length, and a white middle stripe from the front middle
eyes as far back as the posterior eyes.   The abdomen has lateral
and middle white stripes connected in front; the lateral stripes are
broken in their hinder half into two white spots, and the middle
stripe is sometimes broken into spots at the end.   The second,
third, and fourth legs are irregularly ringed with gray and white.
but the first pair are highly ornamented with long black hairs and
white spots, Pl. XII, fig. 3 a. 3 b.   The first leg has the femur black with
short hairs like the other legs, the patella white with a crest of
white hairs above and long black hairs below, the tibia black with
a white spot on the upper side near the end, and long black hairs
above and below, the metatarsus and tarsus white.   The third legs
have no peculiar modifications of the patella or tibia.   The palpi
have the tarsus black and the patella white.

In marsh grass and under sticks and stones along the shore,
Ipswich, Mass., Long Island, New York.

Males and females mature about August 1.   In dancing before
the female, the male holds the front legs out sidewise with the
tibia nearly horizontal and the tarsus turned downward, and walking
on the other six legs, approaches her by short quick steps without
much movement from side to side until near enough to touch her
and then quickly retreats.

**Pellenes borealis,** Banks. 1895.
**Habrocestum cristatum,** Pkm. Attidae of N. A., 1883. (Plate XII. figures 4 to 4 c.)

The female is 5—6 mm. long, the male 4.5—5 mm. The female is light gray and brown like the sand, while the male is deep black with white markings. The legs of the male have no peculiar modifications either of the first or third pairs. The markings of the female are very indistinct; the cephalothorax is varied with white, sometimes suggesting two white lines from the lateral eyes backward. The abdomen has a white line across the front and two pairs of short lines at the sides. Toward the end there are two middle spots, sometimes connected, and the usual two small white spots just in front of the spinnerets. The male has the cephalothorax black with long black hairs on the front of the head. The abdomen is black and has the same markings as the female, but much whiter and more distinct. The legs are pale, but the color is darkened by black hairs. The face below the eyes is white in the female, and in the adult male is thinly covered with small white scales, but in the young male before the last moult, this part is bright red, so that it may be mistaken for the young of *P. ca-catus*, which lives farther south. See Psyche, Journal of Cambridge Ent. Club, Vol. II, p. 82, April, 1904.

The epigynum has a large oval anterior opening extending backward at the sides almost as far as the posterior opening. The palpal organ is oval and has a stout supporter of the tube extending along the inner side and but little narrowed toward the end.

This spider is very common along sea beaches in the dry grass and rubbish thrown up by the tide. Adults are found most abundantly about the first of May, but some of them mature in the late summer as early as the last of August. The red-faced young males are found in the summer and fall, and in spring as late as June.

**Chalcoscirtus montanus.**
**Icius montanus,** Banks. Can. Ent., 1896.

The cephalothorax is 1.2 mm. long, the abdomen of the male about the same length, and that of the female longer. The cephalothorax is two-thirds as wide as long, a little flattened above and with the sides nearly straight and parallel. The posterior eyes are half as far from the front eyes as they are from each other and the middle eyes are slightly nearer the posterior than the front eyes. The color differs in the sexes, the male being much darker than

the female. The male is dark brown, almost black, without any markings, and the abdomen is slightly iridescent. The female has the cephalothorax dark brown and the abdomen light brown with pale herringbone markings like the female *Euophrys*. The legs of the female are pale. The fourth leg is longest in both sexes. The male palpi are short with the patella and tibia of equal length, the patella thicker than the tibia. The tarsus is oval and does not cover the bulb, which is thick at the base and extends backward under the tibia nearly its whole length. At the distal end of the bulb a small oval piece is constricted off and turned to one side. and at the tip of it is the small sharp tube. The epigynum re-sembles that of *Neon* and *Euophrys*.

Sifted from moss on the upper part of Mt. Washington range. July 4, 1907.

**Homalattus cyaneus,** Pkm. N. A. Spiders, Trans. Wisconsin Acad. Oct., 1888
**Attus cyaneus,** Hentz. (Plate XI, figure 9, 9 a.)

Female 4 mm. long and 1.5 mm. wide. The part of the cephalo-thorax showing in front of the abdomen is as wide as long, narrowed a little in front. The posterior eyes are very far back, two-thirds as far from the front of the head as they are from each other. The cephalothorax and abdomen are both flattened, and the front of the abdomen covers the cephalothorax about a quarter of its length. The color is metallic green, the cephalothorax roughened and covered with small scales nearly as wide as long, and the ab-domen with small but longer scales.

New Haven, Conn. and Sharon. Mass. under shingle of a barn.

**Peckhamia picata.**
**Synemosyna picata,** Hentz. Journal Boston Soc. Nat. Hist. 1846. (Plate XII, figures 7, 7 a, 9 b.)

This species continues to be rarely found in New England. Adult males and females were found in May. 1906, at Three-Mile Island, Lake Winnipesaukee, N. H., and adult females in July at the same place. They lived on a dry hillside among dead leaves on the ground and were seen walking slowly in and out among the leaves, resembling ants of the same size and color that were wandering over the same neighborhood. · The male figured was 4 mm. long. The dancing of the male of this species before the female has been described by Peckham in the Occasional Papers of the Nat. Hist. Soc. of Milwaukee, Vol. 2, 1892.

**Peckhamia scorpionia.**
**Synemosyna scorpionia,** Hentz. Boston Journal Nat Hist. 1846.
(Plate XII, figures 6, 6 a.)

This little spider was found at New Haven, Conn., in 1883 but was overlooked at the time of publication of the N. E. Attidae. Since then it has been found in considerable numbers at Cold Spring Harbor, Long Island, N. Y., and at Cambridge and Ipswich, Mass., always on fences on which it runs about slowly and irregularly like an ant. When threatened it flattens itself against the wood, holding on so tightly that it is hard to pick it up without injury. The males mature about June 1 and when confined with females dance before them much like *picata,* holding the abdomen up vertically and swinging it toward the advancing side and sometimes turning the feet of that side under the body. The front legs are not turned forward as much as in *picata.*

The females are about 3 mm. long, the males 2 to 2.5 mm. The cephalothorax is twice as long as wide, and widest across the hinder third. The posterior eyes are farther back than the middle of the cephalothorax. The abdomen is oval, slightly widest behind, and both it and the cephalothorax are flattened on the upper side and without any constrictions or indentation.

The legs are short and slender and the first pair thickened in both sexes. The color is dull brown and gray with pale markings. On the cephalothorax there is a transverse light spot just behind the eyes. On the abdomen there are two white stripes across the middle and between them two light spots connected with the anterior band. The space between the light bands is slightly paler than the rest of the abdomen. The legs are pale with a dark longitudinal stripe on the front side. The femora are darkened a little in the first, and less in the second and fourth pairs. In the male the first legs have the tibia and patella thickened as well as the femur, but not flattened on the upper side as they are in *picata.*

**Myrmarachne albocinctus,** Koch.
**Salticus albocinctus,** Koch. 1846. Vol. XIII p. 36.
**Salticus ephippiatus,** Em. Trans. Conn. Acad. 1891,

It is doubtful if this is the **Synemosyna ephippiata** of Hentz neither his description nor figure show the thickened palpi.

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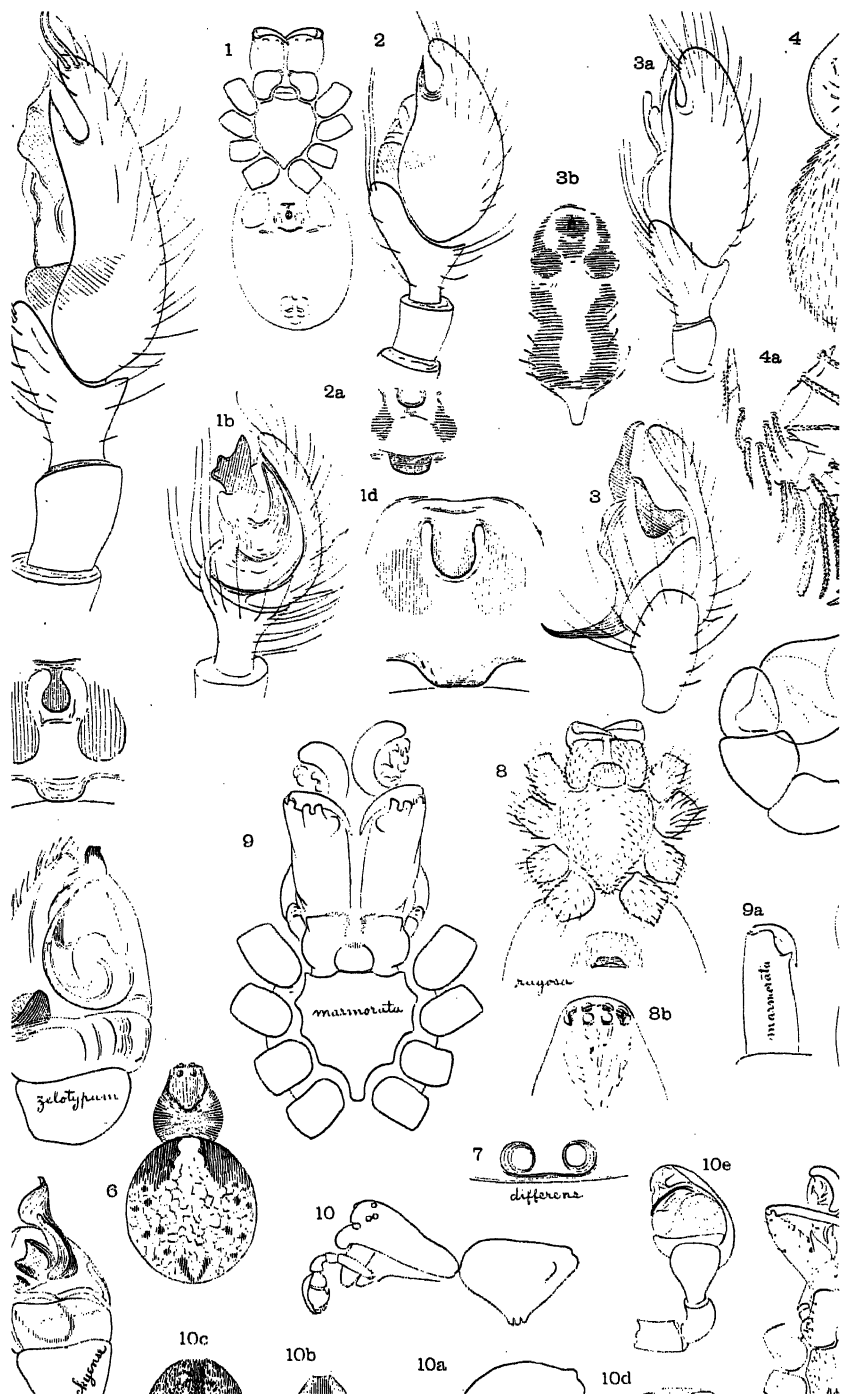
1  1b  1d  2  2a  3a  3b  3  4  4a  *marmorata*  *zelotypum*  *rugosa*  *marmorata*  9  9a  8  8b  6  7  *differens*  10  10a  10b  10c  10d  10e  *hymen*

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anu

nunn

3c

2p          aurum

uni

tatum

+a          6          6b

+ ititutus

6

nt m

eat

na

palludum

ia hten

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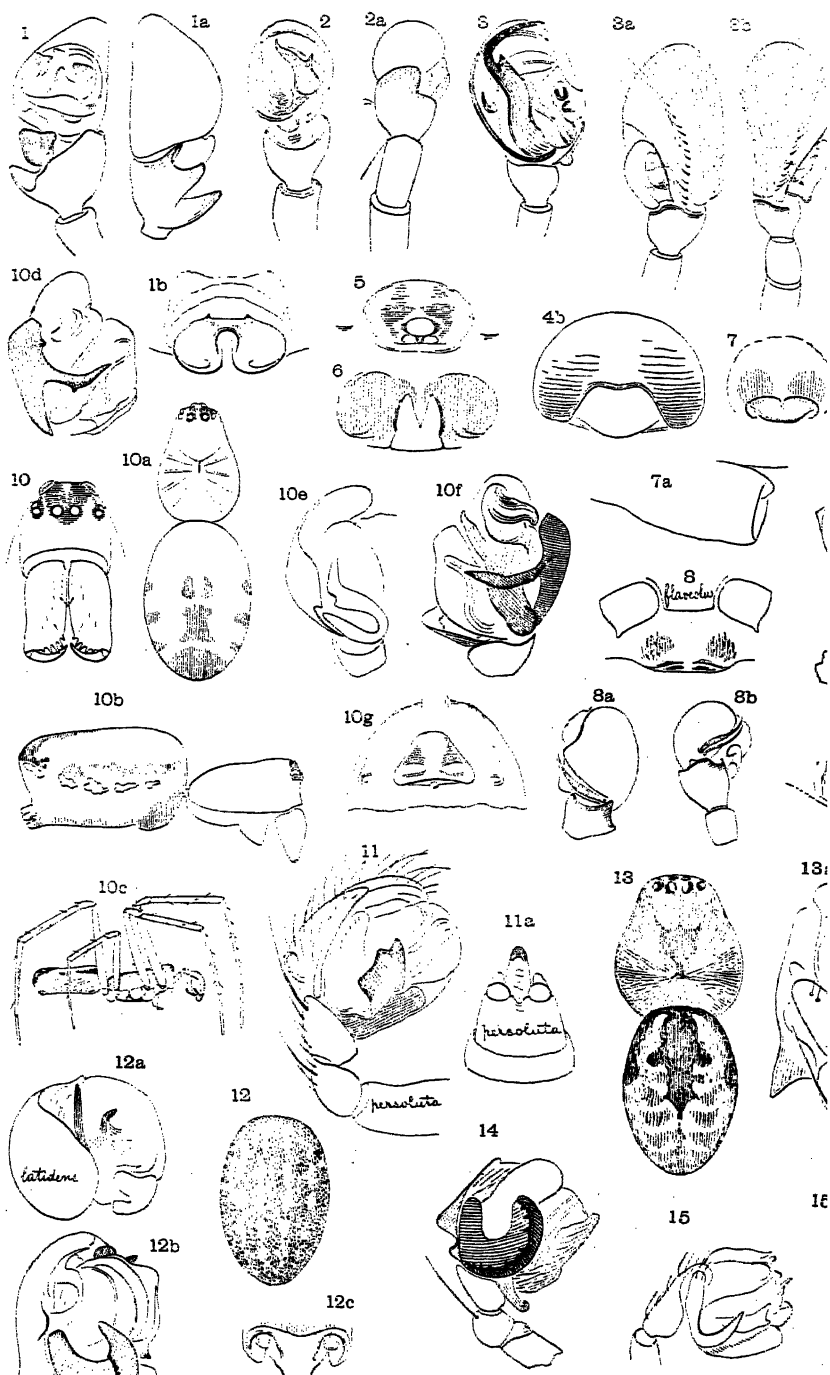
1  1a  2  2a  3  3a  3b

10d  1b  5  4b  7

10a  6

10  10e  10f  7a

8  flavestris

10b  10g  8a  8b

10c  11  11a  13  13a

persoluta

12a  12  11  persoluta  14

latidens  12b  12c  15

# PLATE V

1

2

4b   notata u.

4c   atrica

4d   x notata

1a

Thael u.

# PLATE VI

# PLATE VII
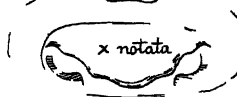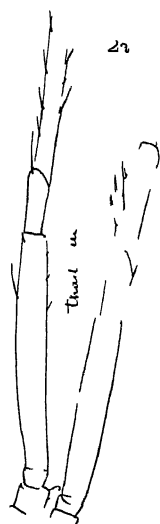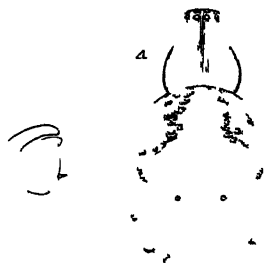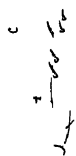
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1b

1

1c

3d

3

3e

pubes ♂

nidifex ♂

pubes ♀

Ctisonaria

retrum

1a

2

3b

3c

nidifex

nidifex

vara

2a

7a

4d

4

striata

6d

7

6

punctulaty

6

5

trifunculatus

bispunctatus

7d

6b

7c
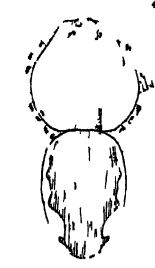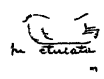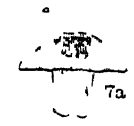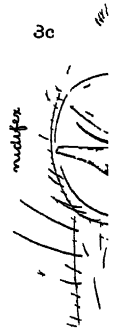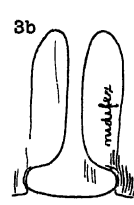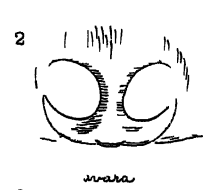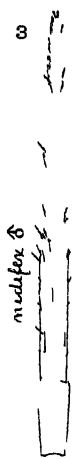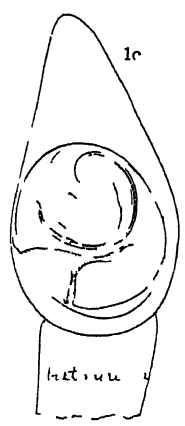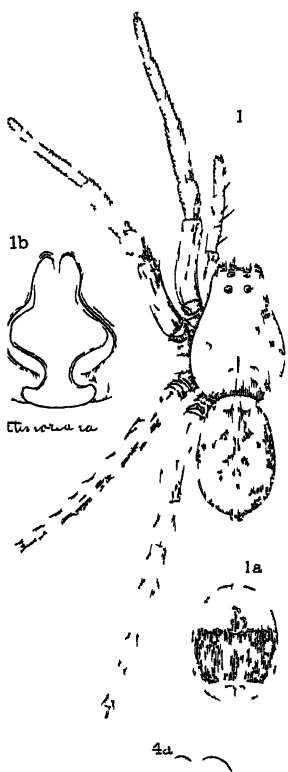
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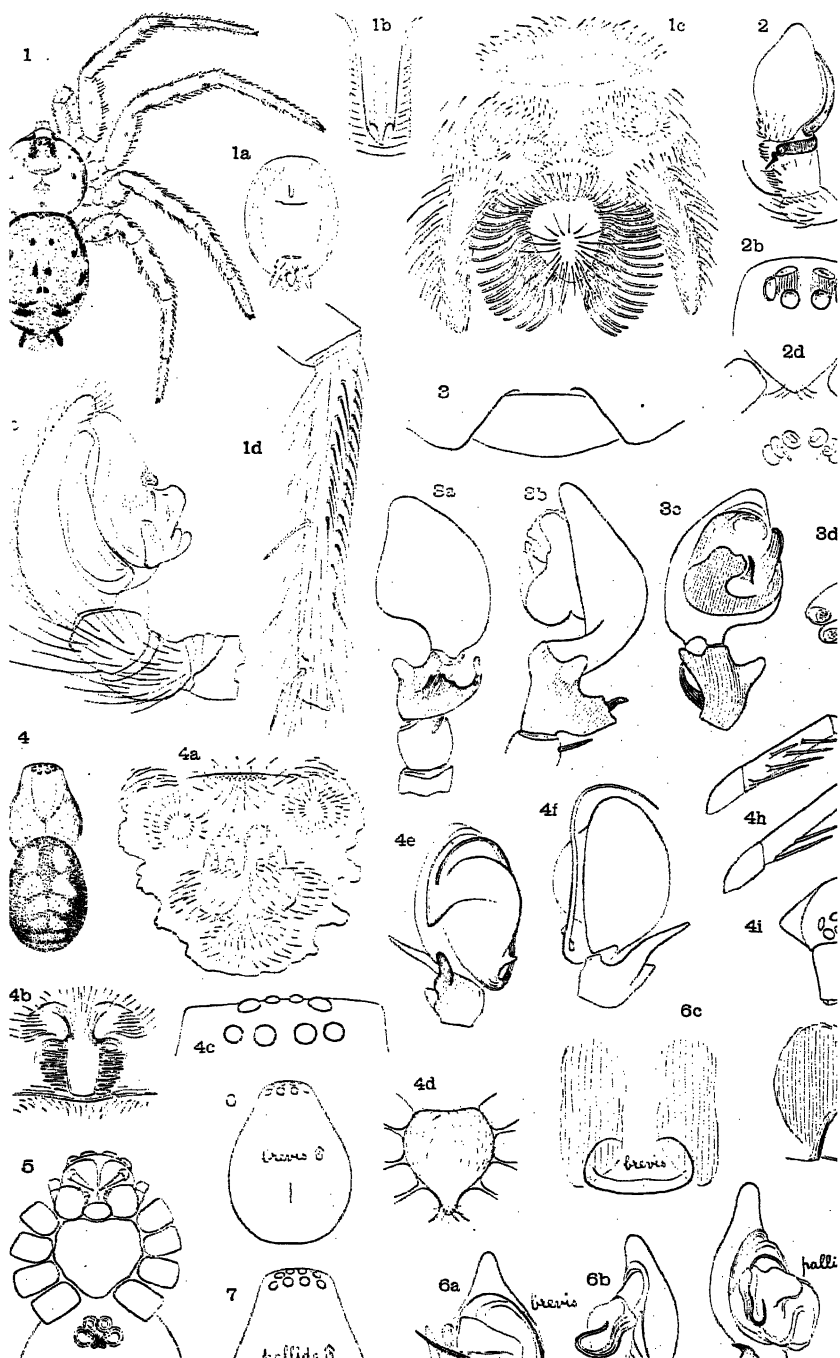
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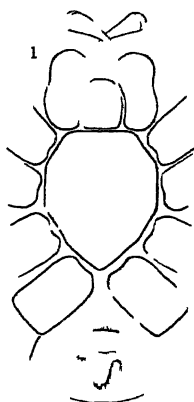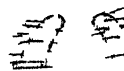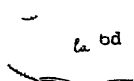
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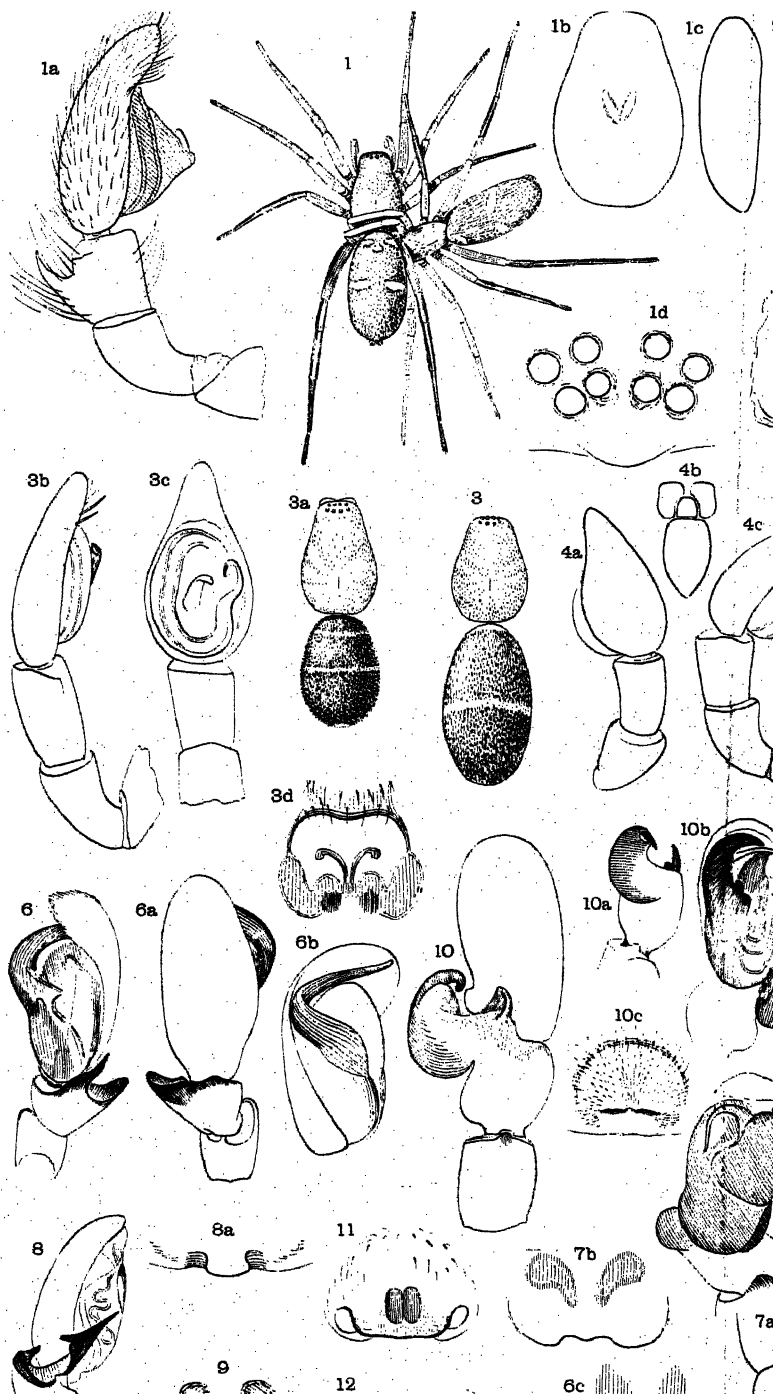
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2

2a

2c

2b

2d

brunneus

3

3a

4

3c

3e

militaris

3b

3d

jeffersoni

jeffersoni

flav: pedes

6

6a

7

7a

7b

8a

7c

8
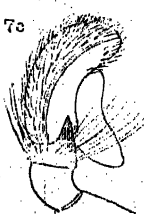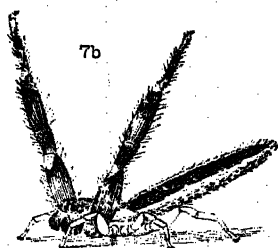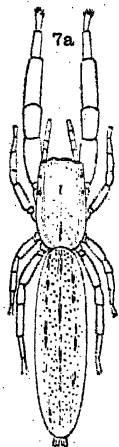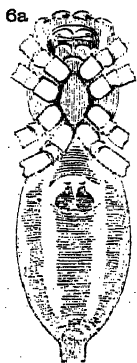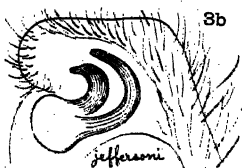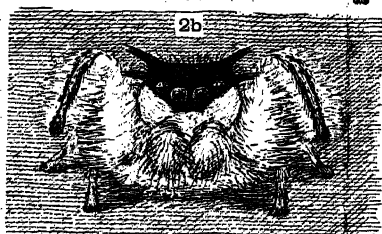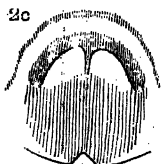
# PLATE XII

Figure 1.—*Phidippus Whitmani*, male.

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Figure 9. - *Chalcoscirtus montanus*, female. 9a male. 9b male palpus.
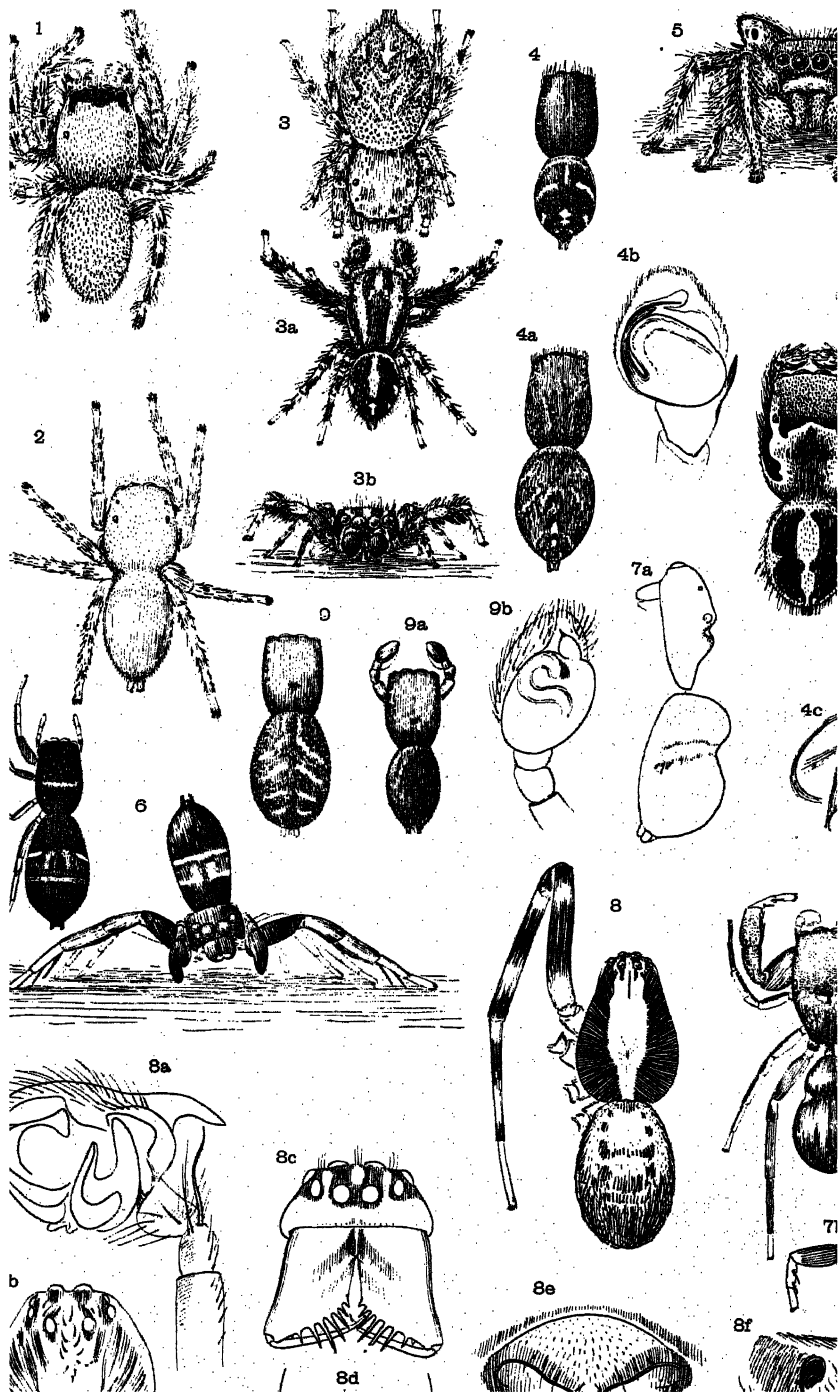
# The Poems of Thomas Third Lord Fairfax

## From MS. Fairfax 40

In the Bodleian Library, Oxford    .

BY

EDWARD BLISS REED

NEW HAVEN, CONNECTICUT

1909

## IV. The Poems of Thomas Third Lord Fairfax.

In the annals of England the name of Thomas, third Lord Fair-
fax, is deservedly illustrious. As a general, he was an intrepid
fighter and a skilful commander; in his private life, a man of
scholarly tastes, happy in his country estates, which he preferred
to the court. Policy and self-advancement were far from his thoughts,
despite his great opportunities for aggrandizement; and the simplic-
ity of his character, at which his enemies sneered, was but a proof
of his sincerity. To sketch his life in detail is unnecessary, yet his
poems will gain significance if, in the briefest manner, we review
his interesting career.

The son of Fernandino, second Lord Fairfax, and Mary, daughter
of Lord Sheffield, he was born at Denton, Yorkshire, in 1612, of
a family long distinguished for its soldierly qualities. In 1620 his
grand-father, Thomas, first Lord Fairfax, then a man of sixty, joined,
with two of his sons, the single regiment sent by James I to the
support of the Elector of the Palatinate. He was obliged to return
to England to take part in the Parliamentary elections, but his two
sons died at Frankenthal at the head of their troops. Fernandino
did not make this campaign, and his father spoke of him as a
"tolerable country justice, but a mere coward at fighting"[1]; yet
Fernandino took the field against Charles I, and certainly did not
deserve this taunt.

The early years of our poet were spent in Yorkshire, and he un-
doubtedly enjoyed in his first studies the guidance of his great
uncle, Edward Fairfax, the translator of Tasso. In 1626 he entered
St. John's College, Cambridge, where he remained four years, and
then, following the family traditions, he went to the Low Countries,
to serve under Lord Vere against the Spaniards. Another young
volunteer in the same camp was Turenne. After witnessing the
capture of Bois-le-Duc, he traveled and studied in France for eighteen
months, returned to England in 1632, and requested permission to
volunteer under Gustavus Adolphus, but his family opposed it, and
he retired to the Yorkshire estates to live the life of a country gentle-

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[1] *A Life of the Great Lord Fairfax*, by Clements R. Markham, London,
1870, p. 12

man. In 1637 he married the daughter of his commander, Anne Vere, a woman of energy and courage, who followed her husband to the field, shared his dangers (she was once taken prisoner by the Royalists) and, in no small measure, determined his career.[1]

In the two brief and inglorious Scottish compaigns, Fairfax joined the King's army, but when in 1642 Charles came to Yorkshire to seize the supplies at Hull, and raise troops against Parliament, the Yorkshire gentry who opposed the King looked to Fairfax for leadership. He was entrusted with a formal protest against the King's actions, and, despite repulses, succeeded in laying this document on the royal saddle at Heyworth Moor, where Charles was endeavoring to win over the gentry of the shire. Fairfax thus showed the world on which side he would be found, and in the Yorkshire campaign that followed, he fought with the greatest courage. Undaunted by defeat, fearing no odds, on at least one occasion he attacked a force that outnumbered his own by four to one. When surrounded, he cut his way through the enemy. At Marston Moor he found himself carried by the tide of battle into the thick of the enemy's ranks. Taking from his hat the white badge worn by the Parliamentary forces, he calmly rode through the ranks of the Royalists, regained his troops, and led another attack.[2] So fearless was he that on several occasions he narrowly escaped death. In 1644 a musket-ball pierced his shoulder, another broke his arm. Hardly recovered from these wounds, he was again struck at the siege of Pomfret Castle. His skill as a leader, his bravery in action, had attracted the attention of all England, and in 1645, when but thirty-three years of age, he was made Commander-in-chief of the Parliamentary forces, having as his first task the organization of the New Model army. While in the popular opinion it was Cromwell who was "the leading spirit of the war," to quote Sir Clements Markham, the biographer of Fairfax, "it was Fairfax who organized the new army without the smallest assistance from Cromwell. It was Fairfax whose genius won the fight at Naseby, and whose consummate generalship concluded the war, and restored peace. Cromwell was his very efficient general of horse, but nothing more : and indeed he was generally employed on detached duties of secondary importance."[3] At Naseby, Fairfax was conspicuous for his daring; at the surrender of Oxford, he placed a guard about the

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[1] *Ibid.,* p. 108.
[2] *Ibid.,* p. 171.
[3] *Ibid.,* Preface, p. iv.

Bodleian and saved it from destruction, as he had spared the minster at the siege of York.[1]

With Charles hopelessly defeated, Fairfax was unwilling to depose him, wishing the King to rule, with the constitution safeguarded from encroachments of the crown. He hotly resented the seizure of Charles by Joyce, and through his insistance Charles was allowed to see his friends, and above all, his children—a favor for which he repeatedly thanked Fairfax.[2] In the political intrigues which preceded the execution of Charles, Fairfax took no part; but when the Royalists made a last stand, he laid siege to Colchester, captured the town, and crushed the insurrection. It was at this time that Milton addressed to him his noble sonnet:

> Fairfax, whose name in arms through Europe rings,
> Filling each mouth with envy or with praise,
> And all her jealous monarchs with amaze,
> And rumours loud that daunt remotest kings,
> Thy firm, unshaken virtue ever brings
> Victory home, though new rebellions raise
> Their Hydra heads, and the false North displays
> Her broken league to imp their serpent wings.
> O yet a nobler task awaits thy hand
> (For what can war but endless war still breed?)
> Till truth and right from violence be freed,
> And public faith cleared from the shameful brand
> Of public fraud. In vain doth Valour bleed,
> While Avarice and Rapine share the land.

Though appointed one of the Commissioners to try the King, Fairfax refused to be present at the trial, and opposed it in vain. Surely there are few more dramatic moments in history than when Lady Fairfax rose in the gallery of Westminster Hall to protest against the trial, and to defend her husband's name. Indeed, so well known was Fairfax's opposition to the execution of the King that Cromwell accused the general of planning to rescue Charles.

In 1650 Lord Fairfax resigned his command, and returned to his estates at Nunappleton. On the death of Cromwell he decided that there would be anarchy unless Charles II returned and ruled. Lambert, with a disciplined army of ten thousand men, was on the

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[1] *Ibid.* p.271. Fairfax bequeathed to the Bodleian 28 manuscripts. See W. D. Macray: *Annals of the Bodleian Library, Oxford.*

[2] *Ibid.* pp. 290, 298.

field to oppose Monck, who, with an army of seven thousand, was on the point of declaring for Charles. Though ill and suffering intensely, Fairfax sent word to Monck that he would take the field in support of Charles. When he appeared, Lambert's troops deserted and flocked to their old commander, and thus, without a shot being fired, the Restoration was accomplished. It was fitting that Lord Fairfax should head the commission sent by Parliament to the Hague to invite Charles to return. No honors were conferred on him by the Merry Monarch—he sought none—and he retired to Yorkshire, where he died November 12, 1671, three years before the death of Milton.

It is not surprising that the letters of Fairfax, and his two *Short Memorials of the War*, should have been published, but it is strange indeed that a manuscript of 656 pages of verse, all in his own handwriting, should never have been carefully examined. This manuscript passed from the possession of the Fairfax family, and was owned successively by Ralph Thoresby, the Duke of Sussex, and Dr. Bliss of Oxford, from whose collection the Bodleian library, its present owner, purchased it in 1858. Archbishop Cotton, in his *Editions of the Bible and Parts thereof in English from the year MDV to MDCCCL,* Preface to the second edition, 1852, gave a table of contents of the manuscript, then in the possession of Dr. Bliss, and reprinted one of the paraphrases of the Psalms. Sir Clements Markham, in his *Life of Fairfax*, already cited, went further; for in the text of his work he reprinted three of Fairfax's poems,[1] and in an appendix gave ten more, wholly or in part, but as a historian, interested in the political, and not the literary life of the times, he made no study of the manuscript. Since Markham, I can not find that any one has examined these poems or published them.

We have no means of dating the poems, with the exception of the following:

*Upon the New-built House at Apleton* (1650), *To the Lady Cary upon her Verses on my deare Wife* (1665), *On the Fatal Day* (1649), *Upon the Horse which his Majestie Rode upon att his Coronation*

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[1] *Life of Fairfax*, p. 352: *On the Fatal Day, Jan. 30, 1648*; p. 365, *Upon the New-built House at Apleton*: p. 384, *Upon the Horse which his Majestie Rode upon Att his Coronation*. Appendix A, pp. 415—427 contains the following: *Preface to the Psalms, Honny dropps* (excerpts), *The Solitude, The Christian Warfare* (excerpts), *Life and Death Compared together, Shortness of Life, Of Beauty, Upon a Patch Face, Upon an ill Husband*, and two of the *Vulgar Proverbs*.

(1660).[1] As these poems are written down in this order, it will be seen that their position gives no clue to the time of their composition, indeed, the very last poem in the manuscript is an eclogue, *Hermes and Lycaon*, by Edward Fairfax, who died in 1635.[2] If we refer Fairfax's translations from "good old Mantuan" to his student days, the poems certainly cover a period of forty years.

A perusal of the manuscript shows us at once that Fairfax is not a poet, but rather a man of poetic tastes, an admirer of verse. We have, then, no discovery of a neglected genius, and there will be no call for the Complete Works of Thomas Fairfax. It will occasion no surprise, therefore, that we have omitted a considerable amount of his poetry.[3] It will readily be seen that the chief defect in these poems is their poor technique. Fairfax had very little sense of rhythm; at times his ear seems absolutely untrained, and, though a multitude of corrections in the manuscript show how hard he struggled to improve his lines, yet his revisions are generally as awkward as his first rude draft. Few of his poems have any metrical charm, and when in his *Honey Drops* or *Vulgar Proverbs* he seeks to become epigrammatical, he lacks both point and finish. His best writing is seen in such a poem as *David's Lamentation*, or in the straightforward couplets of the *Christian Warfare*; however, it is not for his skill as a writer that Fairfax deserves attention, but for certain conclusions that may be drawn from the subject-matter of his lines.

Fairfax divided his poetry into religious and secular verse, the former occupying 551 pages out of 650, 388 of these being devoted to a metrical paraphrase of the Psalms. From the days of Wyatt and Surrey in England and Clément Marot in France, to "translate" the Psalms, or indeed to turn any part of the Scriptures into verse, was a pastime indulged in alike by the devout and by the profligate. A complete list of English writers who from 1500 to 1700 made metrical versions of any portion of the Bible has never been compiled. It would be a surprisingly large one, and, though Fairfax was a devout man, he was following a literary fashion as well as his own inclination in his paraphrase which offers so little that is

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[1] The *Epitaph on A. V. dieing Younge* might be dated, were we sure that V. stands for Vere.

[2] As Markham published this in *Miscellanies of the Philobiblon Society*, vol. 12, 1868—9, I have not reprinted it.

[3] See table of contents of the MS. on page 249. With the exception of the Psalms, I have a copy of the whole MS. It is at the disposal of any one interested in it.

interesting that I have reprinted but four Psalms, enough to show his method.[1] In his hymns we notice most of all that he writes in an impersonal style, for we have in them no picture of his own mind, no account of his spiritual conflicts, his doubts, his defeats, or his victories. Religious verse is valuable in proportion as it shows us the soul of a man, and this is precisely what Fairfax does not attempt to do.

This same lack of the personal element in his writing is a marked defect of the secular verse also, for he gives us practically nothing of his own life, even in remote allusion. When we consider the great scenes he had witnessed, the part he had played in shaping the destinies of England, it is rather surprising that he should choose to write on *Envy, Temperance, Anger.* Surely he might have written with more spirit on Liberty, Tyranny, or Valor. He collects many popular proverbs, but he does not jot down the song of his soldiers. For a fighting man, how faint and unrealistic are such lines:

> As men besieged with mines about
> Ready to spring and ruing [*sic*] all,
> Hearing the alarm beat, runne out
> To th' assault and gard ther wall,
> And by the blast in ruins sinke
> Vanquist are when they least thinke.[2]

And yet they are quite unusual, so rarely does he refer to the shock of battle. As Fairfax does not tell us what he has felt, so he has little desire to write down what he has seen. Apart from all considerations of the immeasurable distance that separates Andrew Marvell's work from that of Fairfax, it is yet surprising that Marvell should describe Appleton House and the estates so fully, and that Fairfax, who delighted in them, should give us but a few faint lines on the new-built house. Similarly we should expect the sympathetic picture of the last moments of Charles to

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[1] Markham, in his *Life of Fairfax*, p. 369, mentions another copy of Fairfax's version of the Psalms, owned by Mr. Cartwright of Aynho. I have not attempted to trace this. At the end of the MS. of the *Short Memorial*, at Leeds Castle, are versions of the 18th, 24th, 30th, and 85th Psalms. He prefaces Psalm 18 with the following: 'That I chuse this 18 Psalm let none think that I arrogate anything to myself, for farre be it from me to applie it otherwise than as David's triumph over his enemies.' See Markham's *Fairfax*, p. 415.

[2] *A Hymne to Christ the Messiah.*

come from the pen of the general rather than from the tutor of his daughter.[1]

To observe for one's self, to describe one's feelings, demands a certain amount of originality, and this is precisely what Fairfax lacked. The greater part of his religious verse was paraphrase, and we naturally look for translation in his secular poems. Pages 602–10 of the manuscript are taken, he tells us, from the French, the Italian, the Latin. With the exception of the Mazarinades, all these translations are directed against Rome, showing his strong Puritan sympathies. It is interesting to notice that when he translates Petrarch he does not choose the sonnets to Laura, but *The Character of the Romish Church.*[2] Petrarchism, brought in by Wyatt and Surrey a century before, had spent its force, and the lyrics of Philip Ayres, 1687, fill the last book that shows the old sway of the founder of the modern lyric.[3] As confirmation of Fairfax's lack of skill in writing, it is noticeable that he is unable to reproduce the sonnet form, and turns the quatorzains into poems of twelve lines.

Eight pages of translation, however. constitute but a small part of his secular verse. As we read it, we are impressed by the contrasts it shows, contrasts that can not be explained by assuming that certain poems are separated by long intervals of time. Lady Carey had written to Fairfax a metrical epistle on the death of his wife, and he felt called upon to answer it. Knowing his devotion to Lady Fairfax, we expect him to rise above himself under the inspiration of his grief, but his thought is so trivial, and so feebly expressed, that *To the Lady Cary Upon her Verses on my deare Wife* is one of the poorest poems. A few lines will show this more plainly than any comment:

Madam
    Could I a Tribute of my thanks express
    As you have done in love and purer verse,
    On my best selfe then I might Justly raise
    Your Elogy t' Encomiums of your Prayse
    And soe forgett the Subject that did move
    Me to a thankfulness as 't did you to love.
    O 'twere to great a Crime but pray allow

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[1] See Marvell's *Horatian Ode upon Cromwell's Return from Ireland.*
[2] Sonnets, *De Vario Argumento*, Nos. 14 and 16.
[3] *Lyric Poems, made in Imitation of the Italians, London*, 1687.

Wher I fall short but you have reached to,
Making that Good wisest of Kings hath said,
Th' Living's not soe Prayse-worthy then [*sic*] the dead.

A few pages further on, we come to a more formal elegy on Henry of Navarre.

Ah is itt then Great Henry soe fam'd
For taming men, himself by death is tam'd!
Whatt eye his glory saw, now his sad doome,
But must desolve in Teares, sigh out his Soule,
Soe small a shred of Earth should him intombe
Whos acts deserv'd pocession of the whole.

Though this poem has its defects, it is, on the whole, a better piece of writing than the elegy on Lady Fairfax. This consideration, together with the fact that Henry of Navarre was assassinated two years before Fairfax was born, and that there seems to be no special reason why he should lament his death, makes one suspect that the lines are a translation from the French. Such is the case, for I find that the poem is taken word for word from an elegy by Anne de Rohan which Fairfax read at the end of Agrippa d'Aubigné's *Histoire Universelle*, published 1626, for d'Aubigné does not quote the whole poem, and Fairfax translates only as much as he gives.[1] With this hint I have looked in the French literature of the period for the originals of the other poems. *On a Fountain* is a translation of an epigram of Malherbe that was a favorite one,[2] to judge from its appearance in a French anthology (*Les Délices de la Poésie Française*, 1615), while Fairfax's best poem, the one that gives the manuscript its title, is a translation of Saint-Amant's *La Solitude*. Other sources I have not found, but I feel convinced that many of the poems are translations, as for example, *Of a Faire Wife, to Coregio*, which is probably taken from the Italian. Others better read in Continental literature of the period may discover his models.[3]

We are now in a position to see the significance of these poems. They are not fine examples of English verse; they are rather to be regarded as documents that show us what an English gentleman

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[1] *Histoire Universelle* par Agrippa d'Aubigné (Paris, 1879). Vol. 9, pp. 472–75.

[2] See *Oeuvres complètes* de Malherbe (Paris, 1862), Vol. 1, p. 225.

[3] Mr. Lewis C. Everard, Yale Phi Beta Kappa Fellow 1908—1909, has searched in the Bibliothèque Nationale, Paris, for other French originals, but without results.

of the Caroline and Commonwealth period read and thought. They are like an old diary in which a great man has jotted down a list of the books he owns, or of poems he has memorized; they are like a package of old letters, in which the writer tells us of his favorite authors and his literary tastes. It is to be observed that this moralist, who mentions but one English writer—his great-uncle—turns to French literature. *La Solitude* is certainly not only Saint-Amant's best piece of work, but one of the finest French poems of the period, and the evident admiration of Fairfax for it speaks well for his taste. Though Saint-Amant had twice visited London and was possibly known there as a poet, only two other unimportant translations of his verse have been noticed in English literature.[1] It is worthy of mention that Saint-Amant himself had some very pronounced opinions concerning Fairfax, who probably never read the Frenchman's *Epigramme Endiablée sur Fairfax.*[2]

There is another interesting point concerning *La Solitude.* It is well known that in 1650 Andrew Marvell came to Appleton House as a tutor for Mary Fairfax. He had already written verse, but it had not been nature-poetry; his grotesque *Flecnoe* and his absurd verses *Upon the Death of Lord Hastings* have nothing of the meadow

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[1] See A. H Upham, *The French Influence in English Literature from the Accession of Elizabeth to the Restoration*, New York, 1908, pp. 345, 405, 409, 412 It is interesting to read Saint-Amant's brief reference to Ben Jonson in his *L'Albion.*

[2]    Je crois qu'il doit bien estre en peine,
    L'execrable tyran qui preside aux enfers,
    Quand, dans les feux et dans les fers,
    Il songe au noir object des foudres de ma haine;
    Son vieux sceptre enfumé tremble en sa fiere main;
    Il redoute Fairfax, ce prodige inhumain;
      Il craint que ce monstre n'aspire
    Au degré le plus haut de son horrible empire,
    Le degré le plus haut est celuy le plus bas,
      C'est où ce prince des sabats,
    Des endroits les plus clairs aux endroits les plus sombres,
      Tomba pour regner sur les ombres;
    C'est la, dis-je, qu'il craint que par quelque attentat,
      Que par quelque moyen oblique,
    Fairfax n'aille du moins renverser son estat
      Pour en faire une republique.
    Et voila les raisons qui l'ont fait hesiter
      Jusqu'à cette heure à l'emporter.
*Oeuvres Complètes de Saint-Amant* (Paris, 1855), vol. 1, p. 472.

in them. During the two years he spent at the home of Fairfax, Marvell wrote those nature-poems that determined his fame—*Upon the Hill and Grove at Billborow, Upon Appleton House, On a Drop of Dew, The Garden*—poems that show an observation, an apprecation of the earth, of flowers, birds and trees unsurpassed in all the works of his predecessors in English poetry, not excepting the very greatest, Chaucer, Spenser, and Shakespeare. That these poems were inspired not only by the beauty of Nunappleton, but by its owner's love and appreciation of poetry, there can be little doubt. We may go even further, and see in Marvell's nature-poems some hints from Saint· Amant. Marvell's verse is richer and deeper; where Saint-Amant is vague in his descriptions or conventional in his thought, Marvell is concrete and original; for it is the Englishman, and not the Frenchman, who uses *le mot précis*, and yet Saint-Amant's theme—to lose one's self in Nature—is the theme of *The Garden* and of the finest lines in *Appleton House*.

We see now the significance of the poems of Fairfax. They throw light on the character of a great Englishman; they remind us that the literary influence of *la ville lumière* was still powerful in England, that it had not died with the sonneteers; and they give us the atmosphere in which Andrew Marvell lived and wrote the tenderest, the sincerest, the deepest nature-poetry of the seventeenth century.

Yale College,                               Edward Bliss Reed.
February 19, 1909.

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† Of a Faire Wife to Coregio.　p. 564.

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[p. ii]                    The Preface to the Psalmes.

Vaine Fancy whether now darst thou aspire
W^th smoky Coales to light the holy Fire
Could thou indeed as wth the Phenix burne
In perfum'd flames & into Ashes turne
Thou might'st hope (vaine hope) yet once againe
To rise wth purer notions in thy Braine
But t'would nott serue for they would still be darke
Till from thyn Alter Lord I take a sparke
I need not then assend up any higher
In offring this to fetch another fire
Inspired thus may on my Muse distill
Dewes nott from Parnass but Herman's sweet Hill.


[p. 1]                          Psal 1

Blest is the man in walking daly shuns
Pernitious Councel that from th' wicked Comes
Nor to the sinners paths his steps doth bend
Or he yitt up to Scorners chaire assend
Who in the early morne & euening laite
On lawes deuine makes choyse to meditate
As by the runing streames the well sett tree
His fruit in season yeild, the iust shall be
Whos leafe shall neuer fade & what he doth
Shall thriue as itt & shal be fruitfull both
But wth the wicked itt is diffrent farre
As chaff tost in the Ayre, So they are
Nor shall he stand fore th' impartial Judge
Or mongst the iust who in sins way doe trudge


[p. 38]                         Psal 19

The heauens Lord the siluer studed frame
They are the Curious works thy hands declare
Time vnto time itt doth recount the same
To places most remote, ther voyce it heares
Ore all the earth ther arched Sphers extends
The Tun on's throne ther rises ther desends

As cherfull brid-grome in his nuptial state
Or actiue men to race wth ioy Come out
From East to West so runs he at that raite
Till his cirquitt rownd he'as gone about
All parts euen to thé wide Earths extreames
Both light & heat takes from his radent beames

[p. 39]   Thy law ô Lord to soules perfection giues
They that are simple by thy words madr wise
They shall reioyce who in thy precepts liues
Thy Statutes pure inlighten's the blind eyes
To feare the Lord will vs preserue for euer
Whos iudgments true & rightious altogether

More sweete then honny yea or gold refin'd
Thy seruants setts them att a hier prise
They great rewards in keeping them do find
But ô alas who ist his errors spies
My faults vnseene ô let ther none remaine
From bold-fac'd sins thy seruant Lord restraĩe

[p. 40]   O let not sin wth it's tyranick might
Ére gitt a iuri[s]diction ouer mee
So in my soule shall I then be vpright
And from the great transgression guiltless be
So shall my words & thoughts acceptance find
W^th thee my strength redeemer of man-kind


[p. 49]                    Psal 23

How can I want the Lord my sheapard seemes
Who to the verdant pasturs leads me outt
By flowry bankes wher waters gently streams
My soule he doth refresh he sets my foot
In paths of truth & eaqual Iustice both
This only for his owne name sake he doth

Al Though I through death[s] shady vale doe goe
No terrors.ther shal makes me yitt affraid
His rods my guide his staff my strength also
Before myn foes my table he doth spread
W^th oyle my head full cups my hart doth chere
Him in his house for euer I'le serue ther

[p. 104]                          Psal 46

If in distress ᵒ Lord thou 'lt giue me ayde
What need I feare though rocks in seas be throwe
Though by ther rage the hills on hills be layd
Thou still preseruest thos that are thyn owne
In thes o're turnings shal noe fear cease them
For God was ther, his help in season Came

When furious rage procest the Heathen world
Thou was to vs as a strong Towre in War
Thou spake the word & Earth on heaps was hurld
Come se then ther what great vastations are

[p. 105]  T'is he when wars arise Can stop ther Course
This he ther weapons breake ther Chariots fire
Wait thou on him know he's a God of force
Did he not rule the world t' would soone expire
He mongst the Heathens will exalted be
But Jacobs Gods the Towre to whom we flee

[p. 390]              Songs of the Old & New
                            Testiment

                        Moses Songe
                        Exodus 15

Vnto the Lord let prayse be sung
    Who gloriously triumphed hath
For he into the sea hath flung
    Both Horse & Rider in his wrath

The Lord my strength & songe shall be
    Who my sure saluation
Mine & my father's god is he
    Soule be his habitation

[p. 391]  A man of Warr's the Lord renown'd
    His name is by Jehouah knowne
Who in the Sea hath Pharoah drownd
    Downe Captains Horse & Chariots throwne

This goodly Traine wᵗʰ fury drunke
    The waues as Couerings did Containe

Wher to the bottome they are sunke
As stones that neuer rise againe

Thy hand o Lord has done this deed
Glorious in Powre art thou become
Thy hand I say when ther was need
Th' insulting Foe has ouercome

[p. 392] They that agaīst thee did Combine
Thy wrath has broke in thy defence
As stuble th'are before the winde
So powrefull is thyn' excelence

Thy Nosthrills w<sup>th</sup> a blast haue layde
The liquid Seas on sollid heapes
The floating waues ther w<sup>th</sup> were stay'd
As Ice Congealed in the depths

Pursue o'retake th' enimy said
Ther spoyles let vs mongst vs deuide
Whilst w<sup>th</sup> ther Swords they hauack made
And lust as law to them was guide

[p. 393] But in a happy howre thou did
The Treasures of thy winde display
So sunk they as the heauy lead
And vnder watry-Monntains lay

Amongst the Gods who's like to thee
O Lord in Holiness & Prayse
The fearfull wonders w<sup>ch</sup> we see
Doe Trophyes to thy Glory raise

Thou stretcht thy hand & they were gone
The gapinge earth deuourd them quite
To th' Holy mountaine thou leddst on
The chosen Flock of thy delight

[p. 394] Nations hard this w<sup>th</sup> pale-fac'd looks
And horred feare amazed stood
Edom Moab & Syrian Dukes
Ditt melt away w<sup>th</sup> Canan's broode

Thy Glorious Name did soe apall
Ther trimbling Harts w<sup>th</sup> feare & dread
That as a stone lie still they shall
Till those pass ouer thou dost lead

To Zions mount thou didst them bring
Didst plant them in its firtil soyle
The place wher thou delightsl in
A sanctuary freed from toyle

[p. 395]     Raigne Lord for euer vn-opposd
For Pharoa's Horse & Men are drownd
Him & his force hath sea inclosd
Whilst Israel marches on dry grownd

Miriam the Prophetiss a Timbrel takes
Wher in their Circulinge-dances round
The Virgin-Traine such Musick makes
As th' Hills about w<sup>th</sup> Ecchoes sound

Then Miriam answered them & sunge
The Lord triumphd in Glory hath
Proud Pharoah into th' sea has flunge
W<sup>th</sup> Horse & Rider in his wrath


[p. 418]      Dauids Lamentation for Saule & Jonathan.

2 Sam: 1

Israel has lost her ornement
Alas for itt lement
How are her Mighty, falne & laine
& on Mount Gibea slaine
Ô let itt nott in Gath be knowne
Or told in streets of Askelon

O lett nott Lord our ancient Foes
Joying Deride our woes
Least daughter of th' vncircomcis'd
Triumph o're vs dispis'd
[p. 419] Noe more lett fruitfull showres distill
Or dewes on Gibeas direfull Hill

Nor e'er may any thither bringe
More a Heaue-Offringe
Ther th' Mighty fell, Saule lost his sheild
In this shamfull feild
On him regardless they did treade
As if noe oyle had touch'd his head

Sharp Arrowes shott from Jonathans Bow
Drunk wth the blood of Foe
Nor did Sauls sword rebate a Jott
Till he'ad his[1] enimys smote
[p. 420] How louely-pleasant are you tow
Death Could not loue disjoyne in you

Swifter then Eagles wch th' Ayre peirce
Both stronge as lions feirce
Israel's daughters lement the fall
Of your valiant Saule
Who you in Purple & Scarlet deckt
And did from Foes your land protect

How pleasant was itt to behold
Your orniments of Gold
Thy worthys by the sword, how are
They thus cutt off in war

[p. 421] O Jonathan my harts delight
Slaine in the bloody fight
Mount Giboa saw the woefull day
Thou mongst her Rockes ther wounded lay

How can I Deare Jonathan express
For thee my sad distress
Noe Woman's loue reach'd thatt degree
As thou once loued mee
How is the Mighty falne, is Crusht
And Israels Worthys rould in dust

[p. 422] Hezekiahs-Songe
Isaiah 38

In Cuttinge off my days I said
Must I goe downe to deaths cold shade
Youth's flowre noe sooner Budd but Blast
Be Cropt and to obliuion cast
Mongst liuing Lord must I noe more
Lift vp myn eyes & thee adore

---

[1] Fairfax has written over this line "his foes had smote."

Or longer in this vniuerse
W^th Man-kind haue noe more Connerse
Farwell then Suns chearful light
Whose Rayes expells the shades of Night
[p. 423] Adeiu deare siluer-Horned Moone
By step & step our time setts downe
Yee Stars farwel that in Night appears
Runing in your apoynted Spheres
Who from your orbs soe far from hence
Throwes downe on vs your influence
Stay when you will your Constant Course
For ouer death you haue noe force
Farwel my Friends, farwel delight
Deuided by Eternal Night
My flitting years how soon th'are spent
Remoued as a Sythian Tent
Here to day to morrow dead
[p. 424] Cut off like to a weauers thread
In morning when new hopes began
Er' euening pinning sickness came
Yitt did'st nott heare my sad groanes
But lyon-like brake all my bones
O whatt a little space is this
T'wixt Being & not Beinge is
Euen from th' Eueninge to the Day
My wasting Sperits faide away
As Crane or Swallow sett alone
To the ô Lord I make my mo'ne
And as the Doue that trembling sitts
When Hawke aboue doth sores his pitch
[p. 425] So faints my hart so failes myn eyes
In seing such sad miseryes
But thou in Mercy hast noe piere
O help me in this troubled feare
What shall I say but sure thus much
Thy Word & Truth keep perfait touch
For sin my soule shall all itts days
Walke softly in my pensiue wayes
By these things Lord doe Mortals liue
New life by these things thou dost giue
Lo, Peace to me dost thou restore
And Joy for Greefe I had before

Thou pluckt me from destrctions Pitt
[p. 126] And all my sins didst thou remitt
For who in death can offrings binge
Or in the Graue thy Prayses singe
Of All to Shades beneath repare
Does any hope for Mercy ther
The liuinge 'tis the liuinge They
Shall Prayse thee as I doe this day
Father to sonne relate shall this
How faithfull are thy Promises
Since Lord thou hast prolong'd my days
On Warbling Harpe I'le giue thee prayse
And in thy Courts wth Holy Fire
Of Zeale pay thanks till I expire


[p. 431]                    Simeon's Songe

As thou hast said soe Lord pray I
In peace now lett thy seruant die
Sence my blest eyes haue seene i'th end
Saluation from thy Throne desend
Which thou before earth frame was layd
To saue Man-kind decreed had
A light to guide the Gentiles ways
Of Israel's sones to be the prayse


[p. 435]                    [The Songe of Salomon]
                           Chap 2

I am the Rose of Sharon's fruitfull feild
The Lilly wch the humble vallyes yeild
In midst of thornes as Lilly appear's aboue
Soe mongst the youthfull Virgins is my loue
As Apple-trees 'mongst trees o'th Forrest growe
Amongst the sones of Men my loue is soe
Vnder whose shade is my delightfull seat
And to my tast his fruit is pleasant meat
Then to the house of wines he brought me in
Wher Loue like banners was a Couer in
Stay me wth flaggons wth Apples Comfort giue
Who's sick of Loue may yitt haue hope to liue

        Vnder my head his left hand stretched out
        And wth his Right h' imbraceth me about
[p. 436] O Zions daughters I strictly you adiure
        By the swift Hynde & fearfull Roe be sure
        Noe stir by noyse you make for' to disease
        Or wake my loue before the time he please
        Behold I hear his Voyce o're Hills & Downes
        My loue Comes skiping ouer Mounts & bounds
        Like th' Hart or nimble Fawne & triping Roe
        Standing behind our Wall Behold him Loe
        Through trelest windows how he looketh out
        His Church wth watchfull care he vews about
        Thus speaking to me I my loue did heare
        Arise my faire one Come away my deare
        Lo winters past wth her stormy showers
        Th' Earth now shew's her various coulred flowrs
        Chirping of birds a signe the spring grows near
[p. 437] We in the land the mourning Turtle heare
        The Figg-tree budding green her Figgs disclose
        The tender Grapes of Vines smell as the Rose
        Arise my faire one Come away my loue
        Whom Cliffy Rocks doe hid Come out my Doue
        Shew me thy Face myn eares let thy Voyce meet
        Thy Countinance is Comely, Voyce most Sweet
        Take th' Fox & little Foxes in thy Toyles
        That thus our tender grapes & Vinyard spoyles
        My deare is myne & I am his who 'monge
        The Lillyes feed till shades of Night be gone
        Turne my beloued turne like th' Roe that trips
        Or nimble Hinde that in Mount Bether skips.




[p. 480]                    Honny dropps.

            (Under this title Fairfax has written
one hundred and twenty five couplets and thirty five quatrains)

        Why good men haite all sin 'tis understood
        Because tis both gainst god and ther owne good

        To walke wth god tis goodmen's care we see
        But leaves the Care to god wch way 't should be

Noe safty wth out god in Freindshlp weie
Yitt safe wth enimyes if God be there

[p. 182] A good man questionless was never hee
That strives nott allways better for to be

[p. 183] Good Conscience is a name att wch Men tante
But betters a good name then Conscience want

[p. 484] Whatt before men we are affrayd to doe
Fore God to thinke itt should affright us too

Many the Sacred ordinances use
Making noe proffet of them—them abuse

[p. 486] When thou dost well or any good thou can
Prayse nott thy worke, the worke will prayse the Man

[p. 487] The soule by such a Noble sperit moves
Tis nott soe much where't lives as wher it loves

Sure best are they, nott they who most can talke
How Good God is, but who most with him walke

[p. 490] In sweetest Natures this will sure befall
None All can like nor shall be lik't of all

[p. 491] All Earthly things are such as ther's noe doubt
Worst Men may have and best may goe wth outt
Yett wanting them a man may happy be
When others wth them have butt misery

[p. 492] Noe Time in pastime need we Idly wast
For time will pass from us in too much hast

[p. 507] I'th' Sacred Arke Reason of State should lye
But rules of state should nott Religion tye

When men wth wine themselves like beasts abuse
Not wine the Men but the wine misuse

[p. 509] In all thou undertskes be carful still
That none of thee can speake deserved ill
And soe when that is done thou needs not Care
For Ill men's Censure ('Tis the Common fare)

[p. 549]                    A Songe of Prayse

Earth prayse the Lord him Reverence beare
As well for's Thunders that we heare
At w<sup>ch</sup> poore Mortals stand affraid

As four the glotious Maruels which
Such Splendors doth the world inrich
They are the workes his hands hath made

His Prouidential loue lets singe
That w<sup>th</sup> a plentious flowinge springe
Our barren soules hee watered

The East the West tast of his Care
Hott Affrick nor the freezinge Beare
From his al seeinge eye is hidd

[p. 550]    And wast nott he He who did please
W<sup>th</sup> seuerall kinds to store the Seas
Of Fish beyond account Nay more

Made Woods & Hills that Cataile yeilds
Gaue flowry Pasturs verdent feilds
That bringe both Corne & wine great store

But how doe we his mercy wronge
He sees wee still in Sin grow stronge
And day by day his patience moue

Yet as a Father ready is
To pardon faults he sees in his
Such are the tokens of his loue

[p. 551<sup>a</sup>]    In vs Affections ôh tis strange
W<sup>th</sup> our light humor suddaine Change
As in a moment they grew old

They w<sup>th</sup> the Wind are easely driuen
But his is alweyes firme & euen
And to Eternity doe hold


Finis

[p. 551 c]          THE RECREATIONS OF MY SOLITUDE

                                T : F


[p. 552]                     THE SOLITUDE

O how I loue these Solitudes
And places silent as the Night
Ther wher noe thronging multituds
Disturbe wth noyse ther sweet delight
O how myn eyes are pleas'd to see
Oakes that such spreadinge branches beare
W^ch from old Time's netiuity
And th'enuy of so many yeares
Are still greene beautifull & faire
As att the world's first day they were

Naught but the highest twiggs of all
Wher Zephyrus doth wanton play
[p. 553] Doe yett presage ther future fall
Or shew a signe of ther decay
Times past Fawnes Satyrs Demy-Gods
Hither retird to seeke for Aide
When Heauen wth Earth was soe att odds
As Jupiter in rage had laide
O're all a Deluge these high woods
Prescru'd them from the sweling floods

Ther vnder a flowry Thorne alonge
Of Springs delightfull plant the Cheife
Sadd Philomela's mournfull songe
Doth sweetly entertaine my greefe
And to behold is noe less rare
[p. 554] These hanging Rocks & Precepies
W^ch to the wounds of sadd dispare
Are soe propitious to giue ease
When soe oprest by cruel fate
Death's sought for att another gate

[LA SOLITUDE [1]

A Alcidon.

Que j'ayme la solitude!
Que ces lieux sacrez à la nuit,
Esloignez du monde et du bruit,
Plaisent à mon inquietude!
Mon Dieu! que mes yeux sont contens
De voir ces bois, qui se trouverent
A la nativite du temps,
Et que tous les siecles reverent,
Estre encore aussi beaux et vers,
Qu'aux premiers jours de l'univers!

Un gay zephire les caresse
D'un mouvement doux et flatteur.
Rien que leur extresme hauteur
Ne fait remarquer leur vieillesse.
Jadis Pan et ses demy-dieux
Y vindrent chercher du refuge,
Quand Jupiter ouvrit les cieux
Pour nous envoyer le deluge,
Et, se sauvans sur leurs rameaux,
A peine virent-ils les eaux.

Que sur cette espine fleurie,
Dont le printemps est amoureux,
Philomele, au chant langoureux,
Entretient bien ma resverie!
Que je prens de plaisir à voir
Ces monts pendans en precipices,
Qui, pour les coups du desespoir,
Sont aux malheureux si propices,
Quand la cruauté de leur sort
Les force à rechercher la mort!

---

[1] This is not in the MS. See pp. 246–248.

How pleasant are the Murmuring stream
In shady Vallyes runinge downe
Whose raginge torrents as itt seemes
Just measurs keepe in skpps & bounds
Then glidinge vnder th' arbored banks
As windinge Serpents in the grass
The sportfull Naides playes ther pranks

[p. 555] Vpon the watry plaines of Glass
The christal Elements wherin
These watry Nimphes delight to swime

The quiet Marshe I loue to see
That bounded is w<sup>th</sup> willowes round
With Sallow, Elme, & Popler tree
W<sup>ch</sup> Iron yett hath giuen noe wound
The Nimphes that Come to take fresh Ayre
Here Rocks & Spindles them prouide
Mongst Sedge & Bulrush we may heare
The lepinge Froggs Se wher they hide
Themsclues for feare when they espye
A Man or Beast approachinge nye

[p. 556] A hundred thousand Fowle her lye
All voyd of feare makinge ther Nest
Noe treachrous Fowler here Comes nye
W<sup>th</sup> mortal gunnes to breake ther rest
Some ioying in the sunn's warme beames
Ther fethers buisily doe plume
Whilst others findinge Loue's hott flames
In waters allsoe can Consume
And in all pastimes Inocent
Are pleased in this Element

How pleasant is itt to behold
These ancient Ruinated Towers
[p. 557] 'Gainst w<sup>ch</sup> the Giants did of old
W<sup>th</sup> Insolence imploye ther Powers
Now Sayters here ther Sabath keepe
And Sperits w<sup>ch</sup> our sence inspire
W<sup>th</sup> frightinge dreames whilst we doe sleepe
Noe here againe all day retire
In thousand Chinkes & dusty holes
Lves vgly Batts & Scritchinge Owles

Que je trouve doux le ravage
De ces fiers torrens vagabonds,
Qui se precipitent par bonds
Dans ce valon vert et sauvage!
Puis, glissant sous les arbrisseaux,
Ainsi que des serpens sur l'herbe,
Se changent en plaisans ruisseaux,
Où quelque Naïade superbe
Regne comme en son lict natal,
Dessus un throsne de christal!

Que j'aime ce marets paisible!
Il est tout borde d'aliziers,
D'aulnes, de saules et d'oziers,
A qui le fer n'est point nuisible.
Les Nymphes, y cherchans le frais,
S'y viennent fournir de quenouilles,
De pipeaux, de joncs et de glais;
Où l'on voit sauter les grenouilles,
Oui de frayeur s'y vont cacher
Si tost qu'on veut s'en approcher.

Là, cent mille oyseaux aquatiques
Vivent, sans craindre, en leur repos,
Le giboyeur fin et dispos,
Avec ses mortelles pratiques.
L'un, tout joyeux d'un si beau jour,
S'amuse à becqueter sa plume;
L'autre allentit le feu d'amour
Qui dans l'eau mesme se consume,
Et prennent tous innocemment
Leur plaisir en cet element.
. . . . . . . . . . . . [1]

Que j'ayme à voir la décadence
De ces vieux chasteaux ruinez,
Contre qui les ans mutinez
Ont deploye leur insolence!
Les sorciers y font leur sabat;
Les demons follets s'y retirent,
Qui d'un malicieux ébat
Trompent nos sens et nous martirent;
Là se nichent en mille troux
Les couleuvres et les hyboux.

---

[1] Fairfax omits a stanza here.

These Mortal Augurs of Mischance
Who funerall notes as Musick makes
The Goblins singe & skipp & dance
In valts ore spred wth Toads & Snakes
Ther in a Cursed beame might see
[p. 558]  The horred Skeliton of some poore louer
W^ch for his Mistriss Cruelty
Hanged himselfe sence naught could moue her
Or wth a glance nott once to daine
To ease him of his mortal paine

The Marble Stones here strew'd about
Of Carracters leaue yett some signe
But now are almost eaten outt
By teeth of all deuouring time
The planks & timber from aboue
Downe to the lowest Valts are fau'ne
Wher Toads & Vipers 'mongst them moue
Leauinge theron ther deadly spawne
[p. 559]  And Harths that once were vs'd fvr fyers
Now shaded are wth scratchinge Bryers

Yet lower an Arched-Valt extends
Soe hiddious darke & deepe doth sinke
That did the Sun therin desend
I thinke he scarce Could se a winke
Slumber that from heauy Cares
W^th drowsiness inchants our sence
Sleepes here secure, as far from feares
Lul'd in the Armes of Negligence
And on her back in sluggish sort
Vpon the pauement lyes & Snort

[p. 560]  When from these Ruings I doe goe
Vp an aspiringe Rock nott farre
Whose topp did seeme ast were to know
Wher mists & Stormes ingendred are
And then desending att my leasure
Downe paths made by the storming Waues
I did behold wth greater pleasure
How they did worke the hollow Caues
A worke soe Curious & soe rare
As if that Neptuns Court were ther   .

L'orfraye, avec ses cris funebres,
Mortels augures des destins,
Fait rire et dancer les lutins
Dans ces lieux remplis de tenebres.
Sous un chevron de bois maudit
Y branle le squelette horrible
D'un pauvre amant qui se pendit
Pour une bergere insensible,
Qui d'un seul regard de pitic
Ne daigna voir son amitie.
. . . . . . . . . . . [1]

La se trouvent sur quelques marbres
Des devises du temps passe;
Icy l'âge a presque efface
Des chiffres taillez sur les arbres;
Le plancher du lieu le plus haut
Est tombé jusques dans la cave,
Que la limace et le crapaut
Souillent de venin et de bave;
Le lierre y croist au foyer,
A l'ombrage d'un grand noyer.

Lá dessous s'estend une voûte
Si sombre en un certain endroit,
Que, quand Phebus y descendroit,
Je pense qu'il n'y verroit goutte;
Le Sommeil aux pesans sourcis,
Enchante d'un morne silence,
Y dort, bien loing de tous soucis,
Dans les bras de la Nonchalence,
Laschement couche sur le dos
Dessus des gerbes de pavos.
. . . . . . . . . . . [1]

Tantost, sortant de ces ruines,
Je monte au haut de ce rocher,
Dont le sommet semble chercher
En quel lieu se font les bruïnes;
Puis je descends tout a loisir,
Sous une falaise escarpée,
D'ou je regarde avec plaisir
L'onde qui l'a presque sappée
Jusqu'au siege de Palemon,
Fait d'esponges et de limon.

[1] Fairfax omits a stanza here.

Tis a delightfull sight to see
Standinge on the murmringe shore

When Calmer Seas begin to bee
After the Stormes, w<sup>ch</sup> raginge roare
How the blew Trytons doe appeare
Vpon the rollinge Curled Waues
Beatinge w<sup>th</sup> hiddious tunes 'the Ayre
W<sup>th</sup> Crooked Trumpets Sea-men braues
Att whose shrill notes the winds doe seeme
By keepinge still to beare esteeme

Sometimes the Sea w<sup>th</sup> Tempests rore
Frettinge itt Can rise noe higher
Roulinge or'e the flinty shore
Throwes them vp againe retires

Somtimes through itt's deuouringe Jawes
When Neptun's in an angry moode
Poore mariners finde his Cruel lawes
Made to his finy Subiects foode
But Diamonds Amber & the Jett
To Neptune they doe Consecrate

Sometimes soe Cleare & soe serene
Itt seemes ast were a looking glass
And to our Vewes presenting seemes
As heauens beneath the waters was
The Sun in it's soe clearely seene
That contemplatinge this bright sight

As't was a doubt whether itt had beene
Himselfe or image gaue the light
Att first appearing to our eyes
As if he had falne from the skyes

Thus Alcidon whose loue inioynes
To thinke for thee noe labor paine
Receaue these Rustick Shepheards lines
That's from ther liuinge obiects ta'ine
Sence I seeke only desart places
Wher all alone my thoughts doe use
Noe entertainment but what pleases
The genius of my Rural Muse
But noe thoughts more delighteth mee
Then sweet Remembrances of thee

Que c'est une chose agreable
D'estre sur le bord de la mer,
Quand elle vient à se calmer
Après quelque orage effroyable !
Et que les chevelus Tritons,
Hauts, sur les vagues secouées,
Frapent les airs d'estranges tons
Avec leurs trompes enrouées,
Doat l'eclat rend respectueux
Les vents les plus impetueux.

Tantost l'onde, brouïllant l'arène,
Murmure et fremit de courroux,
Se roullant dessus les cailloux
Qu'elle apporte et qu'elle r'entraine.
Tantost, elle estale en ses bords,
Que l'ire de Neptune outrage,
Des gens noyez, des monstres morts,
Des vaisseaux brisez du naufrage,
Des diamans, de l'ambre gris,
Et mille autres choses de pris.

Tantost, la plus claire du monde,
Elle semble un miroir flottant,
Et nous represente à l'instant
Encore d'autres cieux sous l'onde.
Le soleil s'y fait si bien voir,
Y comtemplant son beau visage,
Qu'on est quelque temps à sçavoir
Si c'est luy-mesme, ou son image,
Et d'abord il semble à nos yeux
Qu'il s'est laissé tomber des cieux.

Bernières, pour qui je me vante
De ne rien faire que de beau,
Reçoy ce fantasque tableau
Fait d'une peinture vivante.
Je˜ne cherche que les deserts,
Où, resvant tout seul, je m'amuse
A des discours assez diserts
De mon genie avec la muse;
Mais mon plus aymable entretien
C'est le ressouvenir du tien.
.   .   .   .   .   .   .   .   .   .   [1]

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[1] Fairfax omits the two concluding stanzas.

[p. 564]                    Of a Faire Wife

to Coregio

Thou thinkst Coregio thou hast gott
An exclent Beauty to thy lott
But yet remember this againe
For pleasure also thou'lt haue paine
No perfect rest can be to thee
When watchfull always thou must be
T'is hard & difficult to keepe
That all the world desire & seeke
Is her beauty much, Then know
Her pride's noe less w^oh she doth show
[p. 565]  Dost thou admire her th'more will shee
For thy esteeme disdainfull be
But is shee faire Consider this
If shee be chast, some doubt it is
As shee in hansomnes exceeds
Soe much of Modesty shee needs
Shee'l alwayes be a Mistress there
Wher only thou Comand should beare
But wouldst thou haue me to define
This rare beauty that is thine
Thy Idoll as thou make's of itt
Much more of Hurt then good thou'le gett
[p. 566]  For th' Adoration by thee giuen
Giues thee a Hell insteade of Heauen
New habits daly shee will axe
And if denyed then shee will vex
And thinke all's nothing in her passion
That's nott in the Mode & fashion
As if her Body were assign'd
To giue Inquietud's to thy minde
Me thinke I see thee rauisht on her
Thou blinde (as Idolizinge) Louer
Ma'as soone begett Ixion's brood
On Juno's Image in a cloude

[p. 567] Why shouldst thou longer thus submit
         To her who to obay's more fitt
         Least when thy Reason once is lost
         Thy Liberty too itt will Cost
         And in the end butt as a slaue
         A soueraine for Companion hauc
         To say noe worse of Beauty I Conclude
         It is but an Ilustrious seruitude


[p. 568]                    Of Beauty

         Beauty's a fraile & brittle good
         Wᶜʰ Sicknes Time & Age doe blast
         The Rose & Lilly in face thatt budd
         Hardly are keept & seldome last
         What hath she then to boast on Saue
         A fragil life & timely graue

         Beauty wher sweet Graces faile
         May be Compared vnto this
         A goodly ship wᵗʰ out her saile
         A spring her fragrant flower doe miss
[p. 569] A day want's Sun or Torch itts Light
         A shrine want's Saint or Starless night

         But how doth Nature seeme to smother
         The Virtues of this louely Flower
         Who is of wanton Lust the Mother
         Of toyinge Vanity a Bowre
         Enimy of Peace the Fount wher Pride doe swime
         Th' Incendeary of Strife of Passions Magazen


[p. 570]                Vpon a Patch Face

         Noe Beauty Spots should ladycs weare
         They but the Spots of beauty are
         Who knowes nott this (saue foolish Sotts)
         That Beauty aught to haue noe Spotts
         Some note a Spot that Venus had
         Admitt itt were in one soe badd
         Yett should nott shee haue Spots vpon Her
         That would be held a Maide of Honor

|p. 571|                  Vpon an ill Husband

All Creaturs else on Earth that are
Whether they Peace affect or Warre
Males ther Females ne're opress
By the Lyon safe lyes the Lyoness
|p. 572|     The Beares ther Mates noe harme procure
W<sup>th</sup> Wolfe the shee Woolfe liues secure
And of the Bull the Earth w<sup>ch</sup> teeres
The tender Heyfer has noe feares
But men then these more brutish are
Who w<sup>th</sup> ther wiues Contend & jarre


|p. 572|                         Of Enuy

In Enuy's Face discerne I this
Of Monsters shee most Monstrous is
A hurtfull glance her eye doth dart
A painfull paine lies att her hart
|p. 573|     Noe Good doe's Man enioy by Right
Her enuious teeth doth nott bitte
To Carracterize her yitt more fitt
Of Erringe blindness shee the Pitt
A Hell to Natures swetest Life
Reuenges Spur the flame of Strife
Her Actions yett bespeake her worse
To Ciuill Peace a vexinge Curse
Temptation's Sargent that's assign'd
The Sentinell of Restless minde
More hurtfull to the soule by farr
Then Vipers to the body are
But in a word t'express this Euell
T'is the Sin peculier to the Diuill


[p. 574]                       Of Anger

Noe Passion's rooted deeper or extends
Her branches furder or that more offends
Then Coller doth of w<sup>ch</sup> no sex or Age
Can boast a full exemption from itts rage
And when it's boundless fury growes
It's high distemper Madnes showes

Soe oft as Man is Angery oh tis sadd
He's nott only weake but blinde & Madd
Error for Truth imbraces & t'wer well
If dearest freinds from enimys he Could tell
A harmeless smile or from the eye a glance
Though vndesign'd puts him into a trance
[p. 575] And when his fury wakes how oft tis seene
Frendships most sacred bonds disolued haue beene
Who doth nott then discerne in sundry fashions
How Man afflicted is w$^{th}$ Angry passions
More feirce then are some Brutes as may apeare
They sometimes yeilds but he's in full Cariere
As Mariners when w$^{th}$ amazement smitt
The Pilots voyce in stormes regards nott itt
Soe men in frenzy ther strange gesters are
Wild as the beasts & Irreguler
The flaminge fire w$^{ch}$ Passions kindle flies
In furious sparkes from his piercinge eyes
His angry face by a reflux of blood
That from his Hart assends becometh rude
[p. 576] His haire w$^{th}$ gastly horror stands vpright
And euery word he speakes he seemes to bitt
His hands & feet in ther excentrick Motions
Breath naught but threats w$^{th}$ rash & bloody notions
His Lookes soe terrible as doe portend
A fatal Change vnto his nearest freind
What must be then's distempred soule w$^{th}$in
Soe vgly outward, but a sinke of Sin

[p. 577] Of Virtue

As wel tun'd Musick sweetly seize
The sences soe doth't Virtue please
The Virtuous, force the Vitious too
Th'admire in others what they should doe
Those best loue virtue & her lawes
That most Contemnes men's vains aplaues
Vertue alone all Grace inhance
And she noe vse doth make of chance
Whose effects are transcent in th' euent
What proceeds from virtue's permenent

Those things itt slights the World doe hold
Pretious as Fortunes Goods & Gold
[p. 578] These hath ther wings & flye away
When Man desireth most ther Stay
The virtious Soule prize most that some
Thinkes but from sheepesh nature Come
And nott from Grace the spring frō whence
Flowes Virtue Goodnes Inocence
Care thou for these sence they'le apeare
Much surer Goods then Riches are
Thy virtious acts goe wher thou will
For Companions thou shalt hauc still
When Men shall faile & freindship both
A better frende w<sup>th</sup> thee then goe'th
Enuy att death shal Cease in Foes
No Post-hume euel Malice knowes
[p. 579] In transendent hight shal vertue shine
Wher feet of Enuy Can not clime
Virtue alone doth death outliue
As't t'wer againe new life doth giue
Whilst Goods of Fortune here haue ends
Virtue alone to heauen assends


[p. 582]                    Nature & Fortune

What thing is nature we may thus define
God draws't through Beings in directst line
Wher as in Fortune soe miscal'd by some
More Crooked is & in Meanders rune
As Natur's rule by prouidence deuine
Soe Fortune too in an obstrucer line
Then Fortune is not blinde as vaine men says
Tis they are blinde discerning not her wayes


[p. 583]                    The Christian War-fare

The marke of note Gods children here doe beare
Is from the World's a different Carrecter
He to th'one for portion here beneath
Doth Losses, Shame & Pouerty bequeath

Yett happy those Afliction, wee account
That to the State Eternal doe amount
The worldly brood if we Caractrize
Th' haue noe Aflictions liue in Paridize
Ther Riches here as they desire augment
Ther Honors too increase to ther Content
But as a dreame these Honers vanish soone
And an eternal woe shal take ther Roome
As fatt of Lambes away they shall Consume
Ther Honor vanish into smoke & fume

[p. 584] T'indure sorrowes & Iniuryes we must
(As Scriptures tel) & be to exile thrust
Then tis a signe indeed heauen is our choyse
When in our Tribulations wee reioyce
T'is Gileads pretious Balme & serues to binde
The wounds & blowes wch here below we finde
Yea happy choyse though thus the World vs treat
Seing that in heauen our reward is great
The Soulder of that name vnworthy is
That trembles att the sight of enimyes
Soe is the Christian wch that title bear's
If he att threats of aduerse destine fear's
But wth a patient calmness lett's receaue
What the Soueraigne hand is pleas'd to giue

[p. 585] The Midle Region or those parts aboue
Are least obscurd nor ther doe Tempests moue
Soe should our soules be raysd boue Passions spheie
Noe Stormes of Tongues Nor Cloudes of enuy feare
In fronts of Batailes we our fortunes Sett
The Ship at Sea wth stormy winds is bett
The Pilot scapt from former gusts noe more
Feare's ship-wrack now then what he did before
The Soulder oft to frequent perills knowne
Neglect's the danger that's soe Comon growne
And soe should we when our Afliction, growe
W^{th} lenghtned Patience learne to beare them too
This Life's a war-fare if sometimes begun
To parly wth our sorrowes t'is soone done
And in th' end when hopes begin to Cease

[p. 586] Proues but a Cessation noe Continu'd Peace
Whilst through cleare skyes the Sun triumphant rides
Vpon a sudden cloudes his splendor hides

Doth health & Pleasure spur our sences on
Soon sickness Comes and all delights are gone
Such is the State of vs vncertanie men
To know in calmes to guide our Vessels then
Is not enough, but t'is when Tempests rise
To steare a Course both Patient, Stout, & Wise
Did our misfortunes soe deuide our share
As some shee would Aflict & others Spare
We might Complaine of her inconstant fitts
Bullets as soon th' Captaine as soulder hitts
The Feauer to the Great a deafe eare hath
[p. 587] As to the meanest both subuerts by death
Soe may the Justice of Impartial fate
For Comfort serue vs in our greatest Strait
Why doe we enuy then aspiringe Men
W^{th} Stormes the Vallyes are less troubled then
The lofty Hills & humble shrubbs belowe
Less danger's in then Oakes that highest growe[1]
See we not how the straitest Popler tree
And spredinge Elme as they vngratfull be
For nurishment) to barreness incline
Whilst prostrate on the ground the Crocked vine
Abundance yeilds or haue we nott seene
From highest plenty men in wants haue beene
How many Kings falne from ther Regall seate
Haue Crack't their Crownes ther Royal Septers breake
[p. 588] Our Wittnesses by cloudes we all may bringe
To shew that splendid honours a vaine thinge
Should they be ta'ne from vs resolue thus much
Ther loss should not be great ther fading's such
Should we aflict ourselues when loss appear's
Our Teares would sooner want then Cause for teares
All you wth heauenly Marks of God indued
Arme to the Fight shew Virtue Fortitude
As Rocks 'gainst wch the raging billowes rore
Keepe firme ther station on the threatned shore
Soe let our Soules be firme & Constant still
Against the threats this World doth make of Ill
Or as a Diamon mongst the dust doth dart
The beauty more in itt's resplendent sparke

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[1] Cf. Horace, *Carm.* II 10.

[p. 589]  In midst of troubles soe lett vs demeane
          As Countinances be pleasant Soules serene
          Remember t'is from high Aflictions fall
          From Prouidence deuine that gouern's all
          Who when he please in turning of an eye
          Turn's Wrath to Mercy Sorrows into Joy
          T'is he who made the firtile Earth produce
          Her anual fruit most meet for humaine vse
          He both the Rose & Violets did Cloth
          T'is he beauty & th'oders gaue to both
          'Twas his Almighty power that did make fall
          Att Israels seige the Jereconian Wall
          That on's Enimyes ruing he might raise
          Trophy on Trophy to inrich his Prayse
[p. 590]  Shal we then those his wonders now less prize
          Or thinke his Power abat's, or hee less wise
          No, hee's as able still Nor shall His want
          Victory on Standards Glory on ther front


[p. 590]              Life & Death Compared together

              Such vulgar thoughts the World doe fill
              To thinke Life good Death only ill
              Then life ill lined noe euell's worse
              Death (dieing well) remoues the Curse
              And tis for certaine truth men tell
              He ne're dies ill that liueth well
              Ill liues doe but ther Ills increase
[p. 591]      But dieng well makes Euells Cease
              Badd men haite death but not soe much
              That itt is Ill, as They arc such
              Moral Men teache vs in their bookes
              That we should dispise death's grime lookes
              T'is Comon sence w<sup>ch</sup> doth inspire
              Ther feares of thatt Good men desire
              Nor Can we truly death define
              By makinge odious what's sublime
              Consider't in th' effects & soe itt will
              Plead much for death be't Good or Ill
              Say itt be Ill yett here's the Good
              To greater Ills it giues a period
              In life what one good thinge is ther

[p. 592]    To keepe our Passions Regular
The many Ills each day is done
Makes Death less fea'd but once to come
But rather thanke Death that's the Cause
Our Ills are not Imortal Lawes


[p 592]                    Vpon a Fontaine

Seest thou how these waters flowe
How soone againe away itt glides
Soe worldly Glory's but a showe
That neuer long wth vs abides [1]


[p. 593]                    Vpon the New-built
House att Apleton

Thinke not ô Man that dwells herein
This House's a Stay but as an Inne
W^ch for Conuenience fittly stands
In way to one nott made wth hands
But if a tıme here thou take Rest
Yett thinke Eternity's the Best


[p. 594]                    Shortness of Life

In Rosy mor'ne I saw Aurora red
But when the Sun his beames had fully spred
She vanisht I saw a Frost then a Dew
T'wixt time soe short as scarce a time I knew
This stranger seemd when in more raised thought
I saw Death Come How soone a life he'ad Caught
Wher in the turninge of an eye he'ad done
Farre Speedier execution then the Sun

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[1]             Pour une Fontaine
Vois tu, passant, couler cette onde
Et s'ecouler incontinent ?
Ainsi fuit la gloire du monde
Et rien que Dieu n'est permanent          Malherbe.

[p. 595]                    Epitaph on A V dieng Younge

O what affront was itt to Nature
And sadder Influence of the Skyes
That in a moment clos'd the Eyes
Of such a machless Creature
But askinge what might be the Reason
That Creuel Fate soe out of season
Had Caried her from vs soe farre
This Answer was to me returnd
Least that the Earth should bee burnd
By th' scorching beames of that bright starr


[p. 596]                    The Lady Caryes
                       Elogy on my deare Wife

O Fatal fall might not those heapes suffice
This Sumer Captiu'd but thou must surprize
The best of Nobels this soe great good Lady
A Vere A Fairfax Honours-Honour, Shee
Did grace her Birth Sex Relate & Degree
& Shee a Non-parell for Piety
Vers't in the Theory of Godliness
The wch she did in Conference express
Its Practick part her life to life did shew
Each way but most excellinge in all vew
Was Faith Submission vnweared pleasantnes
[p. 597] With vniuersal weaknes, Paine Sickness
Many longe lasting Great few euer sence
Soe followed Job in suffringe Patience
But she is now most gloriously exalted
Wher sin & sorrow neuer entred
To Mount Zion heauenly Jerusalem
The City of God to Sperits of Just men
To Church of the first borne to Angels blest
To God to Jesus this Compleats the rest
Her Faith saw this wch made her smile att death
And wth much Joy surrendred vp her breath
Her Body deare her All thats out of Heauen
To Billbrough church as a riche Treasure's giuen
Bilbrough church-yeard daine me a little roome
That after death my graue waite on her Tombe.

|p. 598|                     To the Lady Cary
                    Vpon her Verses on my deare Wife
Madam
            Could I a Tribute of my thanks express
            As you haue done in loue & purer Verse
            On my best selfe then I might Justly raise
            Your Elogy t'Encomiums of your Prayse
            And soe forgett the Subiect that did moue
            Me to a thankfulnes as't did you to loue
            Ô t'were to great a Crime but pray allow
            Wher I fall short but you haue reached to
            Makinge that Good wisest of Kings hath said
      .     Th' Liuing's not soe Preyse-worthy then the dead
            I thinke the Reason's this itts grounded on
            'Cause Mercys are not priz'd till they are gone
|p. 599|  O had not hopes surpast my grosser sence
            My loss Could not haue had a recompence
            Yett such an Influence hath your happy straine
            To bring my buried Joy to life againe
            Vertue Goodnes Loue things Immortalize
            The better part when as the other dies
            True, Soules in Bodyes haue ther being here
            But Loues in Soules haue ther ther proper Sphere
            Then is true loue Compos'd of Nobler fyers
            Then to extinguish when the Life expires
            Butt to Conclude Madam me think you 'spire
            In humblest Thoughts to raise your Trophys higher
            Then Her's you would attend in gelid Mould
            Wᶜʰ for her Friend the lodging seemes too Could
|p. 600|  But were itt soe itt my good happ might bee
            To lye next Her, To you our Quire is free


[p. 600|                      On the Fatal day
                        Jan: 30 1648

            Oh lett that Day from time be blotted quitt
            And lett beleefe oft in next Age be waued
            In deepest silence th' Act Concealed might
            Soe that the King-doms Credit might be saued

But if the Power deuine permited this
His Will's the Law & ours must acquiesse

Curæ loquuntur leues
Ingentes stupent                    .

[p. 601] Of Inpartial Fate

Here we all the Same Danger run
By the like Destin's we are ledd
Same Misfortune to the Shepeard Come
May attack as well the Crowned head
Our dayes are Spun vpon that wheele
The meanest Subiect & greatest Kinge
To like end th' Fatal Sisters bringe
The thread when Cutt both same Sisers feele

[f. 604] A Carracter of the Romish
Church by Francisco Petrarca
Laura Can: 106

Fiamma dal ciel su tue treccie pioua

Heauens dire flame sits on thy Curled tresses
O wrech, from scrip & wallet who's become
Both riche & great through those wch thou oppresses
Soe much reioyces thou when euells Come
A nest of Treasons wher mischeifes bredd
Ther hacht in the o're the World is spred

Wine Bed good Belly chere & pleasant dayes
To All, thy whoredoms to the vttmost shews
[p. 605] Thy seruants younge & old the wanton playes
This fire wth bellowes Bel-ze-bub blowes
Such is thy life thou wicked Epicure
As to the Heauens thy stinch is gone vp sure

Fountaine of Greefe & woe wraths harbor too
Temple of Heresy Pitt of Errors deepe
In elter times we held thee Rome but now
Babel the peruerse for wch wee weepe
A shopp of Cousnage prison of Crueltyes
Wher ills mentaind & wher Goodnes dyes

When founded first wast humble Poore & Chast
Thy homes against thy Founders now thou lifts
|p. 606| O shameles Strumpet wher's thy trust now plast
Is't in th' Adultryes ill gott Goods or Shiffts
Then vnto All great wonder itt will bee
If Christ in th' End powre nott his wrath on thee[1]

|Fiamma dal ciel su le tue trecce piova,
　Malvagia, che dal fiume e dalle ghiande,
　Per l'altru' impoverir se' ricca e grande;
　Poiche di mal oprar tanto ti giova:
Nido di tradimenti, in cui si cova
　Quanto mal per lo mondo oggi si spande;
　Di vin serva, di letti e di vivande,
　In cui lussuria fa l'ultima prova.
Per le camere tue fanciulle e vecchi
　Vanno trescando, e Belzebub in mezzo
　Co' mantici e col foco e con gli specchi.
Gia non fostù nudrita in piume al rezzo,
　Ma nuda al vento, e scalza fra li stecchi:
　Or vivi sì, ch'a Dio ne venga il lezzo.

Fontana di dolore, albergo d'ira,
　Scola d'errori, e tempio d'eresia;
　Già Roma, or Babilonia falsa e ria,
　Per cui tanto si piagne e si sospira.
O fucina d'inganni, o prigion dira,
　Ove 'l ben more, e 'l mal si nutre e cria;
　Di vivi Inferno; un gran miracol fia,
　Se Cristo teco alfine non s'adira.
Fondata in casta ed umil povertate,
　Contra tuoi fondatori alzi le corna,
　Putta sfacciata: e dov' hai posto spene?
Negli adulterj tuoi, nelle malnate
　Ricchezze tante? Or Constantin non torna;
　Ma tolga il mondo triste che 'l sostene.[2]]

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[1] See page 245.
[2] These sonnets are not in the MS.

[p. 612]          Vpon the Horse w<sup>ch</sup> his Ma<sup>tie</sup>
                  Rode vpon att his Coronation 1660

Hence then Dispaire my hopes why should itt bury
Sence this braue Steed Bredd first was in my Query
Now thus aduanc't w<sup>th</sup> highest honors loden
Whilst his that bredd him on by most Mens troden
But t'is noe matter Seing tho' hast gott th' Aduance
Then please the Royal Rider w<sup>th</sup> thy Prance
Soe may thy Fame much rayse thy Prayses higher
Then Chessnut that begott the or Brid-la-dore his Sire

                       Bridla-dore (Anglicè)
                            Golden Bridle


[p. 613]               Vulgar Proverbs

          None to another freind can be
          That to himselfe's an enimy

[p. 614]  Of sence & Money & of Faith
          Where's the Man that too much hath

          Betwixt the Bridle & the Spur
          Reason often lodgeth her

          In th' house of Foes prepose this End
          To gett some Woman for thy freind

[p. 615]  The Hope of Gaine—Abateth paine

          Wouldst thou have all thy troubles cease
          Then see & heare & hold thy peace

          Lait (doe we say) repents the Ratt
          When by the Neck has hold the Catt

          His thoughts are good & ever best
          That carryes Death w<sup>th</sup>in his brest

[p. 617]  A fatt Earth makes a Horse to labour
          But A good Lawyer is an ill Neighbour

          Make Night of Night & Day of Day
          Soe w<sup>th</sup> less sorrow live you may

[p. 618]  Pardon to Men that evel be
          Unto the God's an injury

When Pride on horseback getteth upp
Loss & shame sitts on the Croup

|p. 620| He that would live in healthfulnes
Must dine wth little & supp wth less

|p. 621| As the evening doth the day comend
So life is Praysed by the end

|p. 622| Virtue shewes the greater grace
Shining from a bautious face

|p. 624| Att a rounde Table noe Strife is
Who shal be nearest a good Dish

Dry March Wett Aprel May that's both
Brings plenty wher ther is noe sloth

|p. 625| In a fresh gale
Extend thy Saile

|p. 626| We may be sure still inocence
Beares in itselfe its owne defence

|p. 627| To read & yitt to have learn'd nought
Is like the chase wher nothing's caught

|p. 628| Tis good we should the tongue comand
Speake litle & more understand
For if from us our words once fall
It is too laite them to recall

Humaine Praise—Is a vaine blaze

|p. 631| Sett on a Seat a Foole e're longe
He'le wagg his Legges or sing a songe

|p. 633| Nature made nothing so sublime
Butt Virtue to the topp will clime

When a whit frost on earth doth lie
Tis a presage then raine is nie

[p. 635] On a womans first Counsel rest
Seldome the Second is the best

Bread Butter & good Cheese
A shield 'gainst death be al these

Pardon give to every one
But to thyselfe alow none.

[p. 637] When Italy is w<sup>th</sup>out Fish
When France w<sup>th</sup>out Treason is
In England longe noe war we see
Then w<sup>th</sup>out Earth the World shall bee.

[p. 638] My contry is in all lands wher
I goe & meet w<sup>th</sup> true friends ther.

[p. 611]

The teares of France for the
deplorable death of Henry 1
surnamed the Great

Ah is itt then Great Henry soe fam'd
For taming men himselfe by death is tam'd
Whatt eye his glory saw now his sad doome
But must desolue in Teares sigh out his Soule
Soe small a shred of Earth should him intombe
Whos acts deseru'd pocession of the whole

O t'is but litt for joyes we henceforth mourne
Our songes & mirth into sad plaints we turne
Instead of this great King greefe may raigne here
So thatt in sorrow plung'd our fainting breath
May send our endless sighs to th'highst Sphere
Whilst hopless teares distill vpon the earth

[p. 612]  Yis itt is fitt what else can we returne
Butt teares as offrings to his sacred vrne
W^th them his Sable Marble tombe bedew
No no such armes too weake sence itl apeares
For vs he of his blood too careless grew
Haue we naught else for him butt a few teares

O could our eyes to fontains we distill
T' Would nott abaite the least part of our ill
We oft shed teares for simple wrongs oft weepe
Too Comon oft for things of lesser prise
Then lett vs die att this great Monarchs feet
His Tombe th' Alter, our selues, the sacrifice

But who can die if Sisters Fate denies
A closure to our half death trickling eyes
Hauing shut vp those of this warrlike Prince
Atropos so proud's of her royal pray
Her Cypriss into laurels will turne, Sence
Of this great Victor she hath gott the day

[p. 643]  But sence we are ordain'd to sigh & liue
And after this ther faitall stroke then giue
Liue then complaining this sad shock of Fate
Wher happy days are gone, no ioy appeares
Then mourne & sigh till death our greefe abate
And shew whilst liuing, Life shal wast in teares

[¹ Quoi? faut-il que Henri, ce redouté monarque,
Ce dompteur des humains, soit dompté par la Parque?
Que l'œil qui vit sa gloire ores voye sa fin?
Que le nostre pour lui incessamment dégoutte?
Et que si peu de terre enferme dans son sein
Celui qui méritoit de la posséder toute?

Quoi? faut-il qu'à jamais nos joies soyent esteintes?
Que nos chants et nos ris soyent convertis en plaintes?
Qu'au lieu de nostre roi le deuil règne en ces lieux?
Que la douleur nous poigne et le regret nous serre?
Que sans fin nos sousoirs montent dedans les cieux?
Que sans espoir nos pleurs descendent sur la terre?

Il le faut, on le doit. Et que pouvons-nous rendre
Que des pleurs assidus, a cette auguste cendre?
Arrousons à jamais son marbre triste blanc.
Non, non, plustost quittons ces inutiles armes!
Mais puisqu'il fut pour nous prodigue de son sang,
Serions-nous bien pour lui avares de nos larmes?

Quand bien nos yeux seroyent convertis en fontaines,
Ils ne sauroyent noyer la moindre de nos peines.
On espanche des pleurs pour un simple meschef.
Un devoir trop commun bien souvent peu s'estime.
Il faut doncques mourir aux pieds de nostre chef.
Son tombeau soit l'autel et nos corps la victime

Mais qui pourroit mourir? Les Parques filandières
Desdaignent de toucher à nos moites paupières,
Ayans fermé les yeux du prince des guerriers.
Atropos de sa proye est par trop glorieuse;
Elle peut bien changer ses cypres en lauriers,
Puisque de ce vainqueur elle est victorieuse.

Puisqu'il nous faut encor et souspirer et vivre,
Puisque la Parque fuit ceux qui la veulent suivre,
Vivons donc en plaignant nostre rigoureux sort,
Nostre bonheur perdu, nostre joye ravie;
Lamentons, souspirons, et jusques à la mort
Tesmoignons qu'en vivant nous pleurons nostre vie.

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¹ See page 246. This is not in the MS.

Bewaile bewaile this our great Monarchs fall
Of Judgment perfait humour pleasing all
His equal none a Hart w<sup>th</sup>out all feare
Perfection such t'would but fall short in prayse
Enough to' aue serued a World to' aue admird here
Had nott his equal Justice bound his wayes

Lament lament this Sage & Prudent King
Thatt hight of Bonty, vigelence in him
Thatt hart w<sup>ch</sup> could be mou'd not ouercome
Virtues here rarely found though we inquire
Parts I could sooner much admire then sume
Sence this Achilis a Homer would require

[p. 614] We cañott count the Splendours of his Glorys
Nor number yitt his signal victorys
O no for such a subiect were too great
We aught to prayse what yitt we cannot write
And hold our peace or to good purpose speake
He nothing saith doth not to th' full recite

His famous acts once raisd our drouping heads
His Laurels from the temples was our shades
End of his Combats ended feares wee're in
Him only pris'd dispis'd all other Powers
More gloring to be subiect to this King
Then if we'ad had some other Kings for ours

But now this Glory's clouded w<sup>th</sup> a staine
And now our joy & Mirth ther leaue hath taine
The Lillys faide as we att this sad Fate
Downe to the growne ther drouping heads doe bowe
Seeming as humble as Compassionate
To crowne his Tombe or else him homage doe

[pp. 645, 646 *are blank*]

Plaignons, pleurons sans fin cet esprit admirable,
Ce jugement parfait, cet' humeur agréable,
Cet hercule sans pair aussi bien que sans peur,
Tant de perfections qu'en loüant on souspire.
Qui pouvoyent asservir le monde à sa valeur,
Si sa rare équité n'eust borné son Empire.

Regrettons, souspirons cette sage prudence,
Cette extrême bonté, cette rare vaillance,
Ce cœur qui se pouvoit fleschir et non dompter.
Vertus de qui la perte est à nous tant amère
Et que je puis plustost admirer que chanter,
Puisqu' à ce grand Achille il faudroit un Homere.

. . . . . . . . . . . . . . . . . . . . . .[1]

Pourroit-on bien conter le nombre de ses gloires ?
Pourroit-on bien nombrer ses insignes victoires ?
Non, d'un si grand discours le dessein est trop haut.
On doit louër sans fin ce qu'on ne peut escrire,
Il faut humble se taire ou parler comme il faut,
Et celui ne dit rien qui ne peut assez dire.

. . . . . . . . . . . . . . . . . . . . . .[1]

Jadis pour ses beaux faits nous eslevions nos testes,
L'ombre de ses lauriers nous gardoit des tempestes,
La fin de ses combats finissoit nostre effroi.
Nous nous prisions tous seuls, nous mesprisions les autres,
Estant plus glorieux d'estre subjects du roi
Que si les autres rois eussent esté les nostres.

Maintenant nostre gloire est a jamais ternie,
Maintenant nostre joye est pour jamais finie ;
Les lys sont atterez et nous avecques eux.
Dafné baisse, chétifve, en terre son visage,
Et semble par ce geste, humble autant que piteux,
Ou couronner sa tombe ou bien lui faire hommage.|

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[1] Fairfax omits a stanza here.